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The Physics Case for the International Linear Collider

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1 Couplings of Gauge Bosons

The largest cross sections at the ILC are the production of fermion pairs and the production of gauge bosons, namely single-W and W-pair production. These processes have cross sections of a few pb leading to event samples of a few million each. These samples can be used to make stringent tests of the Standard Model and its extensions.

1.1 Fermion pair production

In the Standard Model, fermion pair production for $f \neq e$ proceeds at tree level via the exchange of photons or Z-bosons in the s-channel. This process can thus be used to measure the couplings of fermions to gauge bosons. All cross sections are given by the product of the initial state $\mathrm{e^+e^-}V$ and the final state $\mathrm{f\bar{f}}V$ couplings. Assuming lepton universality lepton pair production thus measures the leptonic couplings while quark production measures the product of the leptonic and the quark couplings.

Since weak interactions violate parity the vector (v) and the axial-vector (a) couplings can vary independently in general. However they can be disentangled experimentally without major problems. The total cross section is proportional to the squared sum of the couplings ($v^2 + a^2$) while several asymmetries like the left-right asymmetry with polarised beams of the forward backward asymmetry measure their ratio v/a.

The fermion-Z couplings have already been measured with great success at LEP and SLD on the Z-resonance [1]. The comparison of their precise measurements with accurate calculations lead to the prediction of the top quark mass before it was actually discovered [2] and to the current prediction that the Higgs boson is light [1].

At $\sqrt{s} \sim 500\,\mathrm{GeV}$ $\mathrm{e^+e^-} \to \mathrm{f\bar{f}}$ samples of a few million events are expected so that the couplings can be measured on the per-mille level. Since the coupling measurements cannot compete with the measurements on the Z-resonance and not much information is expected from the running between m_Z and 500 GeV no model independent parameterisation of the couplings above resonance is available up to now. The main interest in fermion pair production lays in limits on physics beyond the Standard Model. Apart from photons and Z-bosons all other particles that couple to electrons and the studied final state fermions can be exchanged and thus contribute to the cross section. Some cases are discussed in detail in section **??**. In a more model independent approach limits on new physics can be parameterised in terms of contact interactions

$$\mathcal{L}_{eff} = \sum_{i,j=L,R} \eta_{ij} \frac{4\pi}{\Lambda_{ij}^2} \bar{e}_i \gamma^\mu e_i \cdot \bar{f}_j \gamma_\mu f_j$$

This parameterisation assumes that the mass of the exchange particles is so heavy that details of the propagator are not seen and only the Lorenz structure of the couplings remains visible. Depending on the Lorenz structure and the final state the ILC precision on the fermion pair production translates into contact interaction limits of 20 to 100 TeV, where the lower limits are mostly for identified quarks while the larger limits are for inclusive hadronic final states and for leptons [3].

Another possibility of the ILC to measure Z-fermion couplings is to go back to the Z-resonance (GigaZ) [4]. With a luminosity around $\mathcal{L} = 5 \cdot 10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ a billion Z decays can be collected within a few months of running. The most sensitive quantity to measure the Z-fermion couplings is the left-right polarisation asymmetry $A_{\mathrm{LR}} = \frac{1}{\mathcal{P}}\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$, where $\sigma_{L,R}$ denotes the cross section for left/right handed polarised electron beams and $\mathcal{P}$ the beam polarisation. The left-right asymmetry is sensitive to the ratio of the vector to axial-vector coupling of the electron to the Z ($A_{\mathrm{LR}} = 2g_{\mathrm{V,e}}g_{\mathrm{A,e}}/(g^2_{\mathrm{V,e}}+g^2_{\mathrm{A,e}})$) which in turn measures the effective weak mixing angle in Z decays $g_{\mathrm{V,e}}/g_{\mathrm{A,e}} = 1 - 4q_e \sin^2\theta^l_{\mathrm{eff}}$.

