Max-Planck-Institute für Physik
(Werner-Heisenberg-Institut)

Progress on Heavy Colored Particle Thresholds

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International Linear Collider Workshop
Nov 6-10, Valencia

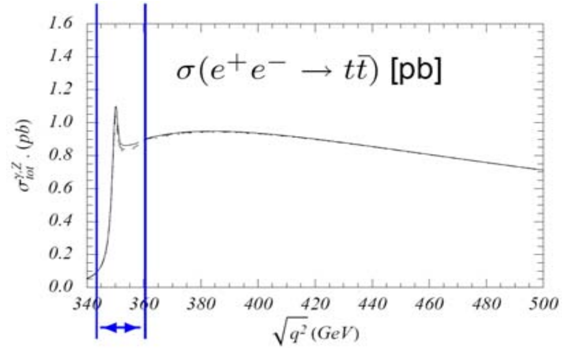
Outline

- Threshold Physics at the ILC
- Review of $t\bar{t}$ production
- vNRQCD for stable quarks / squarks
- NR currents in $d = 3 - 2\epsilon$
- Finite width effects
- Outlook

Threshold Physics at the ILC

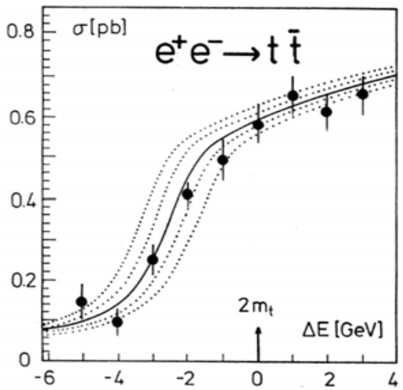
- e+e- linear collider: $E_{cm} = 350\ \mathrm{GeV} - 1\ \mathrm{TeV}$
- Luminosity: $10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \rightarrow 100\ \mathrm{fb}^{-1}/\mathrm{year}$

  $10^5\ t\bar{t}$ pairs $\quad [\sigma < 1\mathrm{pb}]\ (e^+e^- \rightarrow t\bar{t})$



- Threshold scan: center of mass energy variable **[Miquel, Martínez]**

$\sqrt{s} \simeq 350$ GeV (Phase I)



$\rightarrow\ \delta m_t^{\mathrm{exp}} \simeq 50$ MeV

$\rightarrow\ \delta m_t^{\mathrm{th}} \simeq 100$ MeV

$\delta\Gamma_t^{\mathrm{exp}} = 50$ MeV

$\delta\alpha_s(M_Z)^{\mathrm{exp}} = 0.002$

$(\delta\lambda_t/\lambda_t)^{\mathrm{exp}} = 15 - 50\%$

$\Longrightarrow$ theory goal: $(\delta\sigma/\sigma)^{\mathrm{theo}} \leq 3\,\%$

- very good knowledge on intrinsic theoretical uncertainties

$\Gamma_t \approx 1.5\ \mathrm{GeV} \gg \Gamma_{\mathrm{QCD}}$

$\longrightarrow\ t\bar{t}$ is fully **perturbative** at threshold

rf3 mainly uncertainties in the knowledge on the luminosity spectrum
ruizfeme, 5/1/2006

rf4 in contrast to mass reconstruction method
ruizfeme, 5/1/2006

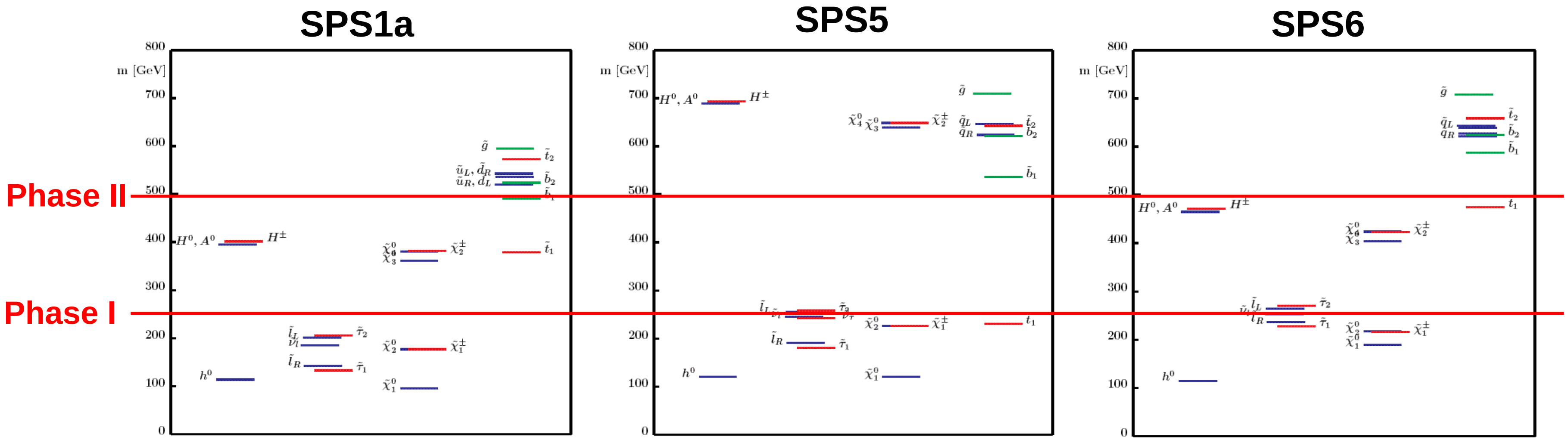
rf5 The stong and Yukawa couplings can be determined from the normalization of the cross section, while the top width determines the sharpness of the peak
ruizfeme, 5/1/2006

rf6 One can rely on perturbative methods because the rather large top quark width suppresses non-perturbative effects and prevents the formation of toponium bound states. So the lineshape can be computed as a functionof the Lagrangian top quark mass in any given scheme without ambiguities.
ruizfeme, 5/1/2006

