The forward-backward asymmetry in electron-positron annihilation

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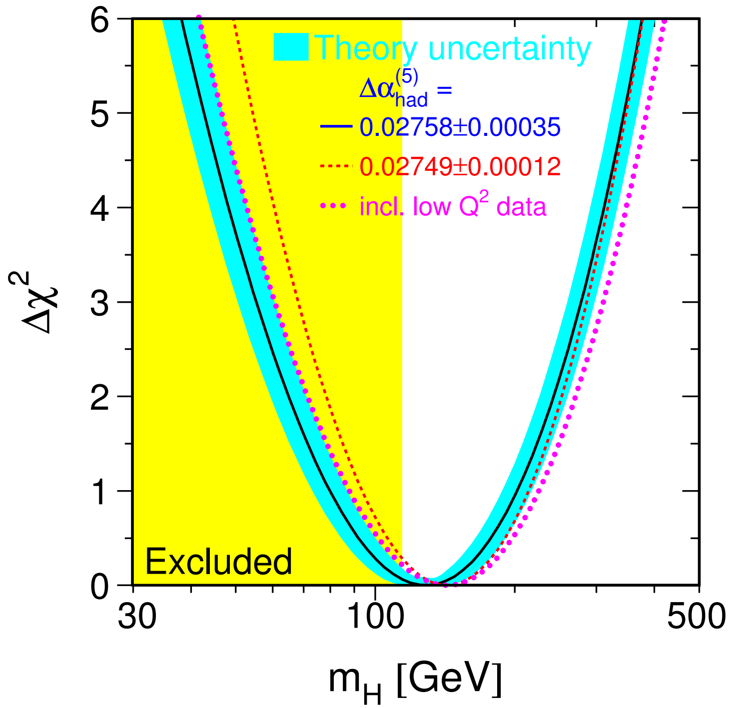
The Standard Model and the Higgs boson

Our current paradigma: The Standard Model

The Higgs boson: The Standard Model predicts a scalar particle, which gives rise to the mass of all other particles.
- yet to be discovered -

Up to now the Higgs boson manifests itself only through quantum corrections!



(Electroweak Working Group, hep-ex/0509008.)

Electroweak precision physics

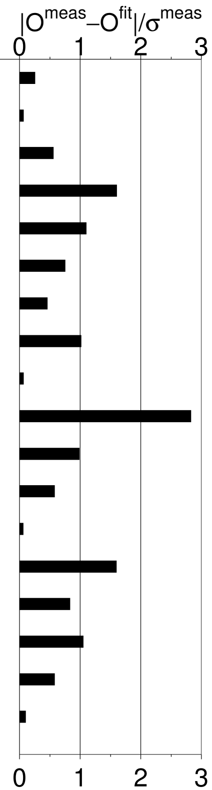
Precision observables allow us to extract the values of the five input parameters for the Standard model at the Z-pole.

Input parameters are:
$\alpha(m_Z^2)$, $\alpha_s(m_Z^2)$, m_Z, m_t, m_H.

Check how individual measurements agree with the result of this fit.

The forward-backward asymmetry for b-quarks shows the largest pull.

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^0 [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



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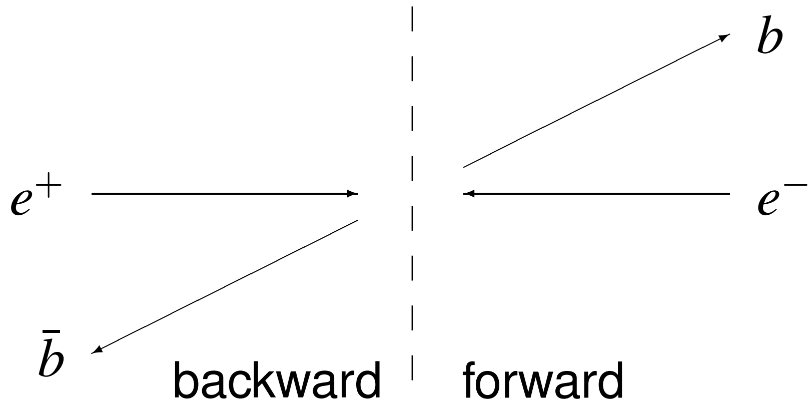
The forward-backward asymmetry



A first definition of the forward-backward asymmetry:

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B}$$

But: Free quarks are not observed, instead hadronic jets are seen in the detector !

