

**Precision calculations for**  
 **$H \rightarrow WW/ZZ \rightarrow 4$  fermions**  
**with PROPHECY4F**

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based on PRD74 (2006) 013004 [hep-ph/0604011] and MPP-2006-138 (in preparation)

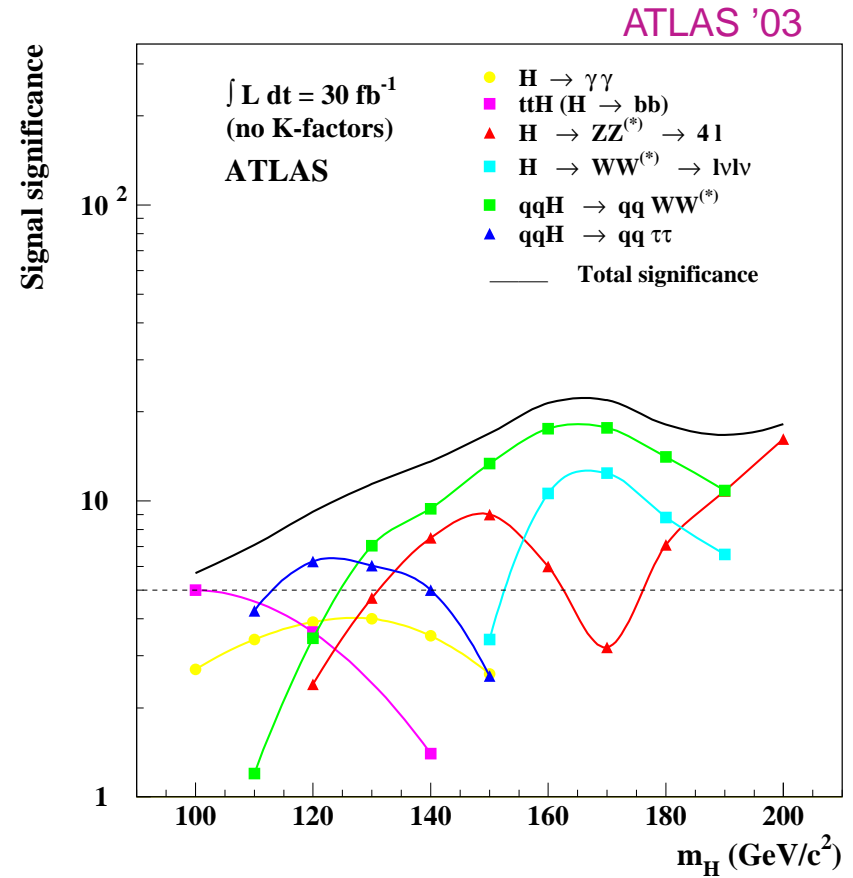
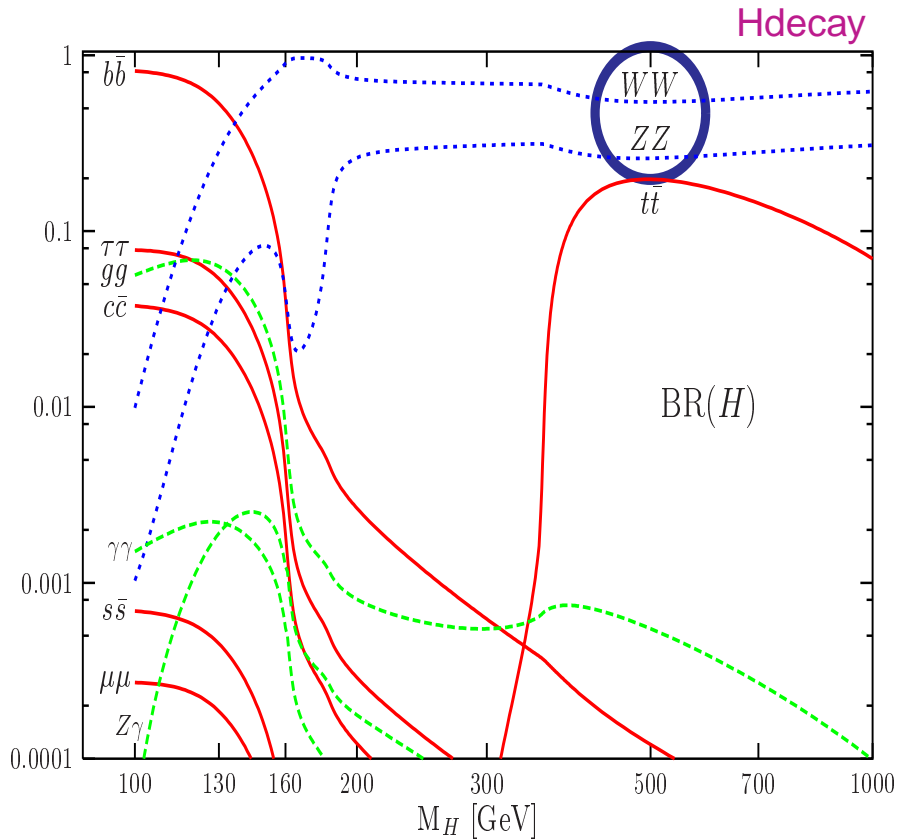