Threshold Physics at the ILC

$$e^+e^- \rightarrow \tilde{q}\tilde{\bar{q}}$$

❑ mSugra scenarios with a light scalar top quark

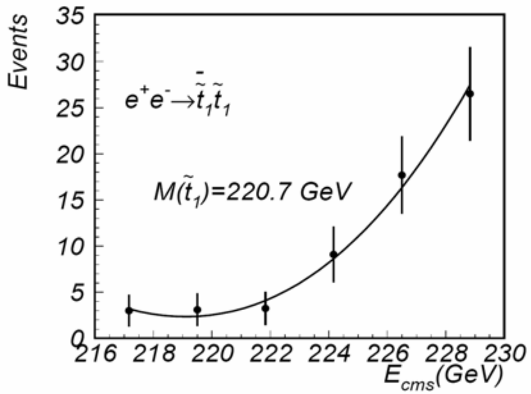


SPS1a	SPS5	SPS6
$m_{\tilde{t}_1} = 396$ GeV	$m_{\tilde{t}_1} = 240$ GeV	$m_{\tilde{t}_1} \simeq 490$ GeV
$\tilde{t}_1 \rightarrow b\chi_1^+, \ldots$	$\tilde{t}_1 \rightarrow b\chi_1^+, c\chi_1^0$	$\tilde{t}_1 \rightarrow b\chi_1^+, \ldots$
$\Gamma_{\tilde{t}_1} = 1.92$ GeV	$\Gamma_{\tilde{t}_1} = 0.04$ GeV	$\Gamma_{\tilde{t}_1} \simeq 3.2$ GeV

Threshold Physics at the ILC

$$e^+e^- \longrightarrow \tilde{q}\bar{\tilde{q}}$$

Threshold scan



$$\Delta m_{\tilde{t}} \sim 2\,\mathrm{GeV}$$

[talk by H.Nowak at ECFA Durham 2rf84]

Improve theoretical uncertainty

→ Precise determinations of stop width, couplings, ... only LO results [N. Fabiano]

- Low-energy dynamics of squarks based on standard QCD
- $t\bar{t}$ / $\tilde{q}\bar{\tilde{q}}$ are nonrelativistic systems
- apply EFT methods: NRQCD for fermion / scalar fields

Slide 5

rf8 simulations of a fit to the total cross section lineshape

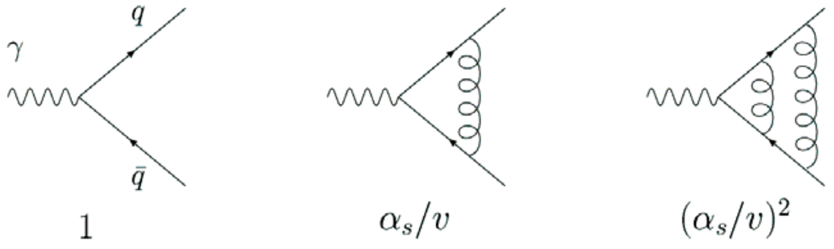
ruizfeme, 5/2/2006

Physics at Threshold

- In the threshold region quarks move at small velocities P13

$$mv^2 \equiv \sqrt{s} - 2m \qquad v \lesssim 0.2 \sim \alpha_s \qquad E_{cm} \simeq 2m_{\tilde{t}} \pm 10\,\mathrm{GeV}$$

- pQCD series has terms $\propto \left(\frac{\alpha_s}{v}\right)^n$



"Coulomb singularities"

✓ count $\frac{\alpha_s}{v} \sim 1$ as LO

✓ perform expansion in v, α_s

✓ resummation of leading terms achieved by means of a Schrödinger field theory

⇒ **NRQCD** **Caswell, Lepage** **Bodwin, Braaten**

P14

P14 this can be implemented sistematically using the factorization properties of non-relativistic QCD (NRQCD)
Pedro, 7/18/2005

P13 in the c.m. frame
Pedro, 7/18/2005

Learning from Top Physics at Threshold

fixed order scheme

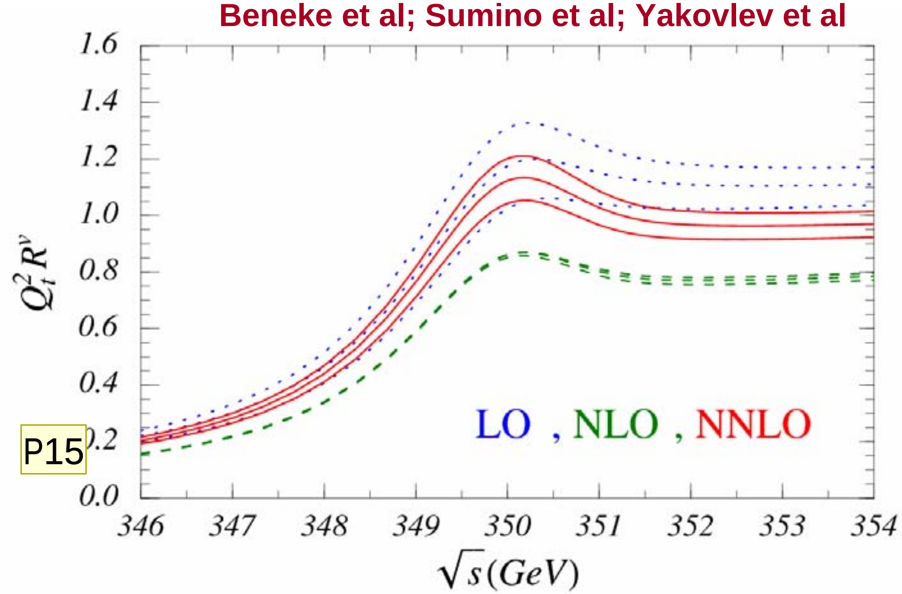
$$\frac{\alpha_s}{v} \sim 1$$

$$\text{LO} \sim \left(\frac{\alpha_s}{v}\right)^n$$

$$\text{NLO} \sim \{\alpha_s, v\} \times \left(\frac{\alpha_s}{v}\right)^n$$

$$\text{NNLO} \sim \{\alpha_s^2, \alpha_s v, v^2\} \times \left(\frac{\alpha_s}{v}\right)^n$$