If electron and positron polarisation is available the cross section for a given beam polarisations is given by

$$\sigma = \sigma_u \left[1 - \mathcal{P}_{\mathrm{e}^+}\mathcal{P}_{\mathrm{e}^-} + A_{\mathrm{LR}}(\mathcal{P}_{\mathrm{e}^+} - \mathcal{P}_{\mathrm{e}^-})\right] \tag{1.1.1}$$

If the sign of the electron and positron polarisation can be flipped independently four measurements with four unknowns are possible so that A_{LR} can be measured without the need for absolute polarimetry. Polarimeters are, however, still needed to measure a possible polarisation difference between the left- and the right-handed state and to track any time dependences of the polarisation which enters in the polarisation product of equation (1.1.1). A_{LR} can be measured with a statistical accuracy around $\Delta A_{\mathrm{LR}} = 3 \cdot 10^{-5}$. The by far largest systematic uncertainty is coming from the knowledge of the beam energy. The slope close to the Z-peak is $dA_{\mathrm{LR}}/d\sqrt{s} = 2 \cdot 10^{-2}/\,\mathrm{GeV}$ due to $\gamma-$Z interference. Not to be dominated by this effect the centre of mass energy needs to be known to 1 MeV relative to the Z-mass which has to be calibrated by frequent scans. If the beamstrahlung is the same in the peak running and in the calibration scans its effect cancels out and the beamstrahlung does not contribute to the systematic uncertainty.

Conservatively a final error of $\Delta A_{\mathrm{LR}} = 10^{-4}$ will be assumed corresponding to $\Delta \sin^2\theta^l_{\mathrm{eff}} = 0.000013$. This is an improvement of more than one order of magnitude compared to LEP/SLD. At the moment this precision would be almost useless due the bad knowledge of the fine structure constant at the Z-scale, $\alpha(m^2_{\mathrm{Z}})$. However measuring the cross section $\mathrm{e}^+\mathrm{e}^- \rightarrow$ hadrons to 1% roughly up to the J/Ψ resonance would reduce the uncertainty of the $\sin^2\theta^l_{\mathrm{eff}}$ prediction to the level of the experimental error [5]. With modest upgrades this is possible using present machines.

If also absolute values of the couplings should be measured one needs to obtain the leptonic width of the Z, Γ_ℓ. The peak cross section $\sigma(\mathrm{e}^+\mathrm{e}^- \rightarrow \ell^+\ell^-)$ for $\sqrt{s} = m_{\mathrm{Z}}$ is proportional to $\Gamma^2_\ell/\Gamma^2_{\mathrm{tot}}$. To measure Γ_ℓ thus, apart from the cross section, the total width of the Z needs to be measured from a scan. Many systematic uncertainties enter the determination of Γ_ℓ. The relative knowledge of the beam energy effects the determination of Γ_{tot}. The knowledge of the total luminosity and the selection efficiency directly enter the cross section measurement. The most severe systematics might come from the beam energy spread and from beamstrahlung. Because the second derivative of a Breit Wigner distribution at the peak is very large the effective peak cross section is strongly reduced by these effects which may well limit the Γ_ℓ measurement. A probably optimistic estimate [4] showed a possible improvement of a factor two relative to LEP.

In addition to the fermion-Z couplings the W-mass can be measured at ILC with as threshold scan to a precision around 6 MeV [6]. Because of a similar structure of the radiative corrections this quantity is usually interpreted together with the coupling measurements. Within a wide range of models the measurement of the W-mass can replace the measurement of Γ_ℓ which is not so well known for the reasons mentioned above.

As a possible application of the precision measurements figure 1.1.1 shows the projected $\sin^2\theta^l_{\mathrm{eff}}$ and m_{W} measurements under different assumptions compared to the prediction of the Standard Model and the MSSM [7]. Within the SM a stringent test of the model is possible. For the MSSM the sensitivity is good enough to constrain some of its parameters. It can also be seen that the precise top mass measurement from ILC is needed for an optimal sensitivity of the comparison.

