Fermionic Corrections to the Matching Coefficient of the Vector Current

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[hep-ph/0607168]

1. Introduction

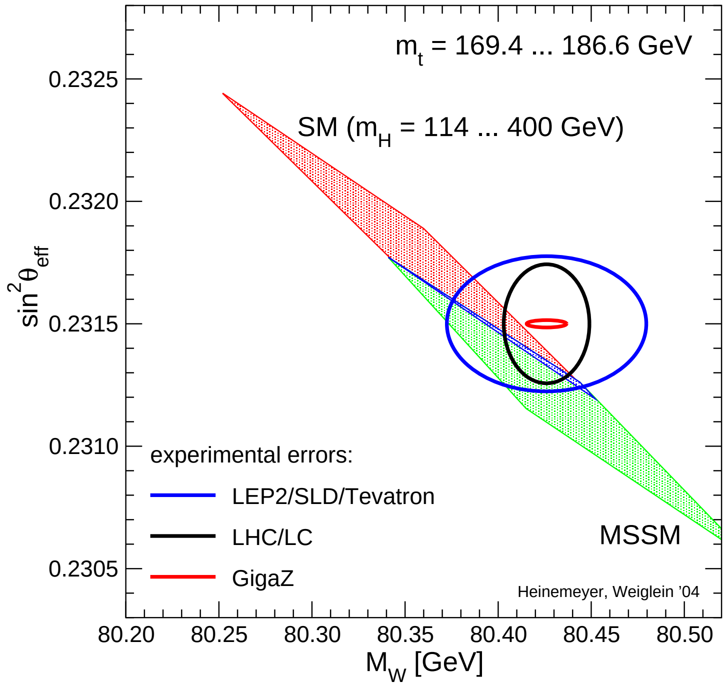
International Linear Collider will allow the measurement of $t\bar{t}$ production cross section at threshold with high accuracy

$\rightsquigarrow$ precise extraction of top quark properties like mass, total width, Yukawa coupling, ..., and of α_s will be possible

[Hoang *et al.*: Eur. Phys. J. directC 2 (2000) 3]
[Martinez, Miquel: Eur. Phys. J. C 27 (2003) 49]

if theoretical precision matches experimental precision

Why is this interesting?



[Heinemeyer *et al.*: Phys. Rept. 425 (2006) 265]

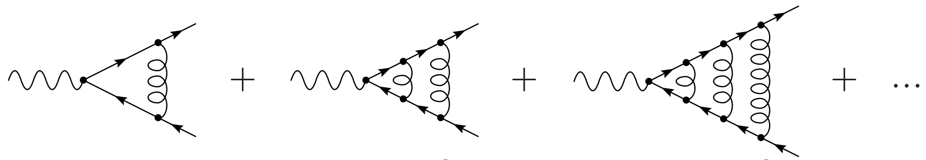
- top quark mass is one of the key parameters of the standard model
  today: $\Delta m_t \simeq 2$GeV
  LHC: $\Delta m_t \simeq 1 - 2$GeV
  ILC: $\Delta m_t \simeq 100$MeV

- non-perturbative effects are suppressed due to large width
  $\leadsto$ no bound states

- $t\bar{t}$ system has to be treated non-relativistically (at threshold)

Non-Relativistic QCD

relative velocity of heavy quarks at threshold is small: $v \sim \alpha_s(m) \ll 1$



$$\frac{\alpha_s}{v} \qquad \left(\frac{\alpha_s}{v}\right)^2 \qquad \left(\frac{\alpha_s}{v}\right)^3$$

$\rightsquigarrow$ singular terms $\sim (\alpha_s/v)^n$ have to be resumed to all orders

$\rightsquigarrow$ non-relativistic effective theory:

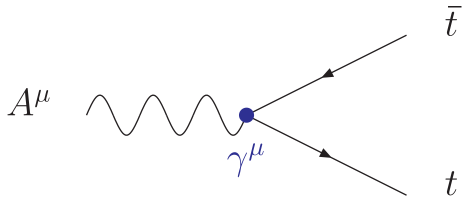
NRQCD

[Caswell, Lepage: Phys. Lett. B 167 (1986) 437]
[Bodwin, Braaten, Lepage: Phys. Rev. D 51 (1995) 1125]

$\rightsquigarrow$ hard modes ($\sim m$) contribute only to matching coefficients, which can be calculated perturbatively

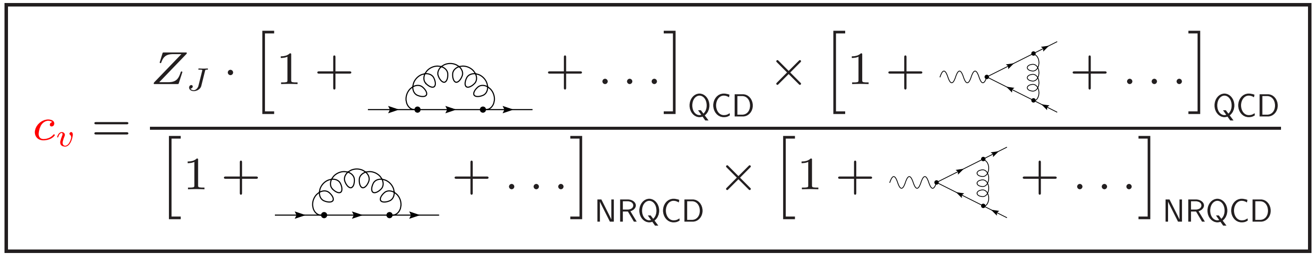
2. The Vector Current

$$\overline{\psi}\gamma^\mu\psi = c_v(\mu)\ \phi^\dagger\sigma^i\chi + \mathcal{O}(v^2)$$



$\rightsquigarrow$ matching equation for c_v:

$$Z_2\,\Gamma \stackrel{!}{=} c_v \tilde{Z}_J^{-1}\,\tilde{Z}_2\,\tilde{\Gamma} + \mathcal{O}(v^2)$$



threshold expansion of Γ_{QCD}: only hard modes contribute to c_v

$\rightsquigarrow$ diagrams have to be calculated at threshold

c_v is known at $\mathcal{O}(\alpha_s^2)$

[Czarnecki, Melnikov: Phys. Rev. Lett. 80 (1998) 2531]
[Beneke, Signer, Smirnov: Phys. Rev. Lett. 80 (1998) 2535]

$t\bar{t}\gamma$ Vertex at Threshold

- tensor structure:

$$\Gamma_\mu = \gamma_\mu F_1 + \frac{[\not{q}\,,\gamma_\mu]}{4m_t} F_2 + \frac{\not{q}\;q_\mu}{q^2} F_3$$

here only F_1 and F_2 are needed:

$$\Gamma = F_1 + F_2$$

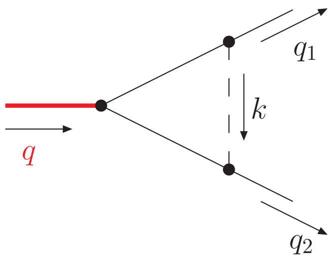
$\rightsquigarrow$ projector on F_1 and F_2:

$$\begin{aligned}\hat{P}_v^\mu \Gamma_\mu &= \frac{1}{8(D-1)m_t^2} \mathrm{Tr}\left\{ \left(-\frac{\not{q}}{2} + m_t \right) \gamma^\mu \left(\frac{\not{q}}{2} + m_t \right) \Gamma_\mu \right\} \\ &= F_1 + F_2\end{aligned}$$

Partial Fractions

- reduction to "2-point functions":