Hoang,Teubner; Penin et al; Melnikov et al.
Beneke et al; Sumino et al; Yakovlev et al



P15

- large NNLO correction

- scale dependence ⟶ large uncertainty in normalization of cross section

$$m_t \sim 175\,\text{GeV} \qquad \mathbf{p} \sim 25\,\text{GeV} \qquad E \sim 4\,\text{GeV}$$

⟶ NRQCD matrix elements, μ? P16

For example: $\alpha_s(m_t)\ln\left(\frac{m_t^2}{E^2}\right) \simeq 0.8$ ⟹ large logs $(\alpha_s \ln v)^n$

Slide 7

P15 two low-energy scales
Pedro, 7/19/2005

P16 all these logs cannot be made small for a single choice of he renormalization scale
Pedro, 7/19/2005

Learning from Top Physics at Threshold

RGE improved computations

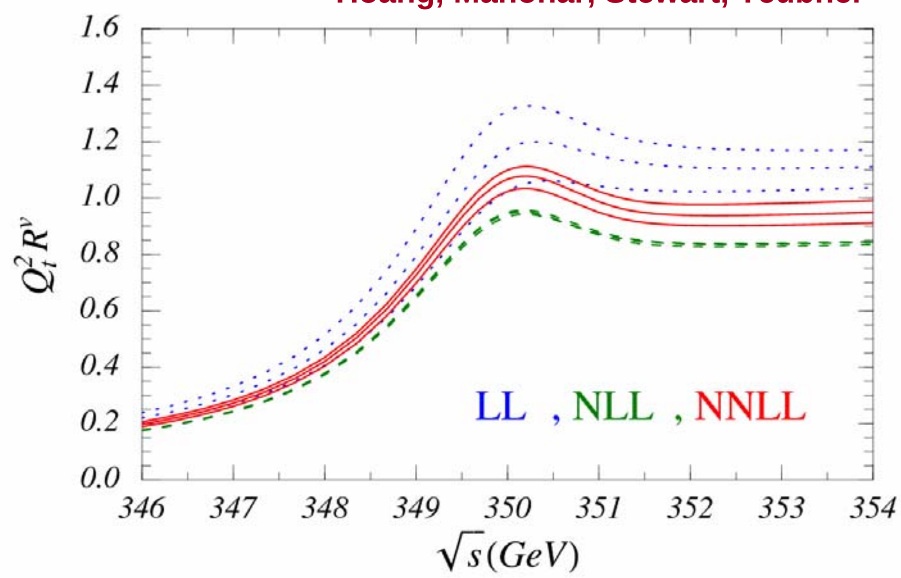
$$\frac{\alpha_s}{v} \sim 1 \qquad \alpha_s \ln v \sim 1$$

$$\text{LL} \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$

$$\text{NLL} \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m \times \{\alpha_s, v\}$$

$$\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 6\%$$

Hoang, Manohar, Stewart, Teubner



rf9

- ✓ log terms summed into coefficients through RGE
- ✓ reduced scale dependence

⇨ **vNRQCD** **Luke, Manohar, Rothstein; Hoang, Stewart**

alternative approach: **pNRQCD**
Brambilla, Pineda, Soto, Vairo
⟶ **talk by A. Pineda**

For heavy $\tilde{q}\bar{\tilde{q}}$ pairs we need the scalar version

rf9 EFT for NR heavy quark pairs with a consistent power counting in v

ruizfeme, 5/2/2006

Effective Theory Framework (stable quarks)

- Scales in the non-relativistic $t\bar{t}$ / $\tilde{q}\bar{\tilde{q}}$ system

$$m \gg \mathbf{p} \sim m\,v \gg E \sim m\,v^2 > \Lambda_{QCD}$$

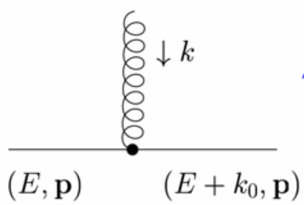
(hard) (soft) (ultrasoft)

- Split heavy quark 4-momentum

$$p^\mu = (m, 0) + (0, \mathbf{p}) + (k^0, \mathbf{k})$$

soft component label ($\mathbf{p}$)

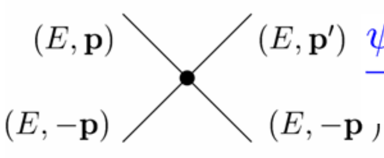
ultrasoft component label ($k^0, \mathbf{k}$)



$$\psi^*_{\mathbf{p}}\, A_\mu(x)\, \psi_{\mathbf{p}}$$

$$\psi \to \sum_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}}\, \psi_{\mathbf{p}}(x)$$

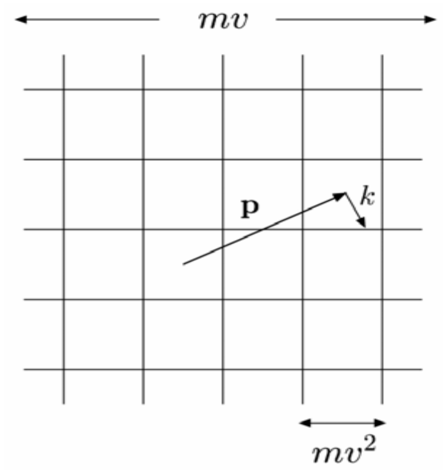
$x \sim 1/mv^2$



$$\frac{\psi^*_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^*_{-\mathbf{p}'}\chi_{-\mathbf{p}}}{(\mathbf{p}-\mathbf{p}')^2}$$