Perturbation theory

Due to the smallness of the coupling constants α and α_s, we may compute an observable at high energies reliable in perturbation theory,

$$\langle O \rangle = \langle O \rangle_{LO} + \frac{\alpha_s}{2\pi} \langle O \rangle_{NLO} + \left(\frac{\alpha_s}{2\pi} \right)^2 \langle O \rangle_{NNLO} + \ldots$$

provided that the observable is infrared-safe!

In particular, it is required that the observable does not change value, if infinitessimal soft or collinear particles are added.

$$O_{n+l}(p_1, \ldots, p_{n+l}) \quad \rightarrow \quad O_n(p_1', \ldots, p_n'),$$

The forward-backward asymmetry is measured experimentally with a precision at the per cent level.

To match this precision the inclusion of QCD corrections in a theoretical calculation is mandatory.

Prior art

Calculation of the NNLO QCD corrections to the forward-backward asymmetry in massless QCD:

$$A_{FB} = A_{FB}^{(0)}\left(1+\frac{\alpha_s}{2\pi}B_{FB}+\left(\frac{\alpha_s}{2\pi}\right)^2 C_{FB}\right)+O\left(\alpha_s^3\right),$$

- G. Altarelli and B. Lampe, 1993;
- V. Ravindran and W. L. van Neerven, 1998;
- S. Catani and M. H. Seymour, 1999.

NLO corrections including mass corrections:

J. Jersak, E. Laermann, and P. M. Zerwas, 1981; J. G. Körner, G. Schuler, G. Kramer, and B. Lampe, 1986; A. B. Arbuzov, D. Y. Bardin, and A. Leike, 1992; A. Djouadi, B. Lampe, and P. M. Zerwas, 1995; B. Lampe, 1996;

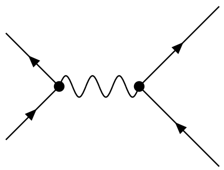
Partial results for mass corrections at NNLO:

W. Bernreuther, A. Brandenburg, and P. Uwer, 2000; W. Bernreuther *et al.*, 2006;

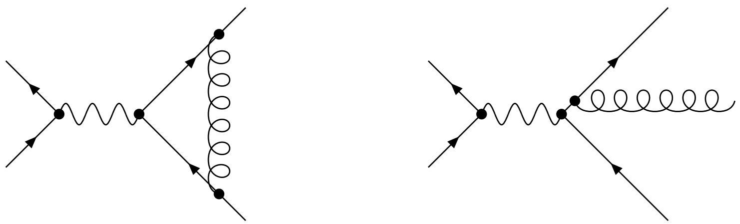
Diagrams

Some examples of diagrams contributing to the various orders in perturbation theory:

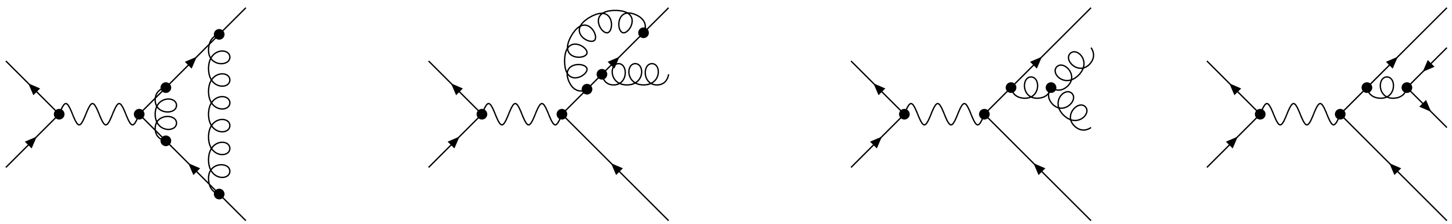
LO:



NLO:



NNLO:



Purely virtual diagrams cancel in the correction to the asymmetry!

Definitions used in the literature

How to define the direction of the b-quark in the presence of additional partons?

- Define the direction by the momentum of the quark.
- Use the thrust axis as direction.

How to treat the $bb\bar{b}\bar{b}$ final state if two b-quarks are tagged?

- Count it once.
- Count it twice.

The experimental analysis seems to have used the thrust axis and counted $bb\bar{b}\bar{b}$ final states with weigth two.

Infrared finiteness

Catani ans Seymour have shown, that none of the combinations thrust axis/ quark axis and weight two/ weight one yields an infrared finite observable.

The divergence is proportional to

$$\int_0^1 dz\, P_{q\to qq\bar{q}}(z)\, \ln\frac{Q^2}{m_b^2}$$

To absorb this divergence one can introduce a b-quark fragmentation function.
This brings along additional uncertainties related to non-perturbative physics.