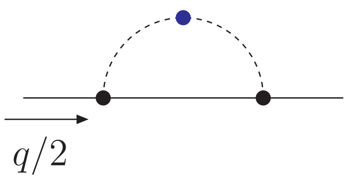
threshold kinematics: $q_1^2 = q_2^2 = m_t^2$, $q^2 = (q_1 + q_2)^2 = 4m_t^2$



$$\int \frac{\mathrm{d}^d k}{\left((k+q_1)^2 - m_t^2\right)\left((k-q_2)^2 - m_t^2\right) k^2}$$

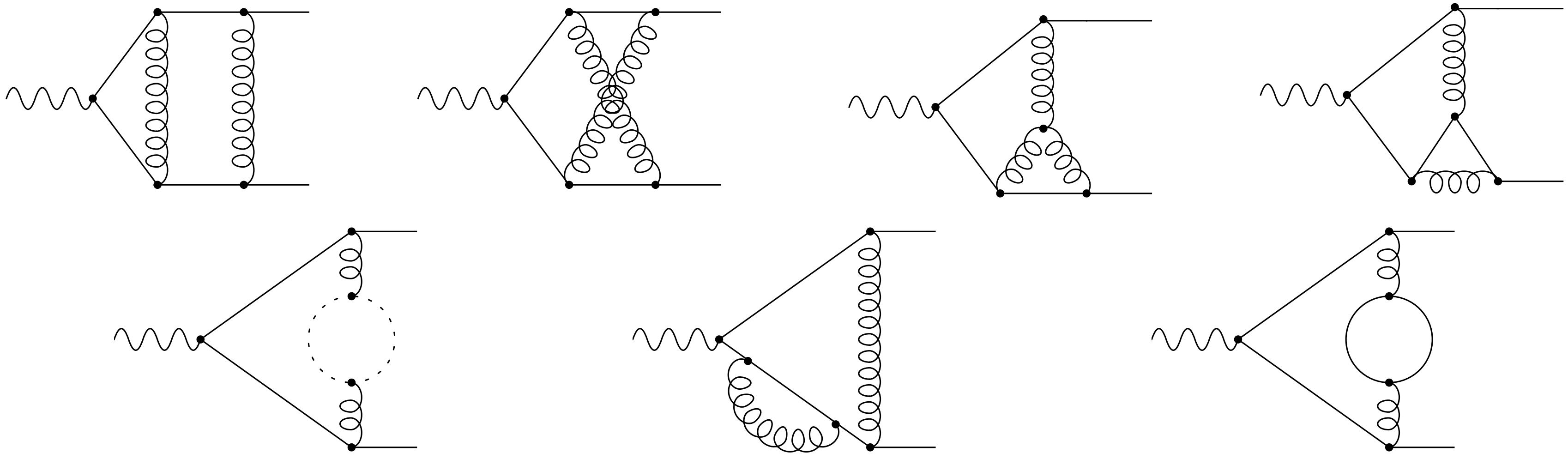
$$= \int \frac{\mathrm{d}^d k}{\left((k+q/2)^2 - m_t^2\right)\left((k-q/2)^2 - m_t^2\right) k^2}$$

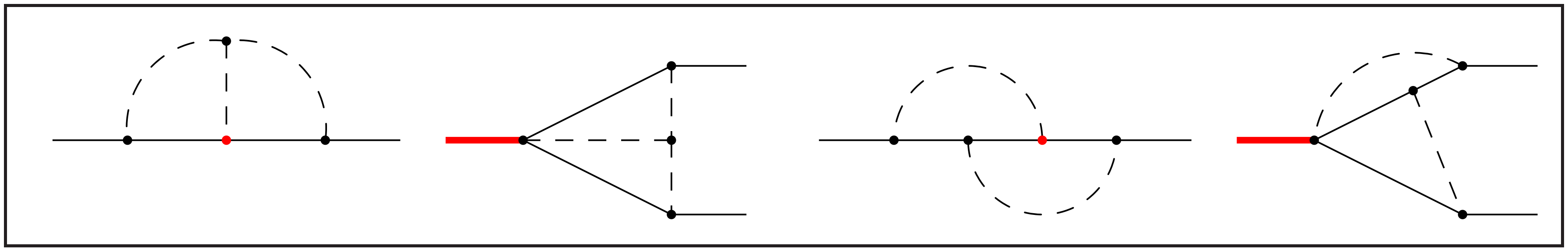
$$= \int \frac{\mathrm{d}^d k}{\left(k^2 + q\cdot k\right)\left(k^2 - q\cdot k\right) k^2}$$