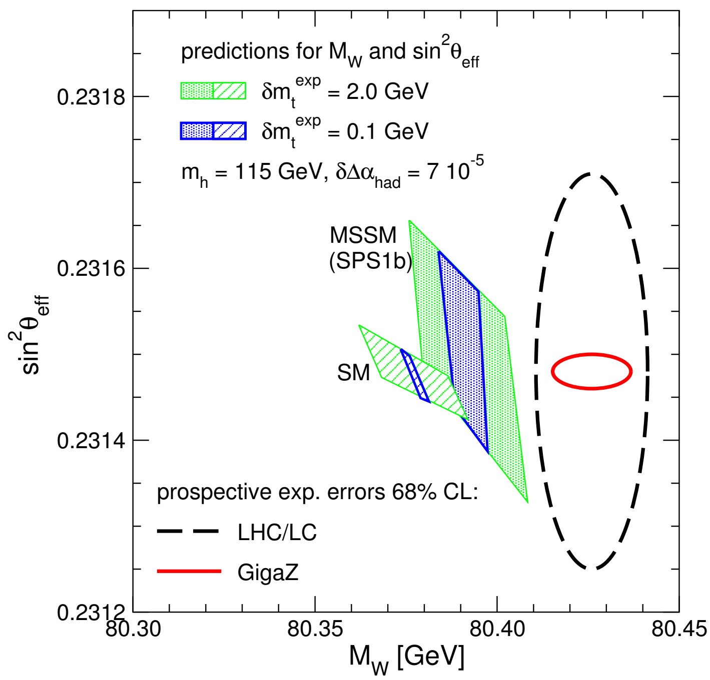

Figure 1.1.1: $\sin^2\theta^l_{\mathrm{eff}}$ *vs* m_{W} *for different experimental assumptions compared to the prediction from the SM and the MSSM [7]*

1.2 Couplings among Gauge Bosons

The couplings among gauge bosons are directly given by the structure of the corresponding gauge group. This structure can thus directly be measured by a measurement of the gauge boson interactions. W-pair production is an especially interesting process in this respect. Without gauge interactions W-pairs are produced in e^+e^- via neutrino t-channel exchange. This process violates unitarity and gets regulated by the photon and Z s-channel exchange involving the triple gauge-boson couplings. Since the exact values of the boson self-coupling-constants as predicted by the $SU(2) \times U(1)$ gauge structure are needed for unitarity restoration small changes lead to a large variation of the cross section. For this reason W-pair production in e^+e^- annihilation is much more sensitive to the triple gauge boson coupling than one would obtain from naively from cross section estimates.

The triple gauge boson couplings are conventionally parametrised as [8, 9]:

$$\begin{aligned} L_{\mathrm{WWV}} &= g_{\mathrm{WWV}}[\\ & \mathrm{i}g_1^{\mathrm{V}} V_\mu \left(W_\nu^- W_{\mu\nu}^+ - W_{\mu\nu}^- W_\nu^+\right) + \mathrm{i}\kappa_{\mathrm{V}} W_\mu^- W_\nu^+ V_{\mu\nu} + \mathrm{i}\frac{\lambda_{\mathrm{V}}}{m_{\mathrm{W}}^2} W_{\lambda\mu}^- W_{\mu\nu}^+ V_{\nu\lambda} \\ & + g_4^{\mathrm{V}} W_\mu^- W_\nu^+ \left(\partial_\mu V_\nu + \partial_\nu V_\mu\right) + g_5^{\mathrm{V}} \epsilon_{\mu\nu\lambda\rho} \left(W_\mu^- \partial_\lambda W_\nu^+ - \partial_\lambda W_\mu^- W_\nu^+\right) V_\rho \\ & + \mathrm{i}\tilde{\kappa}_{\mathrm{V}} W_\mu^- W_\nu^+ \tilde{V}_{\mu\nu} + \mathrm{i}\frac{\tilde{\lambda}_{\mathrm{V}}}{m_{\mathrm{W}}^2} W_{\lambda\mu}^- W_{\mu\nu}^+ \tilde{V}_{\nu\lambda}] \,, \end{aligned} \tag{1.2.1}$$

using the antisymmetric combinations $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and their duals $\tilde{V}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} V_{\rho\sigma}/2$. The overall coefficients are $g_{\mathrm{WW}\gamma} = e$ and $g_{\mathrm{WWZ}} = e \cot\theta_W$ with θ_W being the weak mixing angle.

Electromagnetic gauge invariance requires that $g_1^\gamma(q^2 = 0) = 1$ and $g_5^\gamma(q^2 = 0) = 0$ at zero momentum transfer. In the Standard Model one has $g_1^V = \kappa_V = 1$, all other couplings are equal to zero.

Among the different couplings g_1, κ and λ are C- and P-conserving, g_5 is C and P-violating, but CP-conserving while g_4, $\tilde{\kappa}$, $\tilde{\lambda}$ violate CP.