Questions

Can the introduction of the fragmentation function and dependence on non-perturbative physics be avoided ?

How to define the forward-backward asymmetry in an infrared-safe way ?

What about a jet axis ?

Jet algorithms

The most fine-grained look at hadronic events consistent with infrared safety is given by classifying the particles into jets.

Ingredients:

- a resolution variable y_{ij} where a smaller y_{ij} means that particles i and j are "closer";
- a combination procedure which combines two four-momenta into one;
- a cut-off y_{min} which provides a stopping point for the algorithm.

A typical algorithm:

- for each pair i, j, calculate y_{ij}
- select pair with smallest y_{ij}; if $y_{ij} < y_{min}$, combine i and j
- repeat until the smallest $y_{ij} > y_{min}$

The Durham algorithm

Example: The Durham or $k_\perp$-algorithm for partons, whose flavour is not detected.

(Dokshitzer, 1991)

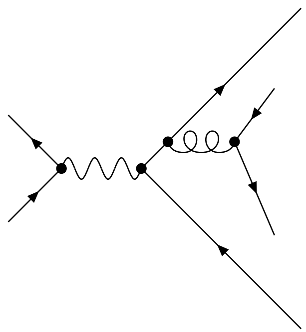
Resolution variable:

$$y_{ij}^{DURHAM} = \frac{2(1-\cos\theta_{ij})}{Q^2}\,\mathrm{min}(E_i^2, E_j^2)$$

Combination procedure:

$$p^\mu_{(ij)} = p^\mu_i + p^\mu_j.$$

Jets with flavour



The Durham algorithm is not infrared-safe for jets with flavour, since at order α_s^2 a soft gluon can split into a soft $q\bar{q}$ pair.

The Durham measure

$$y_{ij}^{DURHAM} = \frac{2(1-\cos\theta_{ij})}{Q^2}\,\mathrm{min}(E_i^2, E_j^2)$$

assumes that parton emission has a soft and a collinear divergence.

However, there is no soft divergence in the $g \to q\bar{q}$ splitting.

The flavour-$k_\perp$ algorithm

In order to account for tagged flavours modify the Durham measure

$$y_{ij}^{DURHAM} = \frac{2(1-\cos\theta_{ij})}{Q^2}\,\mathrm{min}(E_i^2, E_j^2)$$

towards

$$y_{ij}^{flavour} = \frac{2(1-\cos\theta_{ij})}{Q^2} \times \begin{cases} \mathrm{min}(E_i^2, E_j^2), & \text{softer of } i,\ j \text{ is flavourless,} \\ \mathrm{max}(E_i^2, E_j^2), & \text{softer of } i,\ j \text{ is flavoured.} \end{cases}$$

This yields an infrared-safe definition of jets if flavours are tagged.

Banfi, Salam and Zanderighi, (2006).

Definition of the forward-backward asymmetry

- Assign flavour number $+1$ to a b-quark and -1 to a $\bar{b}$-quark.
  All other particles have flavour number zero.

- Cluster particles into jets, using the flavour-$k_\perp$ algorithm.

- If two particles are combined, the flavour numbers are added.

- Select two jet events, where one jet has flavour number > 0.

- The jet axis of this jet defines the direction relevant to the forward-backward asymmetry.

Calculation of the NLO and NNLO corrections

To compute for this definition the NLO and NNLO corrections, a general purpose program for NNLO corrections to $e^+e^- \longrightarrow$ 2 jets is used.

S.W., 2006.

The relevant matrix elements are known for a long time.

T. Matsuura and W. L. van Neerven, 1988; T. Matsuura, S. C. van der Marck, and W. L. van Neerven, 1989; G. Kramer and B. Lampe, 1987; R. K. Ellis, D. A. Ross, and A. E. Terrano, 1981; A. Ali *et al.*, 1979;

Difficulty: Cancellation of IR divergences.

General methods at NLO

Fully differential NLO Monte Carlo programs need a general method to handle the cancelation of infrared divergencies.