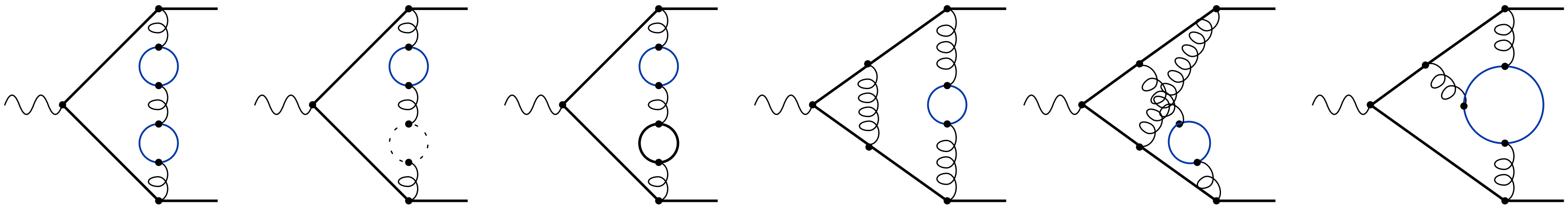
$$= \frac{1}{2}\left(\int \frac{\mathrm{d}^d k}{\left(k^2 + q\cdot k\right)\left(k^2\right)^2} + \int \frac{\mathrm{d}^d k}{\left(k^2 - q\cdot k\right)\left(k^2\right)^2}\right)$$

• 2-loop case:

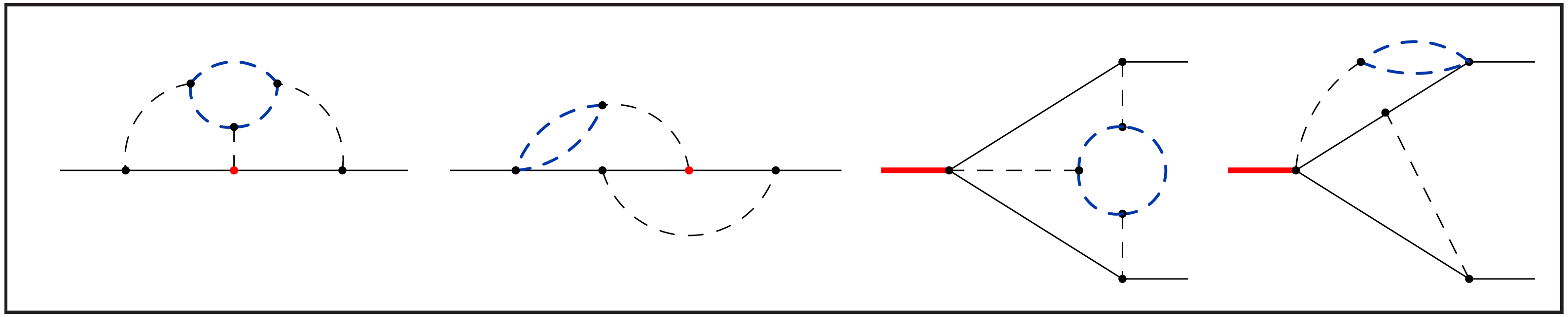


⇒ 

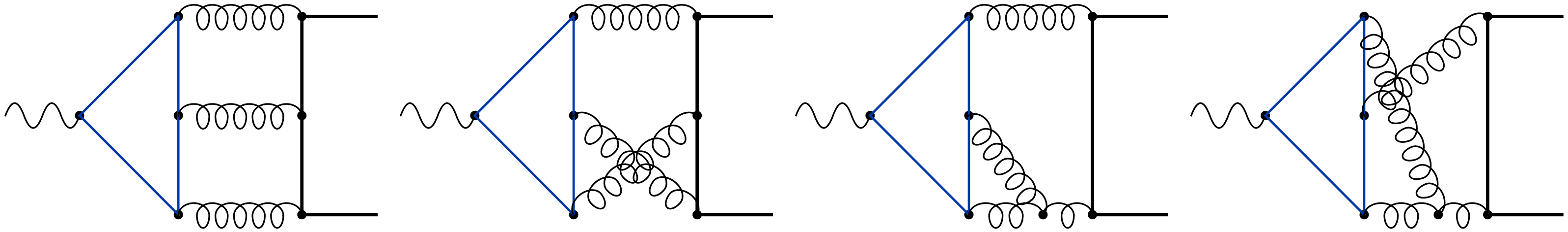
- 3-loop n_l case:



$\Rightarrow$



- new types of diagrams (not yet calculated):