P1	Scale hierarchy allows factorization Pedro, 7/16/2005
P31	Constant, not treated as a dynamical variable Pedro, 10/21/2005
P32	Soft component of the squark 3-momentum Pedro, 10/21/2005
P34	Interactions of heavy quarks with u.s. gluons do not change the soft component, which can only change in the interaction between squark pairs Pedro, 10/21/2005
P35	Procedure to separate soft and usoft scales Pedro, 10/21/2005
P36	\psi_p(x) is a field which describes fluctuations taking place at large distances (x is the Fourier transform of the usoft momentum k) Pedro, 10/21/2005
rf10	Ultrasoft part containing the NR kinetic energy of the squark ruizfeme, 5/2/2006

Effective Theory Framework



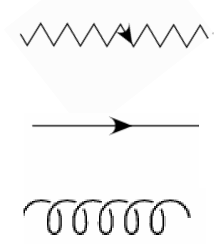
P37

Recall HQET: $p^\mu = mv^\mu + k^\mu$ $\sim \Lambda_{QCD}$

P38

$$\psi \to \sum_v e^{-iv\cdot x}\,\psi_v(x)$$

Resonant modes in the quark-antiquark system



soft gluons	A_q^μ		$(k^0, \mathbf{k}) \sim (\, mv \,,\, mv\,)$
potential quarks	$\psi_\mathbf{p}\,,\ \chi_\mathbf{p}$		$\sim (\, mv^2 \,,\, mv\,)$
ultrasoft gluons	A^μ		$\sim (\, mv^2 \,,\, mv^2\,)$

P68

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P37 The momentum space of size mv is divided into boxes of size mv^2. A point in momentum space is labeled by p and k
Pedro, 10/21/2005

P38 The QCD interactions inside the meson are of order of Lambda_QCD and do not change the velocity of the heavy quark
Pedro, 10/21/2005

P68 Any other light modes (such as light fermions or scalars and ghosts) in the theory must also be divided into soft and usoft fields, as for the gauge fields
Pedro, 10/27/2005

vNRQCD Lagrangian

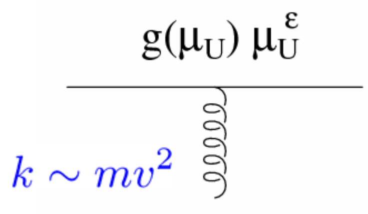
Luke, Manohar, Rothstein;
Hoang, Stewart

$$\mathcal{L} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}$$

P47

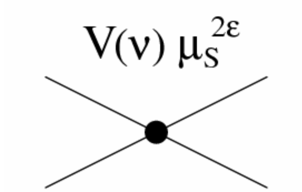
$$\mathcal{L}_{\text{us}} = \sum_{\mathbf{p}} \psi^*_{\mathbf{p}} \left[iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \frac{\mathbf{p}^4}{8m^3} + \dots \right] \psi_{\mathbf{p}}$$

$$D^\mu = \partial^\mu + i\mu_U^\epsilon g_s A^\mu$$



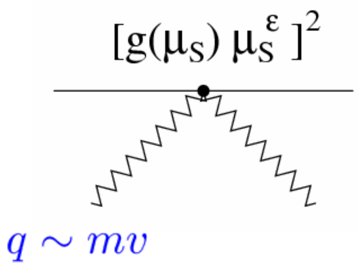
P45

$$\mathcal{L}_{\text{pot}} = -\mu_S^{2\epsilon} \sum_{\mathbf{p},\mathbf{p}'} \left[\frac{\mathcal{V}_c(\nu)}{(\mathbf{p} - \mathbf{p}')^2} + \dots \right] \psi^*_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^*_{-\mathbf{p}'} \chi_{-\mathbf{p}}$$



P46

$$\mathcal{L}_{\text{soft}} = -\mu_S^{2\epsilon} g_s^2 \sum_{\mathbf{p},\mathbf{p}',q,q',\sigma} \left[\frac{1}{2} \psi^*_{\mathbf{p}'} [A^\mu_{q'}, A^\nu_q] U^{(\sigma)}_{\mu\nu} \psi_{\mathbf{p}} + \dots \right]$$



v counting $\longrightarrow$ $(\mu_U)^\epsilon \sim (mv^2)^\epsilon \; , \; (\mu_S)^\epsilon \sim (mv)^\epsilon$

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P45 operators that describe potential-type four-quark interactions (originating from potential gluons and other off-shell modes)
Pedro, 7/20/2005

P46 interactions of quarks with soft gluons
Pedro, 7/19/2005

P47 terms bilinear in the quark field, also contain the interaction with ultrasoft gluons (multipole expanded)
Pedro, 7/19/2005

velocity Renormalization Group

.01

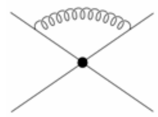
Correlation of scales

$$\mu_U = \mu_S^2/m \equiv m\nu^2$$

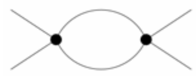
velocity scaling parameter $\nu \,\epsilon [0,1]$

P100

$$\text{potential and soft loops:} \quad \mu_S^{4-D} \int \frac{d^D k}{(2\pi)^D} \sim \ln\left(\mu_S^2/\mathbf{p}^2\right) = \ln \frac{\nu^2}{v^2}$$



$$\text{ultrasoft loops:} \quad \mu_U^{4-D} \int \frac{d^D k}{(2\pi)^D} \sim \ln\left(\mu_U/E\right) = \ln \frac{\nu^2}{v^2}$$



Choosing the renorm. point $\nu \sim v$ all logs are simultaneously small

rf12

⇨ logs resumed in the Wilson coefficients of the EFT

P99

$$C_i(\nu) \sim \alpha_s \sum_k [\alpha_s \ln(\nu)]^k \longrightarrow$$ **"velocity Renormalization Group"**

