NNLO correction to $\bar{B} \to X_s\gamma$

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and

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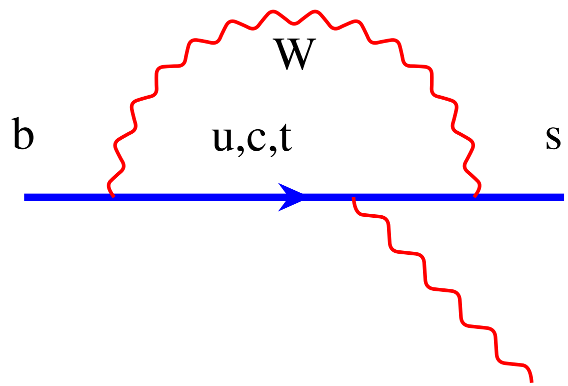
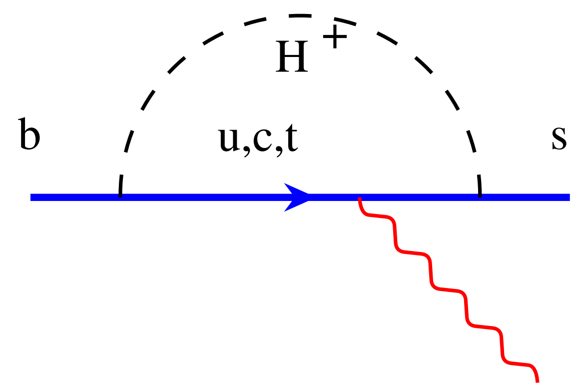
Why $\bar{B} \to X_s\gamma$?

- $$\Gamma(\bar{B} \to X_s\gamma) \approx \Gamma(b \to X_s^{\text{parton}}\gamma) = \Gamma(b \to s\gamma) + \Gamma(b \to s\gamma g) + \ldots$$

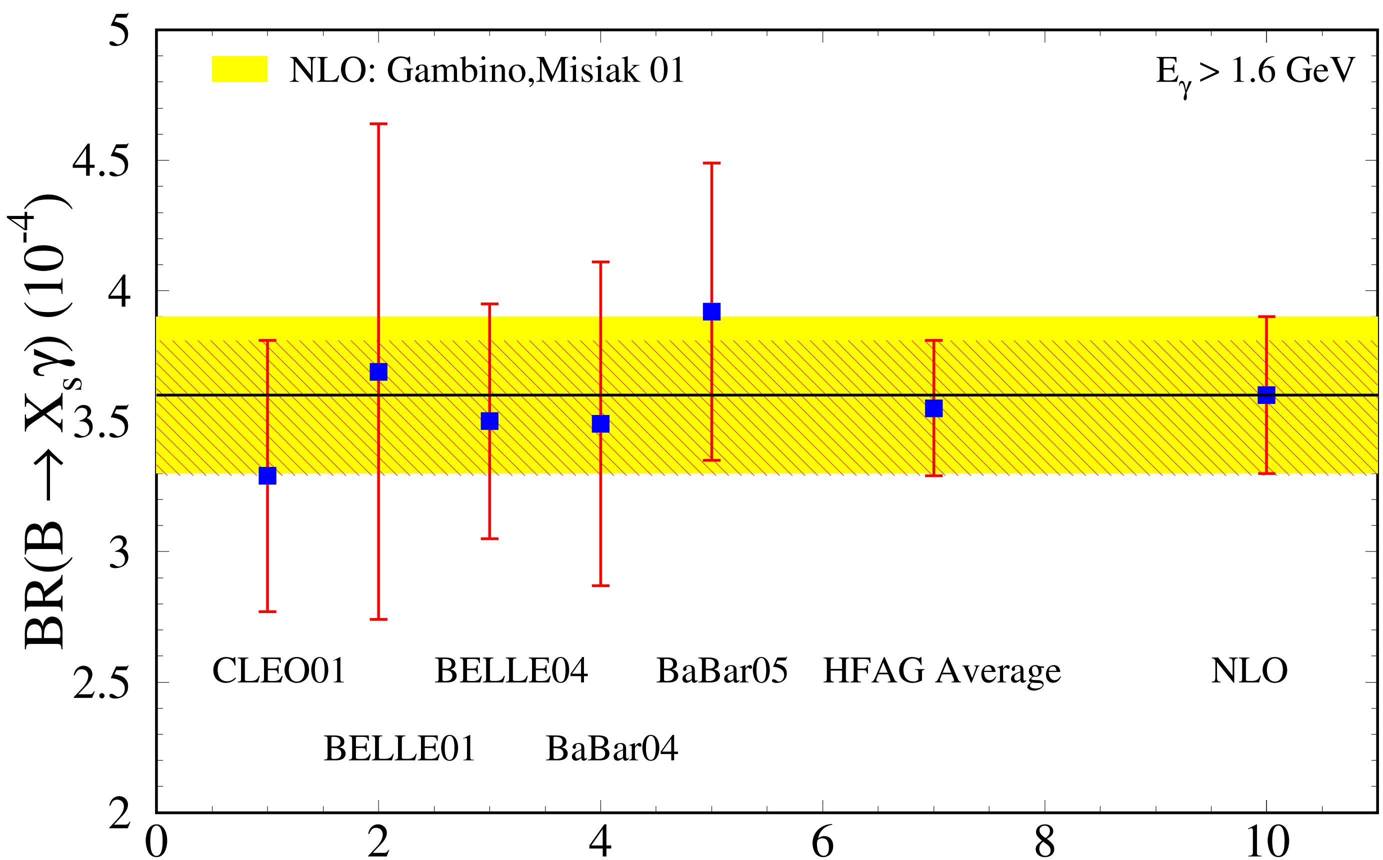
- Loop-induced:



- sensitive to "new physics"



NLO & Experiment



$$\mathcal{B}(B \to X_s\gamma)^{\text{th,NLO}} = (3.60 \pm 0.30) \times 10^{-4}$$ [Gambino,Misiak'01]

$$\mathcal{B}(B \to X_s\gamma)^{\text{exp}} = (3.55 \pm 0.26) \times 10^{-4}$$ [HFAG'06]

Structure of theory prediction

$$\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma > 1.6\mathrm{GeV}} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu})|_{\exp} \left(\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\bar{\nu})}\right)_{\mathrm{LO}} f\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right) \times$$
$$\left\{1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha) + \text{non-pert. corr.}\right\}$$

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$$\left\{1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha) + \text{non-pert. corr.}\right\}$$

- $\mathcal{O}(\alpha_s)$: NLO: $\sim$ 30%
- $\mathcal{O}(\alpha_s^2)$: NNLO: $\sim$ 10%
- $\mathcal{O}(\alpha)$: $\sim$ 4%
- non-pert. corr.:
  - $\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) \sim 1\%$
  - $\mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right) \sim 3\%$
  - $\mathcal{O}\left(\frac{\Lambda}{m_b}\alpha_s\right) \sim 5\%$

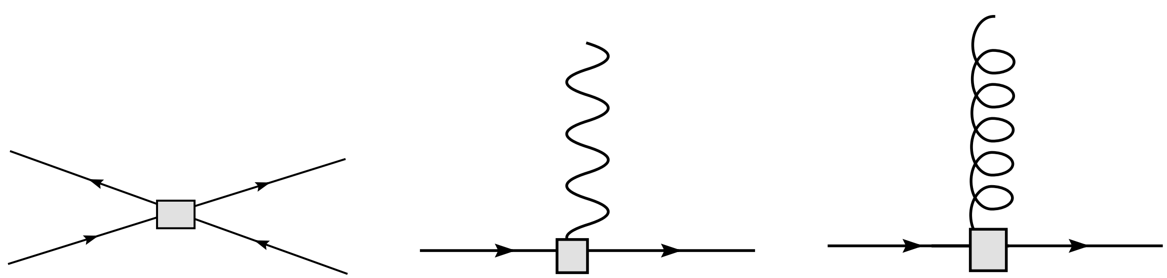
Effective theory

- $m_t, M_W \gg m_b, m_s$
- resummation of logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ necessary

➪ Calculation has to be done in the framework of an effective theory:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \frac{4G_F}{\sqrt{2}} V_{ts}^\star V_{tb} \sum_i C_i(\mu) O_i(\mu)$$

$O_1, \ldots, O_6$ $\quad O_7$ $\quad O_8$



Three Steps

1. Matching: determine $C_i(\mu)$

$\Gamma_{\rm SM} \stackrel{!}{=} \Gamma_{\rm eff.th.}$

$\mu \approx M_W, m_t$

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2. Matrix elements: on-shell $b \to s\gamma$ amplitude,

$\langle s\gamma|O_i|b\rangle$

$\mu \approx m_b$

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3. Mixing: effective theory RGE

$C_i(\mu \sim M_W) \to C_i(\mu \sim m_b)$

resum large logarithms $\left(\alpha_s \ln \frac{m_b^2}{M_W^2}\right)^n$

Preparation for NNLO

1. Matching

- 3-loop matching, O_7, O_8 [Misiak,MS'04]

2. Matrix elements

- O_1, O_2, O_7, O_8, large β_0 [Bieri,Greub,MS'03]
- O_7 [Blokland,Czarnecki,Misiak,Ślusarczyk,Tkachov'05]
- O_7, photon spectrum [Melnikov,Mitov'05], [Asatrian,Ewerth,Ferroglia,Gambino,Greub'06]
- O_1, O_2, interpolation [Misiak,MS'06]

2. Mixing

- 3-loop: $(O_1, \ldots, O_6)$ and (O_7, O_8) sectors [Gorbahn,Haisch'05], [Gorbahn,Haisch,Misiak'05]
- 4-loop: $(O_1, \ldots, O_6) \rightarrow (O_7, O_8)$ [Czakon,Haisch,Misiak, in progress]

Decomposing the branching ratio

$$\mathcal{B}[\bar{B} \to X_s\gamma]_{\mathrm{E}_\gamma > \mathrm{E}_0} = \mathcal{B}[\bar{B} \to X_c e\bar{\nu}]_{\mathrm{exp}} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\mathrm{em}}}{\pi\, C} \left[P(E_0) + N(E_0) \right]$$

$N(E_0)$: non-pert. part

$$P(E_0) = P^{(0)} + \frac{\alpha_s}{4\pi} \left(P_1^{(1)} + P_2^{(1)}(z) \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left(P_1^{(2)} + P_2^{(2)}(z) + P_3^{(2)}(z) \right) + \ldots$$