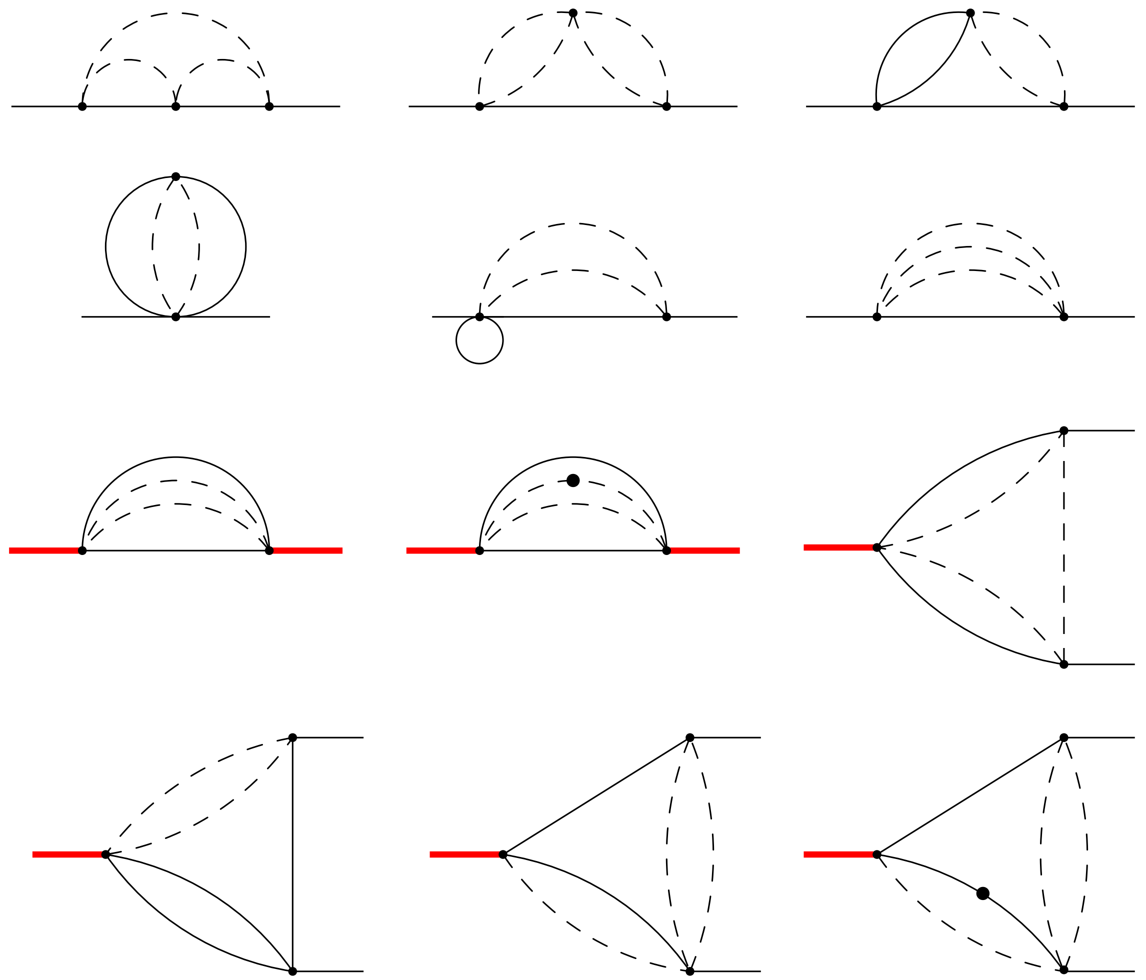
Reduction to Master Integrals

Laporta's algorithm:

`Crusher`

[Marquard, Seidel]

⇒ 12 3-loop master integrals



3. Results

$$
\begin{aligned}
c_v(\mu) \quad = \quad & 1 - 2C_F \frac{\alpha_s(m_t)}{\pi} \\
& + \left(\frac{\alpha_s(m_t)}{\pi}\right)^2 C_F \left[C_F \left\{ \frac{23}{8} - \frac{79}{6}\zeta_2 + 6\zeta_2 \ln 2 - \frac{1}{2}\zeta_3 - \zeta_2 \ln \frac{\mu^2}{m_t^2} \right\} \right. \\
& \qquad + C_A \left\{ -\frac{151}{72} + \frac{89}{24}\zeta_2 - 5\zeta_2 \ln 2 - \frac{13}{4}\zeta_3 - \frac{3}{2}\zeta_2 \ln \frac{\mu^2}{m_t^2} \right\} \\
& \qquad \left. + T_F \left\{ \frac{11}{18} n_l + \frac{22}{9} - \frac{4}{3}\zeta_2 \right\} \right] \\
& + \left(\frac{\alpha_s(m_t)}{\pi}\right)^3 C_F T_F n_l \left[C_F \left\{ 46.7(1) + \frac{25}{18}\zeta_2 \ln \frac{\mu^2}{m_t^2} - \frac{1}{3}\zeta_2 \ln^2 \frac{\mu^2}{m_t^2} \right\} \right. \\
& \qquad + C_A \left\{ 39.6(1) + \frac{37}{24}\zeta_2 \ln \frac{\mu^2}{m_t^2} - \frac{1}{2}\zeta_2 \ln^2 \frac{\mu^2}{m_t^2} \right\} \\
& \qquad \left. + T_F \left\{ \left(-\frac{163}{162} - \frac{8}{9}\zeta_2 \right) n_l - \frac{557}{162} + \frac{52}{27}\zeta_2 \right\} \right] \\
& + \ldots
\end{aligned}
$$

Numerical Analysis

- peak of normalised cross section:

$$R_1 \sim |\Psi_1(0)|^2 \left(c_v^2(m_t) + \frac{C_F^2 \alpha_s(\mu_s)}{12} c_v(m_t)(d_v(m_t)+3) \right) + \dots$$

$$R_1 \approx R_1^{\mathrm{LO}} \left(1 - 0.243_{\mathrm{NLO}} + 0.435_{\mathrm{NNLO}} - \left\{ \begin{array}{ll} 0.268_{\mathrm{N^3LO'}} & \text{(old)} \\ 0.195_{\mathrm{N^3LO'}} & \text{(new)} \end{array} \right. + \dots \right)$$

4. Summary and Outlook

- calculation of higher order corrections to matching coefficient of vector current is necessary for precise extraction of top quark properties at ILC
- $\mathcal{O}(\alpha_s^2)$ result for c_v was reproduced
- non-singlet $\mathcal{O}(\alpha_s^3 n_l)$ contribution was calculated

next step:

- full $\mathcal{O}(\alpha_s^3)$ contribution for c_v