P99	The running in the dimensionless velocity parameter from nu=1 to nu\sim v of order the velocity of the two particle state, sums logs of both the momenta and the energy at the same time, and is referred to as the¨"velocity renormalization group" Pedro, 10/24/2005
P100	Factor of mu_S and mu_U provide an integer v scaling of the integrals. Every loop automatically receives the proper renormalization scale according to the three-momentum flowing through it Pedro, 11/10/2005
P101	nu is a natural scaling parameter for all types of loop integrals in vNRQCD Pedro, 10/26/2005
rf12	natural choice for the velocity scaling parameter in matrix elements ruizfeme, 9/22/2006

Total Cross Section

❑ Theoretical set-up

$$\sigma \;\propto\; \mathrm{Im}\left[\int d^4x\, e^{-i\hat{q}\cdot x} \left\langle 0|T\, J^\dagger_{\mathbf{p}}(0) J_{\mathbf{p}'}(x)\,|0\right\rangle\right]$$

$$\propto\; \mathrm{Im}\left[\, C(\nu)^2\, G(0,0,v)\,\right]$$

external currents

$$J_{\mathbf{p}} = C_{^3S_1}(\nu)\, \psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}\, \chi^*_{-\mathbf{p}} + \;\ldots \quad t\bar{t}\;(^3S_1)$$

- ✓ NLL **Pineda, Soto; Manohar, Stewart; Hoang**
- ✓ NNLL (non-mixing) **Hoang**

$$J_{\mathbf{p}} = C_{^1P_1}(\nu)\, \psi^*_{\mathbf{p}}\, \mathbf{p}\, \chi^*_{-\mathbf{p}} + \;\ldots \quad \tilde{q}\bar{\tilde{q}}\;(^1P_1)$$

- ✓ NLL **Hoang, PRF**

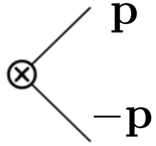
Coulomb Green function

$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s}-2m)\right) G(\mathbf{r},\mathbf{r}') = \delta^3(\mathbf{r}-\mathbf{r}')$$

$$V = \left[\frac{\mathcal{V}_c^{(T)}}{\mathbf{k}^2} + \frac{\mathcal{V}_k^{(T)}\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r^{(T)}(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2^{(T)}}{m^2} + \ldots\right]$$

- ✓ NNLL (quarks) **Manohar, Stewart; Pineda, Soto; Hoang**
- ✓ NNLL (scalars) **Hoang, PRF**

Building NR Production Currents $\left({}^{2S+1}L_J\right)$



Some well-known examples

$$\bar{Q}\gamma^i Q \quad \rightarrow \quad J_{\mathbf{p}} = \psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}\, \tilde{\chi}^*_{-\mathbf{p}} \; ({}^3S_1) \qquad e^+e^- \xrightarrow{\gamma} t\bar{t}$$

$$\bar{Q}\gamma_5 Q \quad \rightarrow \quad J_{\mathbf{p}} = \psi^\dagger_{\mathbf{p}}\, \tilde{\chi}^*_{-\mathbf{p}} \; ({}^1S_1) \qquad e^+e^- \xrightarrow{\gamma} \tilde{q}\tilde{\bar{q}}$$

$$\bar{Q}\gamma^i\gamma_5 Q \quad \rightarrow \quad J_{\mathbf{p}} = \psi^\dagger_{\mathbf{p}}\, [\mathbf{p}\cdot\boldsymbol{\sigma}, \boldsymbol{\sigma}]\, \tilde{\chi}^*_{-\mathbf{p}} \; ({}^3P_1) \qquad e^+e^- \xrightarrow{Z^0} t\bar{t}$$

rf13

Currents with general quantum numbers

Currents must have proper SO(n) transformations in $n = 3 - 2\epsilon$ dim.

$\longrightarrow$ n-dim Spherical Harmonics

spinless case: $J_{\mathbf{p}} = \psi^\dagger_{\mathbf{p}}\, T^{i_1 \ldots i_L}(\mathbf{p})\, \tilde{\chi}^*_{-\mathbf{p}} \quad ({}^1L_1)$

cartesian representation of $Y_{LM}(n, \hat{\mathbf{p}})$

$i_1, i_2 \ldots = 1, \ldots n$

$T^i(\mathbf{p}) = p^i$ P-wave

$T^{ij}(\mathbf{p}) = p^i p^j - \dfrac{\mathbf{p}^2}{n}\delta^{ij}$ D-wave

$\ldots$

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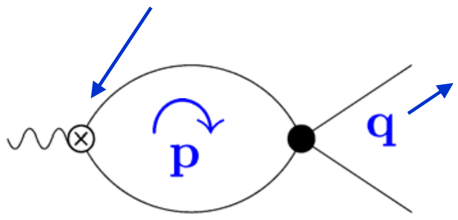
rf13 how to define currents in n-dim, guiding principle is SO(n) rotational symmetry
ruizfeme, 9/22/2006

Nonrelativistic Production Currents

- the use of *n*-dim spherical harmonics is needed to obtain consistent results in dimensional regularization in accordance with SO(n)

$$J_{\mathbf{p}} = \psi^\dagger_{\mathbf{p}}\, T^{i_1\ldots i_L}(\mathbf{p})\, \tilde{\chi}^*_{-\mathbf{p}}$$



$$\propto P_L(n, \hat{\mathbf{p}}\cdot\hat{\mathbf{q}})\, T^{i_1\ldots i_L}(\mathbf{q})$$

First non-trivial example: ***L=2***

$$P_2(n,x) = \frac{nx^2-1}{n-1} \xrightarrow{n=3} \frac{1}{2}(3x^2-1)$$

Pauli σ-matrices in n dimensions

- Consistent description of the spin of a $f\bar{f}$ system:

Pauli matrices

$$\{\sigma^i,\,\sigma^j\} = 2\delta^{ij}$$

	basis in n-dim.		$n=3$	
	$\mathbb{I}$	spin singlet currents	$\mathbb{I}$	
spin triplet currents	σ^{i_1}		σ^{i_1}	
	$\sigma^{[i_1 i_2]}$		$i\,\epsilon^{i_1 i_2 k}\,\sigma^k$	
	$\sigma^{[i_1 i_2 i_3]}$	spin singlet currents	$\epsilon^{i_1 i_2 i_3}$	
	$\sigma^{[i_1 i_2 i_3 i_4]}$	evanescent operators	0	
	$\dots$	evanescent operators	$\dots$	

$$\sigma^{[i_1 \dots i_m]} = \sigma^{[i_1}\sigma^{i_2}\dots\sigma^{i_m]}$$
$$(i_k = 1, \dots n)$$

$$J_{L=0} = \psi^\dagger_{\mathbf{p}}\,\sigma^{[i_1 \dots i_L]}\,\tilde{\chi}^*_{-\mathbf{p}}$$

General Production Currents

❑ form currents describing ${}^{2S+1}L_J$ states combining orbital angular momentum and spin tensors

$\underline{S=1}$

fully symmetric and traceless

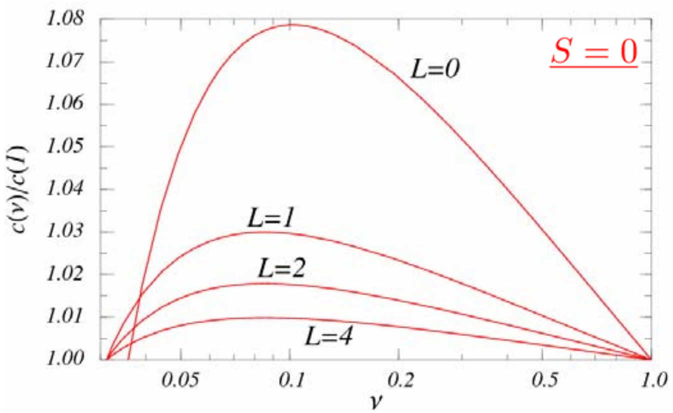
***J* = # indices**

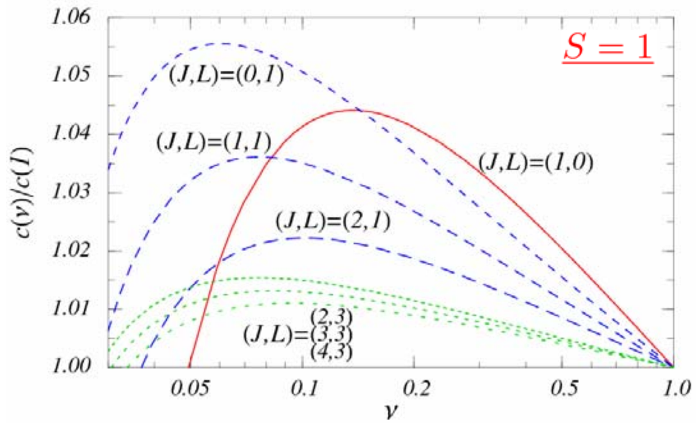
$$J^{i_1\ldots i_{L-1}} = \psi^\dagger_{\mathbf{p}} \left(T^{i_1\ldots i_{L-1}\ell}(\mathbf{p})\,\sigma^\ell \right) \tilde{\chi}^*_{-\mathbf{p}} \qquad J = L-1$$

$$J^{i_1\ldots i_L} = \psi^\dagger_{\mathbf{p}} \left(\sum_{k=1}^{L} T^{i_1\ldots\hat{i}_k\ldots i_L\ell}(\mathbf{p})\,[\sigma^\ell,\sigma^{i_k}] \right) \tilde{\chi}^*_{-\mathbf{p}} \qquad J = L$$

$$J^{i_1\ldots i_{L+1}} = \psi^\dagger_{\mathbf{p}} \left(\sum_{k=1}^{L+1} T^{i_1\ldots\hat{i}_k\ldots i_{L+1}}(\mathbf{p})\,\sigma^{i_k} + \ldots \right) \tilde{\chi}^*_{-\mathbf{p}} \qquad J = L+1$$

✓ NLL anomalous dimensions for arbitrary currents recently computed **Hoang, PRF ´06**





General Production Currents

- different irreducible currents in SO(n) become equivalent in n=3

Example: $({}^3P_1)$ $\quad J^i = \psi_{\mathbf{p}}^\dagger \left(p^\ell [\sigma^\ell, \sigma^i] \right) \tilde{\chi}_{-\mathbf{p}}^*$

We can write another ${}^3P_1$ current: $\tilde{J}^{jk} = \psi_{\mathbf{p}}^\dagger \left(p^j \sigma^k - p^k \sigma^j \right) \tilde{\chi}_{-\mathbf{p}}^* \underset{n=3}{=} i\epsilon^{ijk} J^i$

⟶ their difference is an evanescent operator

- evanescent structures need to be taken into account at subleading order in the renormalization process **Dugan, Grinstein; Herrlich, Nierste**
(they affect matching conditions and anom.dim.)

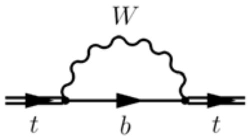
- in the EFT, the choice of basis of currents and potentials defines a renormalization scheme

- transitions between currents containing $\sigma^{[i_1 \ldots i_m]}$ with different number of indices beyond NNLL

vNRQCD for unstable quarks

- No systematic treatment of electroweak beyond **LL**

quark bilinears:



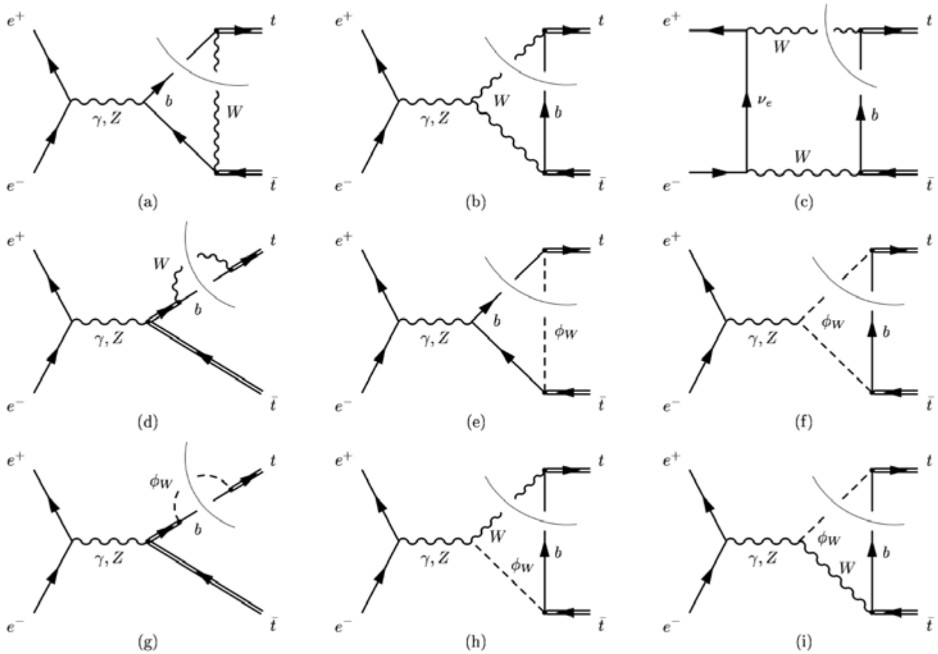
$$= i\Sigma_t \quad \Longrightarrow \quad \mathcal{L}_{\rm us} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \frac{\mathbf{p}^4}{8m^3} + i\,\frac{\Gamma_t}{2}\left(1-\frac{\mathbf{p}^2}{2m_t^2}\right) + \ldots \right] \psi_{\mathbf{p}}$$

- power counting: $\Gamma_t \sim m_t g^2 \sim m_t v^2 \quad \Rightarrow \quad g \sim g' \sim v \sim \alpha_s$
- finite lifetime is LL order, $E \to E + i\Gamma_t$ **Fadin, Khoze**
- top/antitop propagator $\dfrac{i}{p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2}$
- $E \to E + i\Gamma_t$ prescription does not work beyond LL order

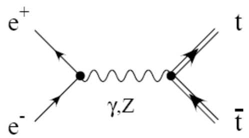
Hoang, Reisser '05 $\Longrightarrow$ inclusive treatment of top decay: finite lifetime effects can be incorporated into the EFT matching conditions to QCD+ew

vNRQCD for unstable quarks

❑ Currents



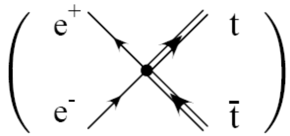
- consider cuts related to top decay (final bW states)
- non- bW -cuts not included
- bW -cuts are gauge invariant



Complex matching conditions

Hoang, Reisser '05

$$J_{\mathbf{p}} = \left[C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + iC^{\rm NNLL}_{bW,\rm abs} + \ldots\right]$$



$\mathrm{Re}\,[C^{\rm NNLL}_{\rm ew}]$ **Kuhn, Guth '92**
Hoang, Reisser '06

$e^+e^- \to t\bar{t}$ Total Cross Section

- EFT Lagragian non-hermitian

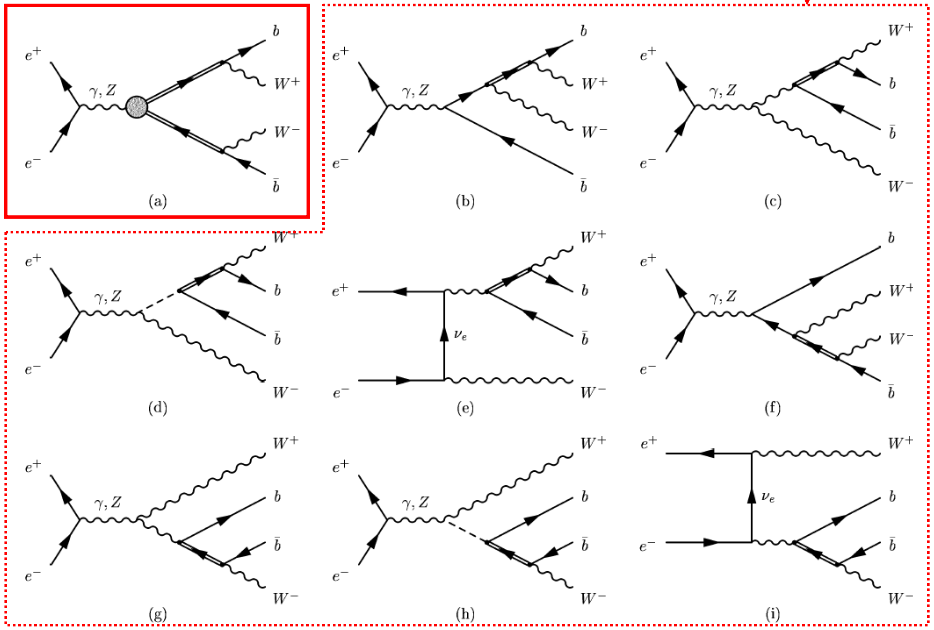
$\Longrightarrow$ total rates through the optical theorem

time dilatation correction

$$\Delta\sigma_{\rm tot}^{\Gamma,1} = 2N_c\,{\rm Im}\left[2C^{\rm LL}\,iC^{\rm NNLL}_{bW,{\rm abs}}\,G_{\rm coul} + (C^{\rm LL})^2\,\delta G_{\rm coul}\right]$$

double-resonant

$e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$



- accounts for interferences between the double resonant amplitudes and the single resonant (NNLL)

- diagrams with no intermediate top quark are v^4-suppressed

single-resonant, v^2-suppressed

$e^+e^- \to t + \bar{b}W^- \to bW^+\bar{b}W^-$

$e^+e^- \to \bar{t} + bW^+ \to bW^+\bar{b}W^-$

Phase Space Divergencies

$$\Delta\sigma_{\rm tot}^{\Gamma,1} = 2N_c\,{\rm Im}\left[2C^{\rm LL}\,iC^{\rm NNLL}_{bW,{\rm abs}}\,G_{\rm coul} + (C^{\rm LL})^2\,\delta G_{\rm coul}\right]$$