$$z = \frac{m_c(m_c)}{m_b^{1S}}$$

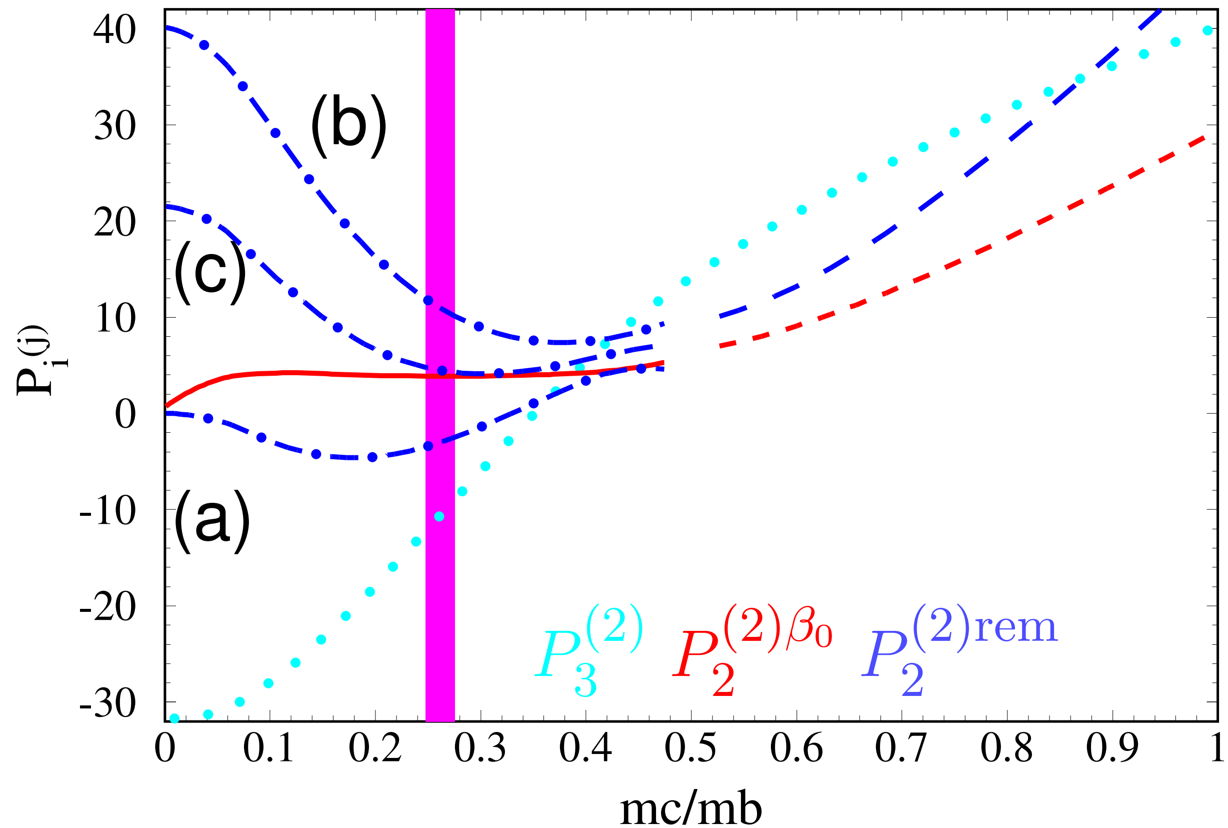
$P_1^{(1)}, P_3^{(2)} \sim C_i^{(0)} C_j^{(1)}$ $\qquad$ $P_2^{(1)}, P_2^{(2)} \sim C_i^{(0)} C_j^{(0)}$ $\qquad$ $P_1^{(2)} \sim C_i^{(0)} C_j^{(2)}, C_i^{(1)} C_j^{(1)}$