- Phase space slicing
  - e^+e^-: W. Giele and N. Glover, (1992)
  - initial hadrons: W. Giele, N. Glover and D.A. Kosower, (1993)
  - massive partons, fragmentation: S. Keller and E. Laenen, (1999)
- Subtraction method
  - residue approach: S. Frixione, Z. Kunzst and A. Signer, (1995)
  - dipole formalism: S. Catani and M. Seymour, (1996)
  - massive partons: L. Phaf and S.W. (2001), S. Catani, S. Dittmaier, M. Seymour and Z. Trócsányi, (2002)

The subtraction method at NNLO

- Singular behaviour
  - Factorization of tree amplitudes in double unresolved limits, Berends, Giele, Cambell, Glover, Catani, Grazzini, Del Duca, Frizzo, Maltoni, Kosower '99
  - Factorization of one-loop amplitudes in single unresolved limits, Bern, Del Duca, Kilgore, Schmidt, Kosower, Uwer, Catani, Grazzini, '99

- Extension of the subtraction method to NNLO Kosower; S.W.; Anastasiou, Melnikov, Petriello; Kilgore; Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; Frixione, Grazzini; Somogyi, Trócsányi and Del Duca;

- Applications:
  - $pp \to W$, Anastasiou, Dixon, Melnikov, Petriello '03,
  - $e^+e^- \to$ 2 jets, Anastasiou, Melnikov, Petriello '04, S.W. '06

The subtraction method at NNLO

Contributions at NNLO:

$$d\sigma_{n+2}^{(0)} = \left({\mathcal{A}_{n+2}^{(0)}}^* \mathcal{A}_{n+2}^{(0)} \right) d\phi_{n+2},$$

$$d\sigma_{n+1}^{(1)} = \left({\mathcal{A}_{n+1}^{(0)}}^* \mathcal{A}_{n+1}^{(1)} + {\mathcal{A}_{n+1}^{(1)}}^* \mathcal{A}_{n+1}^{(0)} \right) d\phi_{n+1},$$

$$d\sigma_{n}^{(2)} = \left(\mathcal{A}_{n}^{(0)*} \mathcal{A}_{n}^{(2)} + \mathcal{A}_{n}^{(2)*} \mathcal{A}_{n}^{(0)} + \mathcal{A}_{n}^{(1)*} \mathcal{A}_{n}^{(1)} \right) d\phi_{n},$$

Adding and subtracting:

$$\begin{aligned} \langle O \rangle_n^{NNLO} = & \int \left(O_{n+2}\, d\sigma_{n+2}^{(0)} - O_{n+1} \circ d\alpha_{n+1}^{(0,1)} - O_n \circ d\alpha_n^{(0,2)} \right) \\ & + \int \left(O_{n+1}\, d\sigma_{n+1}^{(1)} + O_{n+1} \circ d\alpha_{n+1}^{(0,1)} - O_n \circ d\alpha_n^{(1,1)} \right) \\ & + \int \left(O_n\, d\sigma_n^{(2)} + O_n \circ d\alpha_n^{(0,2)} + O_n \circ d\alpha_n^{(1,1)} \right). \end{aligned}$$

Numerical results for the forward-backward asymmetry of b-quarks

Perturbative expansion:

$$A_{FB} = A_{FB}^{(0)} \left(1 + \frac{\alpha_s}{2\pi} B_{FB} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_{FB} \right) + O\left(\alpha_s^3\right),$$