Experimentally the different types of couplings can be disentangled analysing the production angle distribution of the W and the W-polarisation structure which can be obtained from the decay angle distributions. Anomalous WWγ and WWZ couplings give similar signals in the final state distributions. However they can be disentangled easily using beam polarisation. Because of the strong dominance of the left-handed electron state high polarisation values are needed for this analysis. This can also be achieved by increasing the effective polarisation using polarised positron beams.

An analysis using fast simulation has been performed at $\sqrt{s} = 500\,\mathrm{GeV}$ and $\sqrt{s} = 800\,\mathrm{GeV}$ [10]. The results for single parameter fits are shown in table 1.2.1. For the multi-parameter fits correlations are modest for 800 GeV so that the errors increase by at most 20%. For 500 GeV correlations are much larger and the errors increase by about a factor two in the multi-parameter fit of the C,P conserving parameters. For the C or P violating parameters correlations are small at both energies.

coupling	error $\times 10^{-4}$	
	$\sqrt{s} = 500\,\text{GeV}$	$\sqrt{s} = 800\,\text{GeV}$
C,P-conserving:		
Δg_1^Z	15.5	12.6
$\Delta\kappa_\gamma$	3.3	1.9
λ_γ	5.9	3.3
$\Delta\kappa_Z$	3.2	1.9
λ_Z	6.7	3.0
not C or P conserving:		
g_5^Z	16.5	14.4
g_4^Z	45.9	18.3
$\tilde{\kappa}_Z$	39.0	14.3
$\tilde{\lambda}_Z$	7.5	3.0

Table 1.2.1: *Results of the single parameter fits (1σ) to the different triple gauge couplings. For $\sqrt{s} = 500\,\text{GeV}$ $\mathcal{L} = 500\ \text{fb}^{-1}$ and for $\sqrt{s} = 800\,\text{GeV}$ $\mathcal{L} = 1000\ \text{fb}^{-1}$ has been assumed. For both energies $\mathcal{P}_{e^-} = 80\%$ and $\mathcal{P}_{e^+} = 60\%$ has been used.*

Additional information on the triple gauge couplings can be obtained from the eγ and $\gamma\gamma$ options. In these option only the WWγ couplings can be measured without ambiguities from the WWZ couplings. It is often claimed that these options are especially sensitive because of the large cross section and because the leading diagrams contain the triple gauge couplings. However in $e\gamma \to W^-\nu$ and $\gamma\gamma \to W^+W^-$ no gauge cancellations are present so that the sensitivity gets reduced.

Detailed studies show, that for κ_γ the e^+e^- mode is by far superior, while for λ_γ competitive results can be obtained [11, 12].

Figure 1.2.1 compares the κ_γ and λ_γ measurements at different machines. Especially for κ, which is because of its lower mass dimension the more interesting parameter, the ILC is an order of magnitude better than the LHC.

1.2.1 Measurements of quartic couplings

In addition to the triple gauge boson couplings the ILC is also sensitive to the quartic couplings. Two processes are sensitive to these couplings, triple gauge boson production ($e^+e^- \to VVV$) and vector boson scattering ($e^+e^- \to f_1 f_2 VV'$, $f_{1,2} = e, \nu$, $V, V' = W, Z$). In vector-boson scattering the underlying process is $V_1V_2 \to V_3V_4$ where $V_{1,2} = Z$ is strongly suppressed because of the small electron-Z coupling.

In the Standard Model without a Higgs W-scattering violates unitarity at 1.2 TeV. The measurement of the quartic coupling therefore is mainly interesting in the case without a Higgs and thus complementary to the analyses presented in section **??**. The sensitivity to the quartic couplings rises strongly with energy and useful results can be obtained only with the 1 TeV upgrade.

A fast simulation study of both processes has been performed [13] and the results with