$P_2^{(1)}(z)$ and $P_3^{(2)}$: known

$P_2^{(2)\beta_0}$: known

$P_2^{(2)}$: interpolation

m_c dependence of $P_2^{(2)}$ and $P_3^{(2)}$

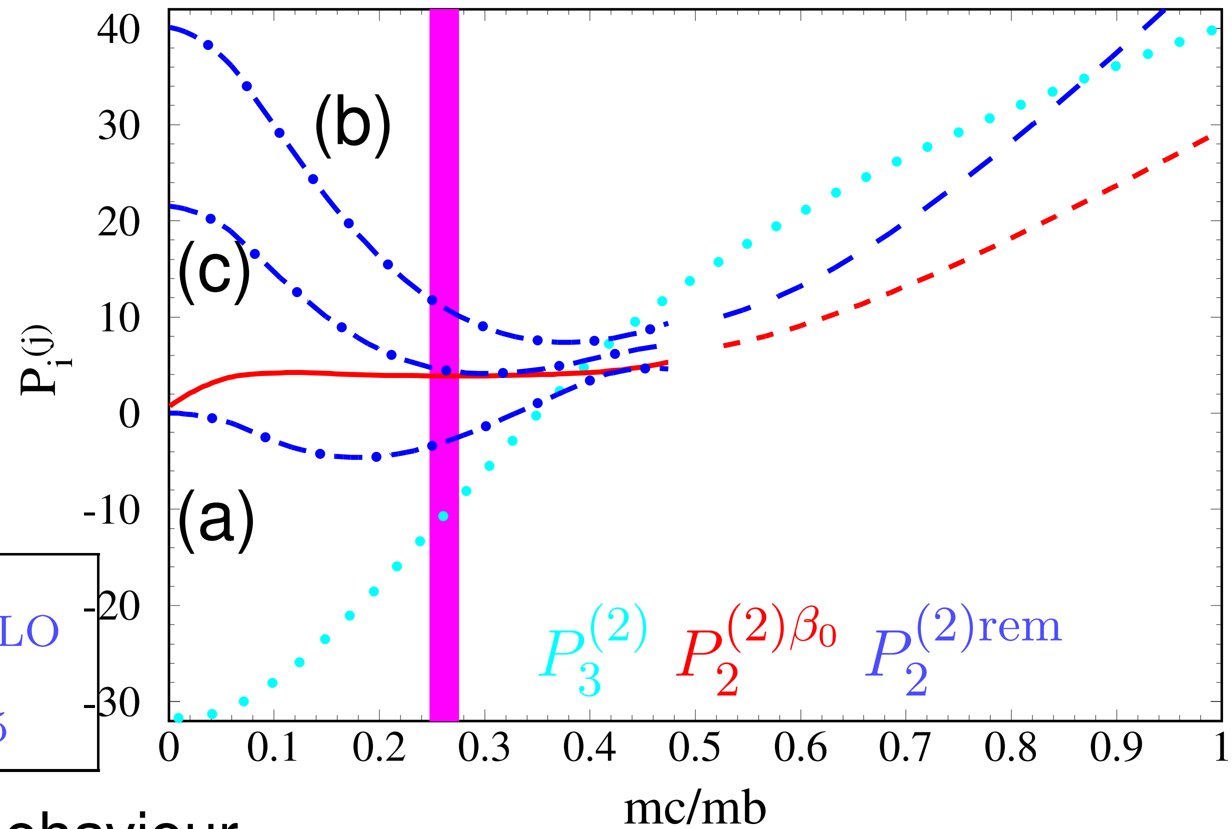


m_c dependence of $P_2^{(2)}$ and $P_3^{(2)}$

Interpolation:

- Compute $P_2^{(2)\text{rem}}$ for $z \gg 1/2$
- Ansatz:

$$P_2^{(2)\text{rem}} = x_1|A_{\text{NLO}}|^2 + x_2 A_{\text{NLO}} + x_3 \frac{\text{d}}{\text{d}z} A_{\text{NLO}} + x_4 P_2^{(2)\beta_0} + x_5$$



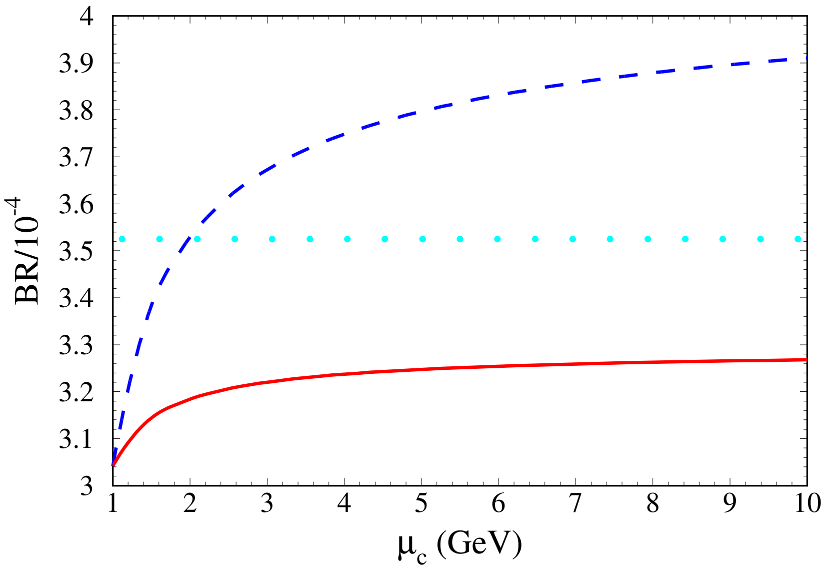
- Determine x_i from behaviour at large $z = m_c(m_c)/m_b^{1S}$ and assumption $\longrightarrow$

(a) $P_2^{(2)\text{rem}}(0) = 0$

(b) $P_1^{(2)} + P_2^{(2)\text{rem}}(0) + P_3^{(2)} = 0$

(c) $P_2^{(2)\text{rem}}(0) = P_2^{(2)\text{rem}}(0)|_{77}$

Dependence on the renormalization scales



LO, NLO, NNLO

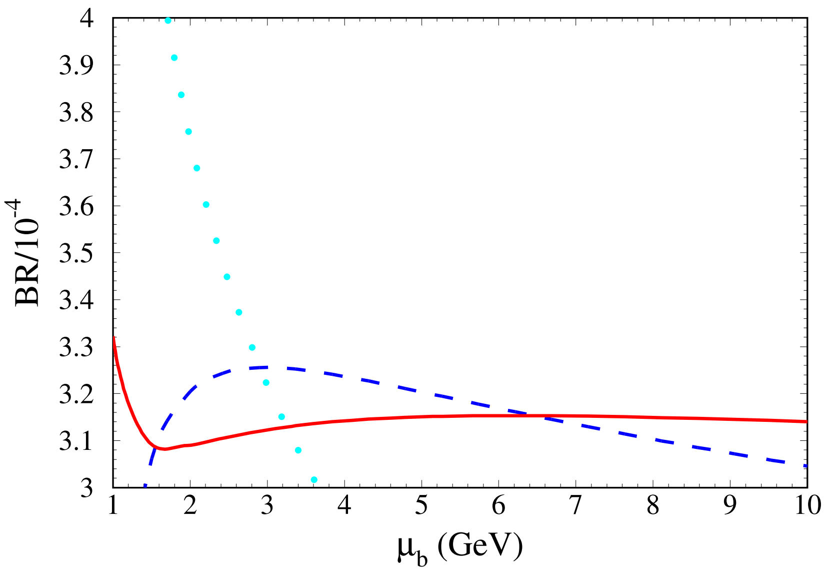
default values:

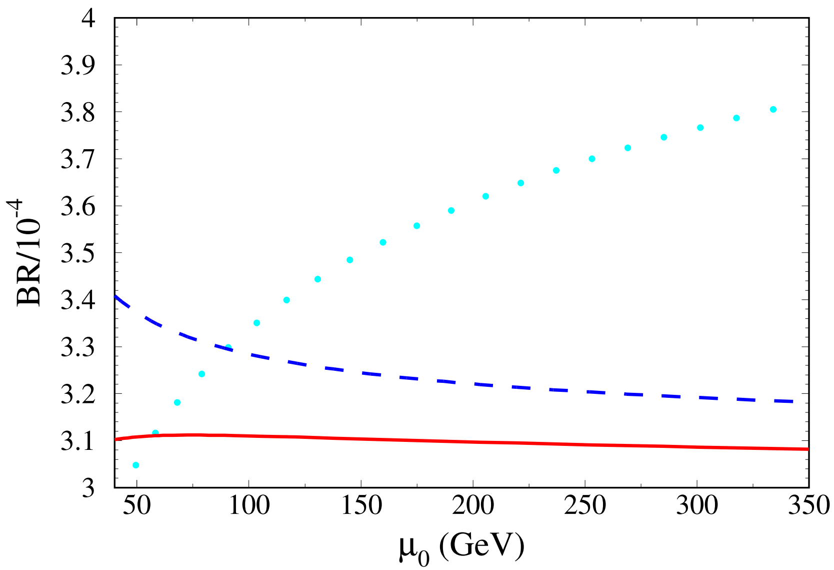
$\mu_c = 1.224$ GeV

$\mu_b = m_b^{1S}/2 = 2.35$ GeV

$\mu_0 = 2M_W$

NNLO: average of case (a) and (b)





NNLO Prediction

$$\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma > 1.6\text{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

[Misiak et al.'06], [Misiak,MS'06]

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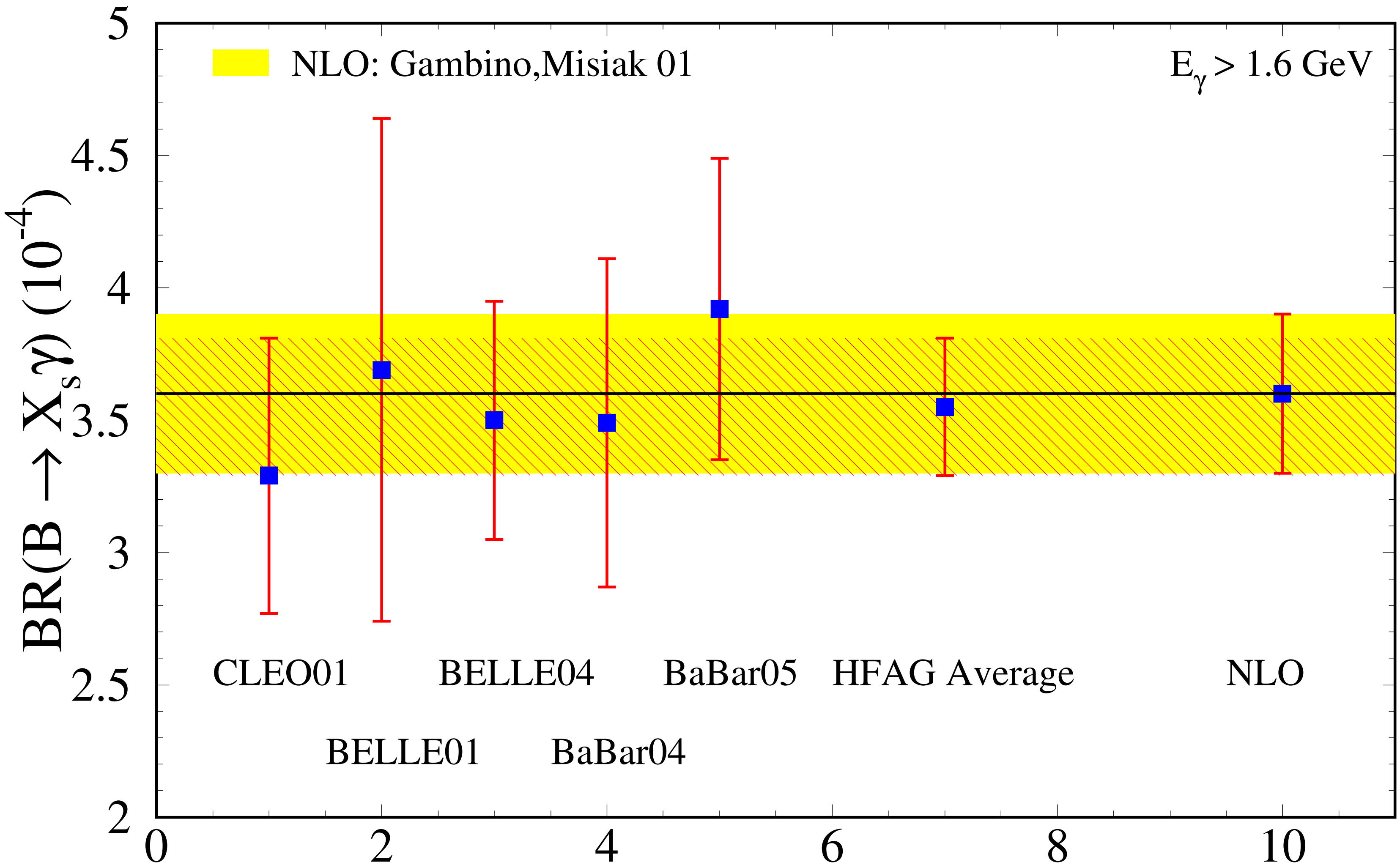
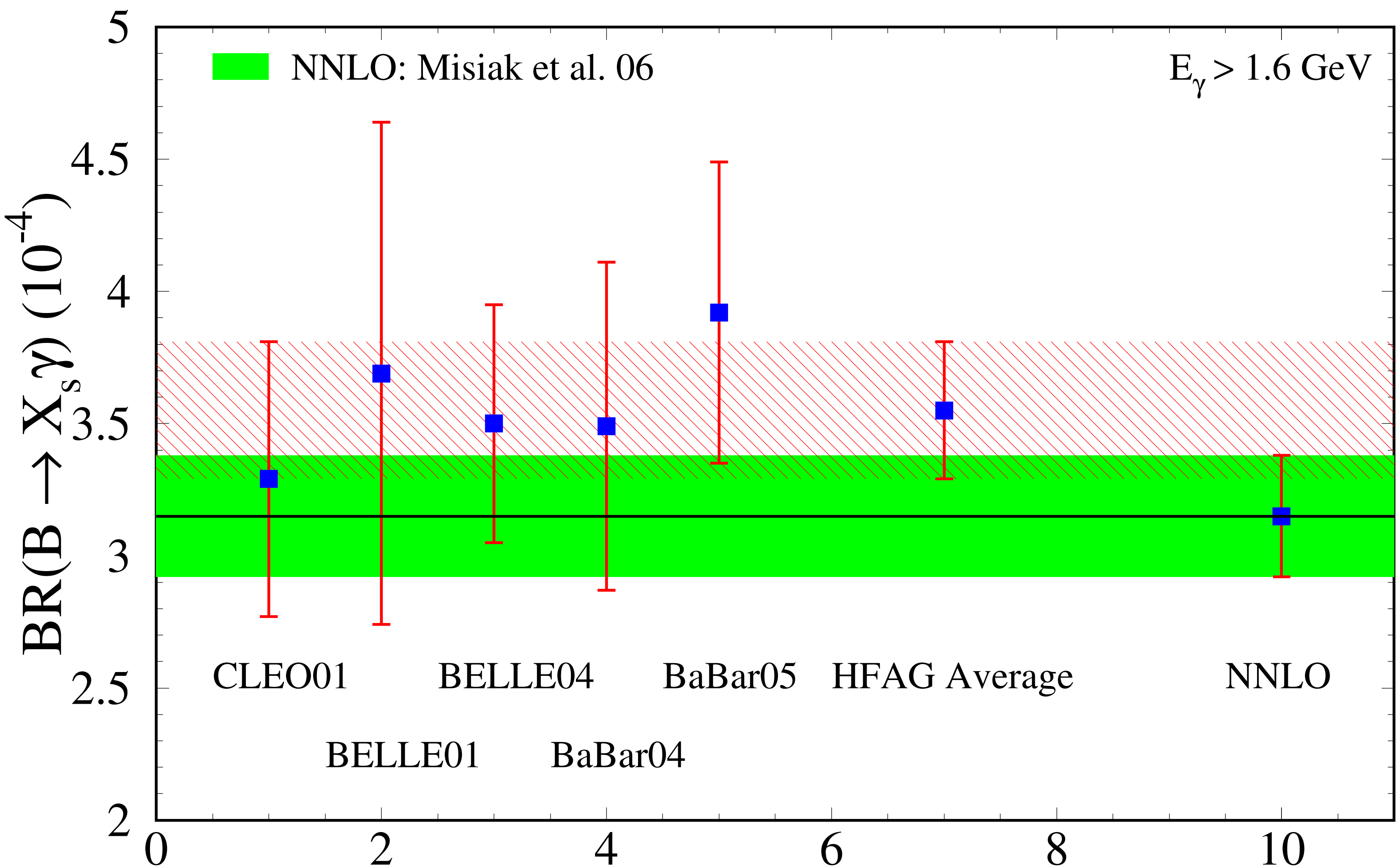
Decomposition of uncertainty:

non-pert., $\mathcal{O}\left(\frac{\Lambda}{m_b}\alpha_s\right)$	5%	(see, e.g., [Lee,Neubert,Paz'06])
parametric	3%	$\alpha_s(M_Z)$, $\mathcal{B}_{\text{SL}}^{\text{exp}}$, m_c, ...
m_c interpolation	3%	
higher order	3%	

NLO & Experiment

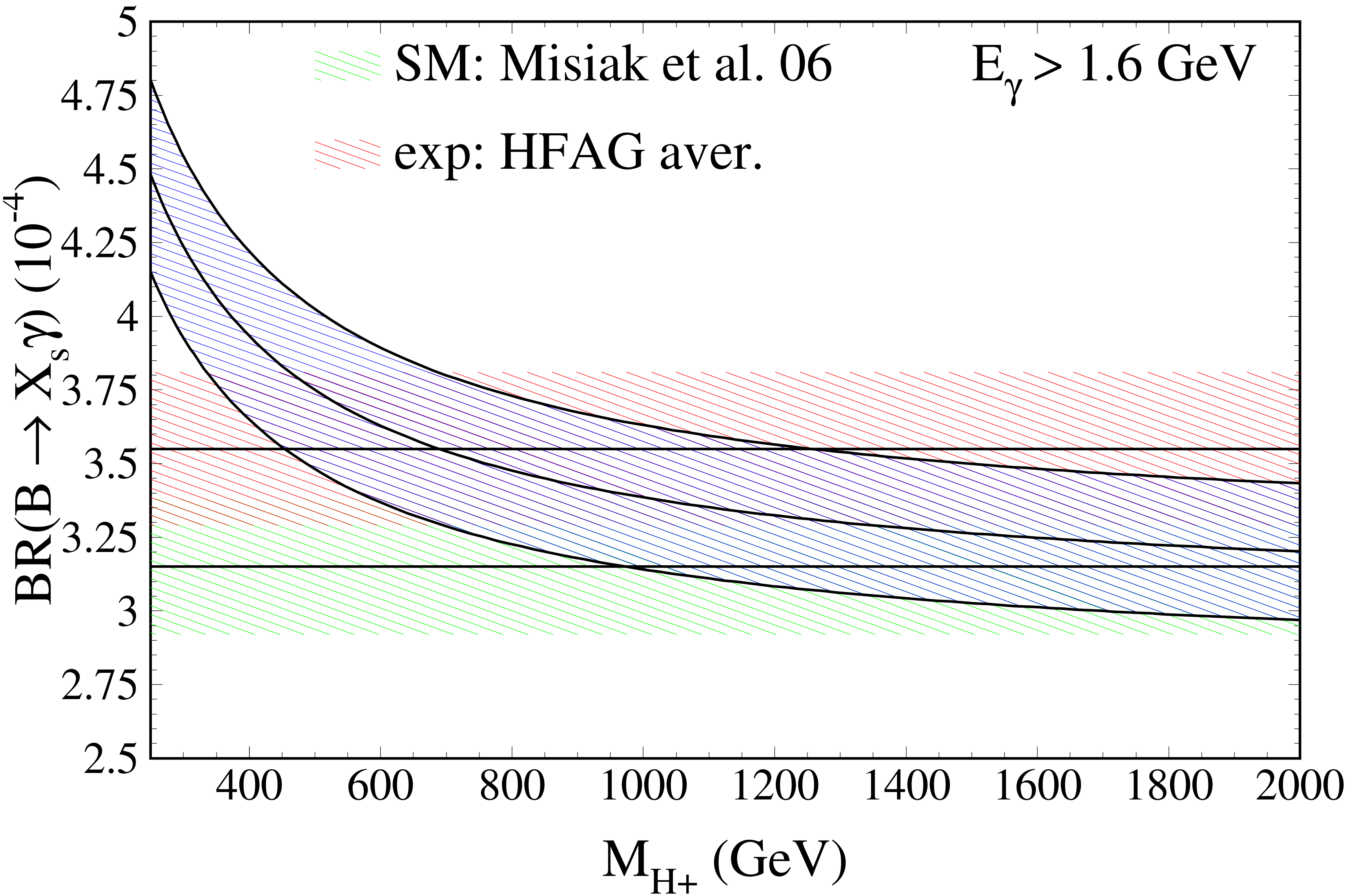


NNLO & Experiment



Very recently: -3% cutoff-related effect announced for $E_0 = 1.6$ GeV [Becher,Neubert'06]

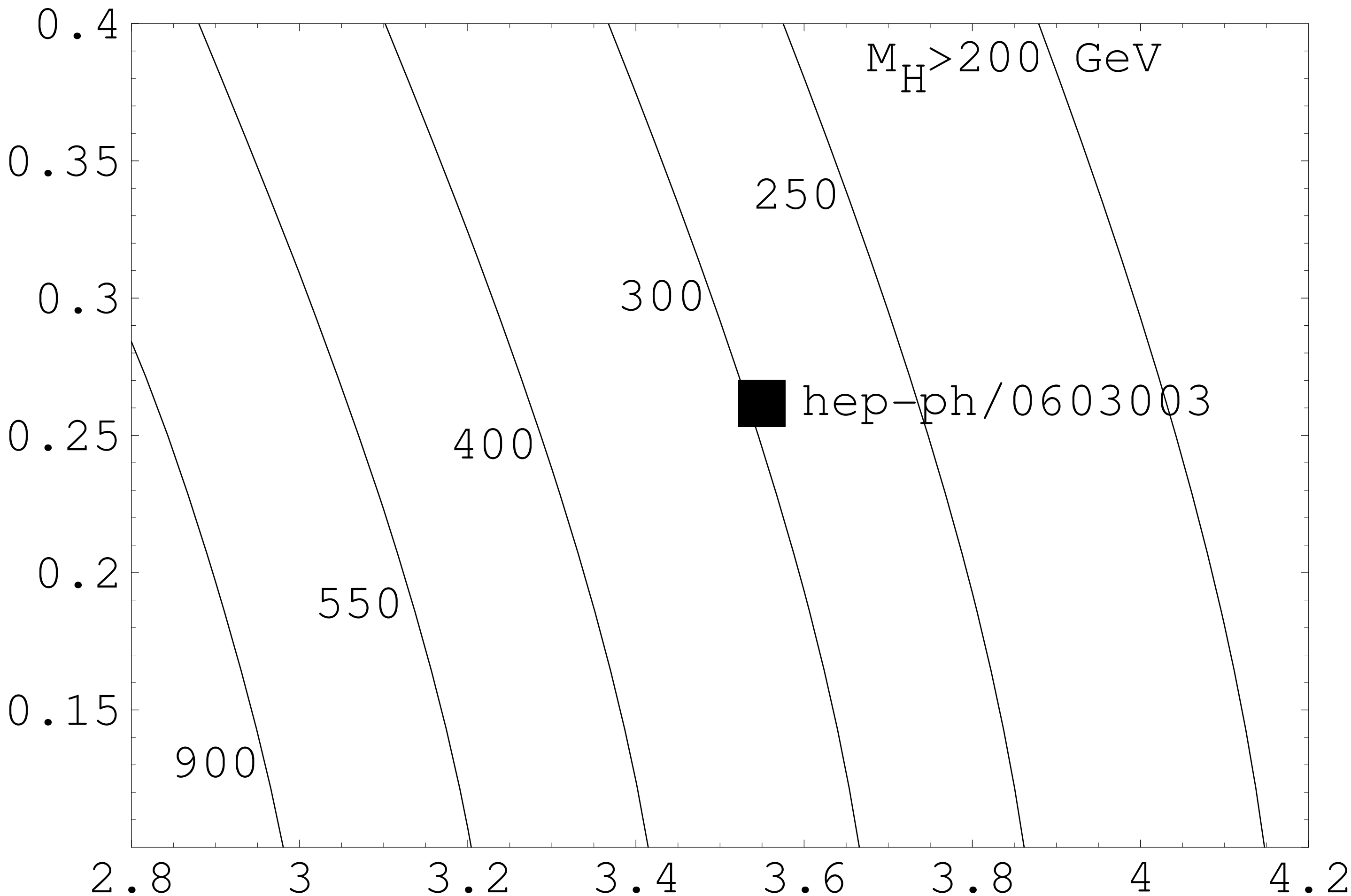
Bounds on M_H^+ (2HDM)