- NNLL corrections from absorptive bW cuts introduce a sensibility to ${\rm Re}\,G_{\rm coul}$

$${\rm Re}\,G_{\rm coul} \sim \frac{m^2 C_F\,\alpha_s}{4\pi}\,\frac{1}{4\epsilon} \quad\Longrightarrow\quad \left(\Delta\sigma^{\Gamma}_{\rm tot}\right) \sim \alpha_s\Gamma_t\,\frac{1}{\epsilon}$$

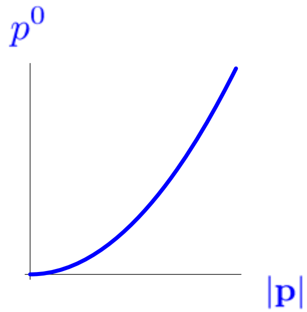
Phase space UV divergencies (NNLL) AK6

AK1

Phase space:

stable tops

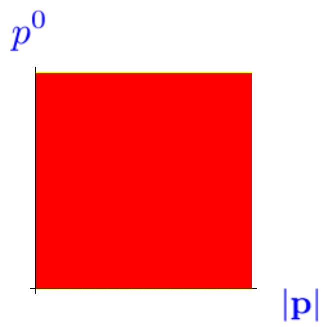
$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\epsilon}$$



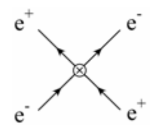
unstable tops

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}}$$

P103



P107

→ anom. dim. for $(e^+e^-)(e^+e^-)$ operator $iC(\nu)\cdot\Big($  $\Big)$

→ optical theorem: additional correction (NLL) $\left(\Delta\sigma^{\Gamma,2}_{\rm tot}\right) = C(\nu)$

Hoang, Reisser '05

Slide 22

AK1 counterterms prop to (e+e-)(e+e-) operators absorb these divergencies
Alois Kabelschacht, 5/5/2006

AK6 comment that these divergencies are only there for Gamma\ne 0; one does not see them in the stable case
Alois Kabelschacht, 5/8/2006

P103 Breit-Wigner-type top quark propagator
Pedro, 11/6/2006

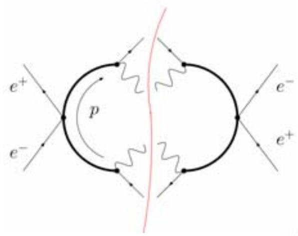
P107 counterterms prop to (e+e-)(e+e-) operators absorb these divergences
Pedro, 11/6/2006

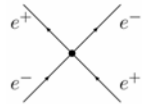
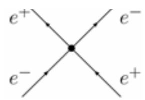
Phase-Space Matching

Hoang, Reisser, PRF

AK8

$C(m_t,\ldots)$ unknown $\longrightarrow$ ***¨phase-space matching¨ (work i.p.)***

$\sigma^{\Gamma}_{\text{tot},\,\Lambda} \sim$ 

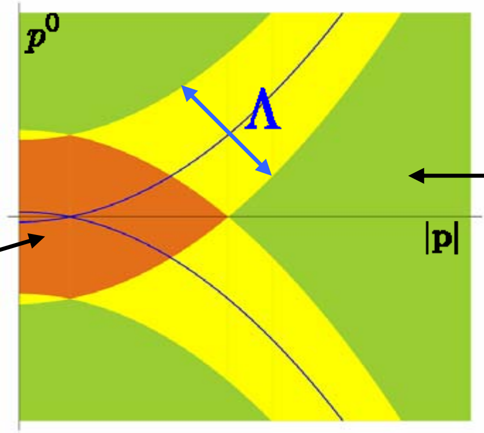
$+ \;\text{Im}\Big[\, iC(\mu,\Lambda)$  $+\; i\widetilde{C}(\mu,\Lambda)\dfrac{\partial_0}{m_t^2}$  $+\;\ldots\Big]$

P108

Λ = cut on t and $\bar{t}$ invariant masses

$(q^2 - m_t^2) \ll m_t\Lambda \lesssim m_t^2$

double resonant
nonrel. expansion valid



unphysical region of EFT
single/non-resonant
subtracted in local expansion
by matching conditions
$C(\mu,\Lambda)$, $\widetilde{C}(\mu,\Lambda),\ldots$

- **P-wave production** $(\,e^+e^- \to \tilde{q}\,\bar{\tilde{q}}\,)$: phase-space divergences are more severe

$$G^{L=1}_{\text{coul}} = m^2\left(v^2 + \frac{C_F^2\,\alpha_s^2}{4}\right)G_{\text{coul}} + \ldots \;\underset{m\,v^2 = E + i\Gamma}{\Longrightarrow}\; \text{Im}\,G^{L=1}_{\text{coul}} \sim \alpha_s\Gamma\,\frac{1}{\epsilon}$$

LL !

Slide 23

AK8 the information about the physical phase-space boundaries must be put back in the EFT
Alois Kabelschacht, 5/5/2006

P108 the matching conditions of the coefficients of the (e+e-)(e+e-) operators account for the difference between the infinite EFT phase space and the true physical one
Pedro, 11/6/2006

Outlook

- Ongoing work to reduce theoretical uncertainties in

  $\sigma(e^+e^- \to t\bar{t})$ $\Longrightarrow$ (almost) full NNLL QCD corrections

  $\Longrightarrow$ electroweak corrections beyond LL

- Analysis of $\sigma(e^+e^-, \gamma\gamma \to \tilde{q}\,\bar{\tilde{q}})$ at threshold possible within scalar NRQCD $\Longrightarrow$ full NLL QCD running completed

- Theoretical developments:

  ✓ consistent treatment of L-wave production of nonrelativistic pairs

  ✘ phase-space divergencies $\Longrightarrow$ ¨Phase-space matching¨