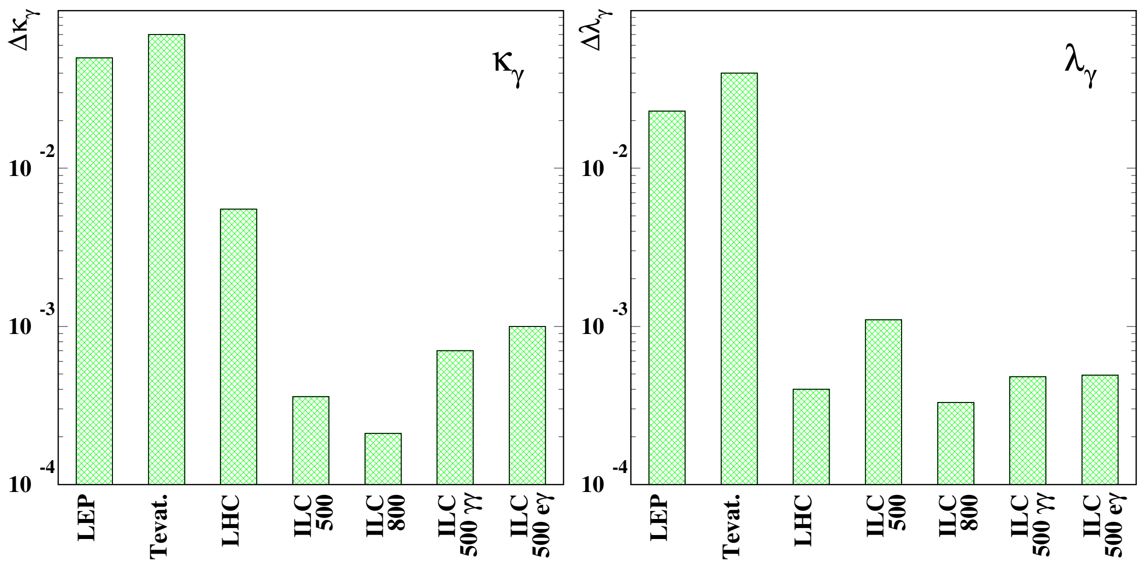

Figure 1.2.1: *Comparison of $\Delta\kappa_\gamma$ and $\Delta\lambda_\gamma$ at different machines. For LHC and ILC three years of running are assumed (LHC:* 300 fb^{-1}*, ILC* $\sqrt{s} = 500\,\mathrm{GeV}$*:* 900 fb^{-1}*, ILC* $\sqrt{s} = 800\,\mathrm{GeV}$*:* 1500 fb^{-1}*). If available the results from multi-parameter fits have been used.*

and without the assumption of custodial $SU(2)$ conservation are shown in figure 1.2.2. The α parameters, taken from the definition in [14], parameterise the deviations of the quartic couplings from the Standard Model predictions. With $SU(2)_c$ conservation LHC obtains similar limits. However since the ILC can, contrary to the LHC, tag the initial and final state gauge bosons the separation of couplings, once $SU(2)_c$ is allowed, is much easier. These limits can be interpreted in terms of heavy resonances. The limits, however depend strongly on the assumptions and vary between 1 and 4 TeV [13].

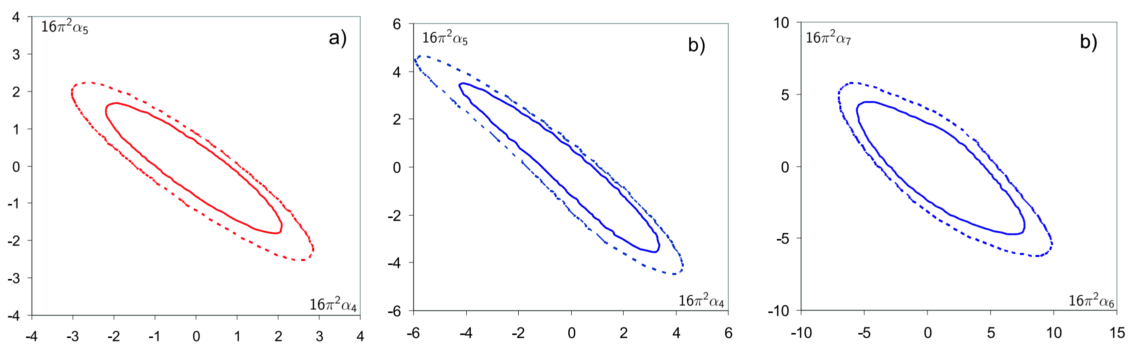

Figure 1.2.2: *Limits of α_4, α_5 assuming $SU(2)_c$ conservation (a) and α_4 - α_7 without this assumption (b) from three-vector-boson production and from vector-boson scattering assuming* 1000 fb^{-1} *at* $\sqrt{s} = 1\,\mathrm{TeV}$*. The dashed line represent 90% c.l. and the solid line 68%.*

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