➪ data favour $M_{H^+} \sim 650$ GeV

Bound on M_H^+ (2)

Exp. error vs. exp. central value

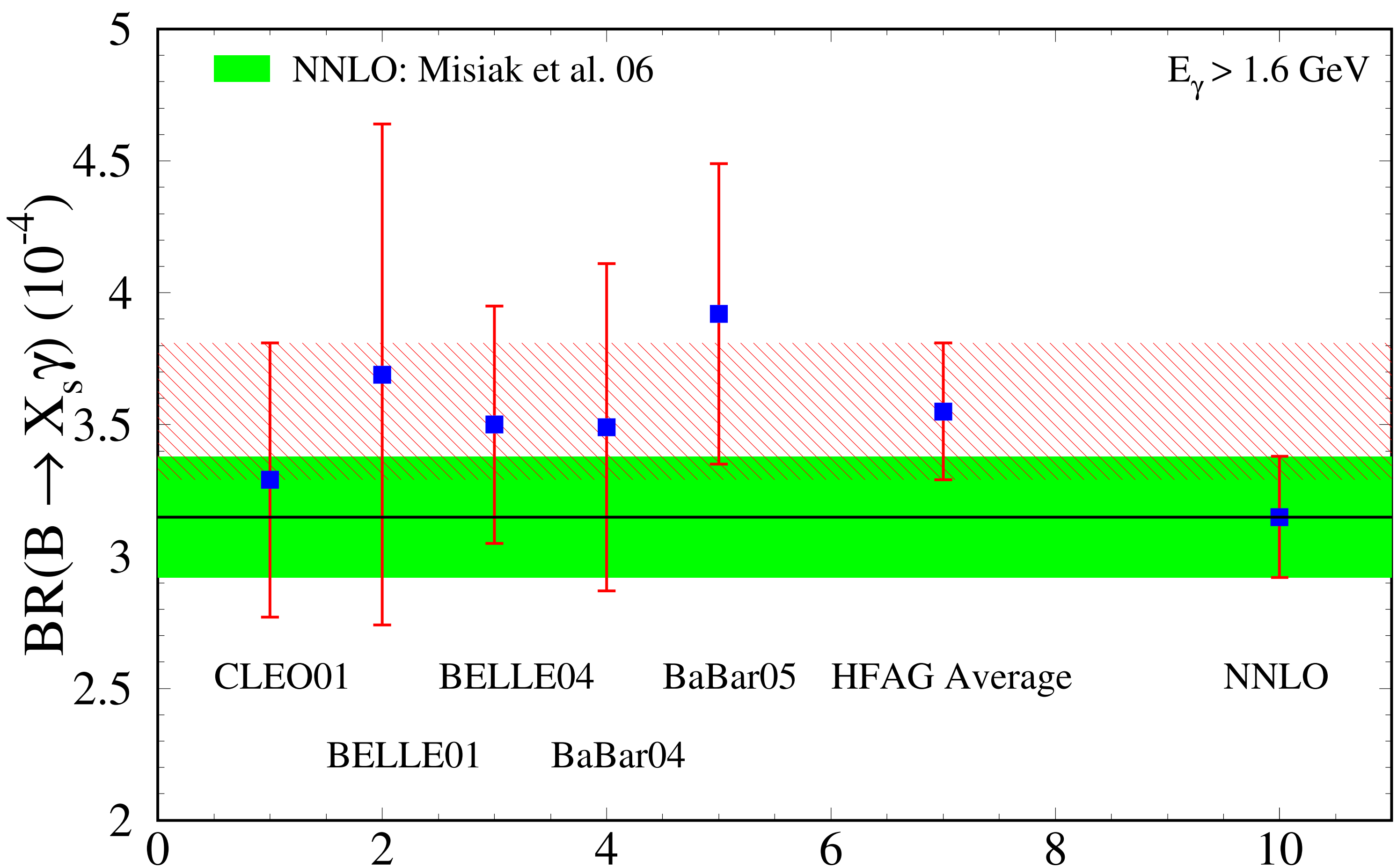


➩ $M_{H^+} > 295$ GeV 95% CL

Conclusions

- NNLO corrections for $\bar{B} \to X_s\gamma$:
  $$\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma > 1.6\text{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$
- dominant uncertainty:
  - non-perturbative: 5%
  - m_c interpolation: 3%
- $\sim 1.5\sigma$ deviation from experimental result
- 2HDM: $M_{H^+} > 295$ GeV 95% CL

$\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma>1.6\mathrm{GeV}}$ to NNLO



$$\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma>1.6\mathrm{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

[Misiak et al.'06], [Misiak,Steinhauser'06]