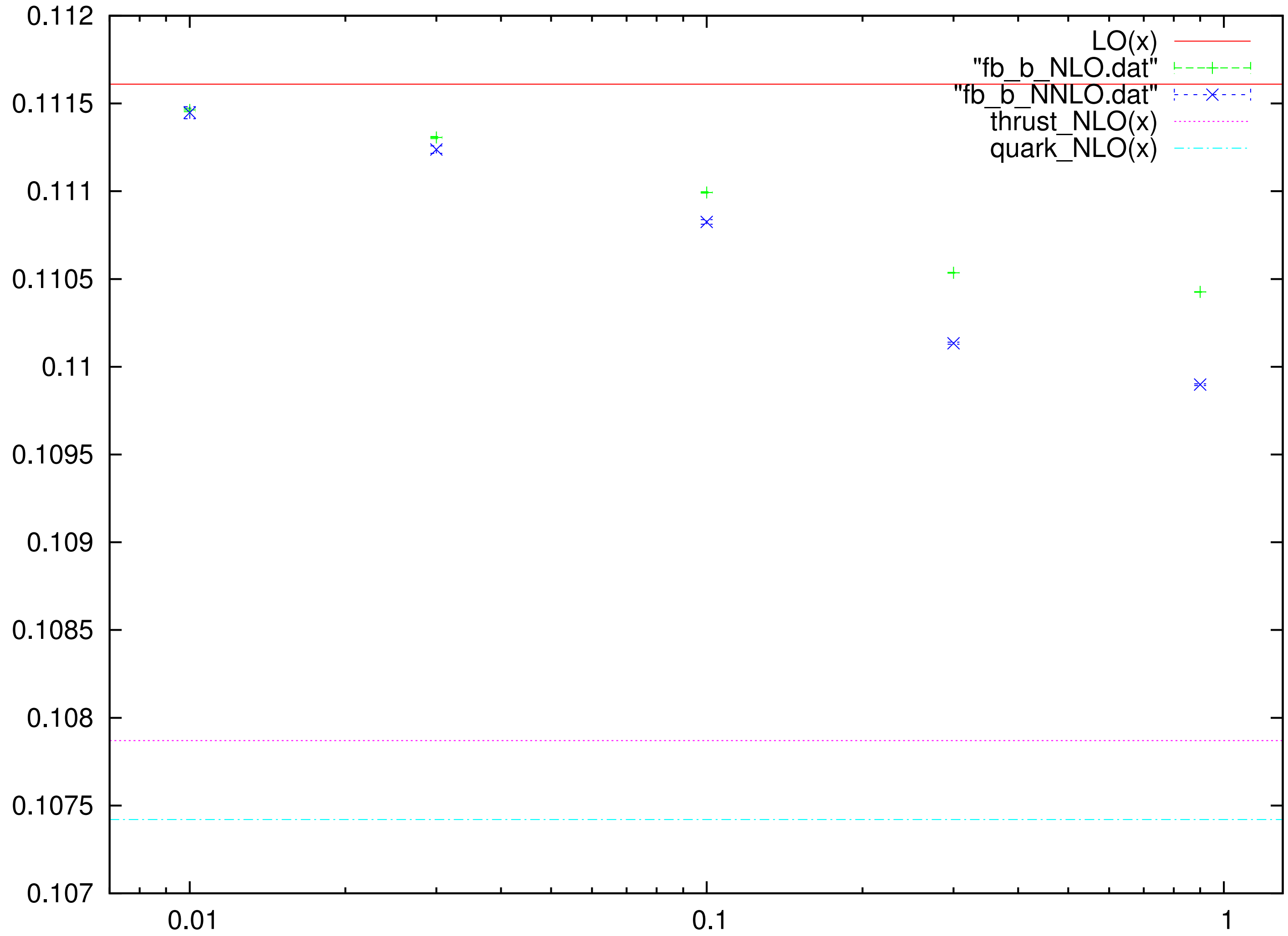
Select two-jet events defined by the flavour-$k_\perp$ algorithm and a given y_{cut}.
Leading order result independent of y_{cut}:

$$A_{FB,b}^{(0)} = 0.11161.$$

QCD corrections:

y_{cut}	$B_{FB,b}$	$C_{FB,b}$
0.01	-0.070 ± 0.005	-0.4 ± 0.8
0.03	-0.145 ± 0.003	-1.7 ± 0.5
0.1	-0.294 ± 0.002	-4.3 ± 0.3
0.3	-0.512 ± 0.001	-10.2 ± 0.1
0.9	-0.565 ± 0.001	-13.4 ± 0.1

Dependence of A_{FB} on y_{cut}



Numerical results for the forward-backward asymmetry of c-quarks

Perturbative expansion:

$$A_{FB} = A_{FB}^{(0)}\left(1+\frac{\alpha_s}{2\pi}B_{FB}+\left(\frac{\alpha_s}{2\pi}\right)^2 C_{FB}\right)+\mathcal{O}\left(\alpha_s^3\right),$$

Select two-jet events defined by the flavour-$k_\perp$ algorithm and a given y_{cut}.
Leading order result independent of y_{cut}:

$$A_{FB,c}^{(0)} = 0.08003.$$

QCD corrections:

y_{cut}	$B_{FB,c}$	$C_{FB,c}$
0.01	-0.070 ± 0.005	-0.5 ± 0.7
0.03	-0.145 ± 0.003	-2.1 ± 0.5
0.1	-0.294 ± 0.002	-4.8 ± 0.2
0.3	-0.513 ± 0.001	-12.1 ± 0.2
0.9	-0.565 ± 0.001	-15.9 ± 0.1

Summary

- The forward-backward asymmetry shows the largest discrepancy in a fit of the Standard Model parameter.
- Experimental analysis based on an infrared-unsafe definition.
- Infrared-safe definition of the forward-backward asymmetry.
- Calculation of the NLO and NNLO QCD corrections.
- The corrections are small, observable useful also for a future linear collider.