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- 1 Introduction — the decays  $H \rightarrow WW/ZZ \rightarrow 4$  fermions
- 2 Calculation of EW and QCD corrections
- 3 Selection of numerical results
- 4 Conclusions



# 1 Introduction — the decays $H \rightarrow WW/ZZ \rightarrow 4$ fermions



Importance of decays  $H \rightarrow WW^{(*)}/ZZ^{(*)}$  at the LHC:

- LHC:**
- most important Higgs decay channels for  $M_H \gtrsim 125 \text{ GeV}$
  - most precise determination of  $M_H$  via  $H \rightarrow ZZ \rightarrow 4l$  for  $M_H \gtrsim 130 \text{ GeV}$
- ILC:**
- measurements of branching ratios at per-cent level
  - full reconstruction of  $H \rightarrow WW$  in semileptonic / hadronic final states



## Theoretical description of $H \rightarrow WW^{(*)}/ZZ^{(*)}$ :

- previous work on partial decay widths:

- ◊  $\mathcal{O}(\alpha)$  corrections to  $H \rightarrow WW/ZZ$  with stable W's/Z's

Fleischer, Jegerlehner '81; Kniehl '91; Bardin, Vilenkii, Khristova '91

- ◊ lowest-order predictions for  $H \rightarrow WW^{(*)}/ZZ^{(*)}$

e.g. by Hdecay (Djouadi, Kalinowski, Spira '98)

- however: proper description of distributions required

- ◊ for the kinematical reconstruction of Z's, W's, and H  
(including radiative corrections, in particular  $\gamma$  radiation)

↪ invariant-mass distributions

- ◊ for the verification of spin 0 and CP parity of the Higgs boson

↪ angular and invariant-mass distributions

Nelson '88; Soni, Xu '93; Chang et al.'93;

Skjold, Osland '93; Barger et al.'93;

Arens, Sehgal '94; Buszello et al.'02; Choi et al.'03

⇒ Monte Carlo generator for  $H \rightarrow WW/ZZ \rightarrow 4f$  with corrections needed

## Recent work and work in progress:

- PROPHECY4F: generator for  $H \rightarrow WW/ZZ \rightarrow 4f$  with EW and QCD corrections

Bredenstein, Denner, S.D., Weber '06

- generator for  $H \rightarrow ZZ \rightarrow 4l$  with QED corrections

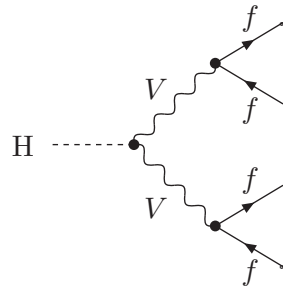
Carloni-Calame et al.



## 2 Calculation of EW and QCD corrections

### Survey of Feynman diagrams

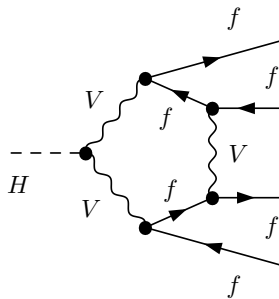
Lowest order:



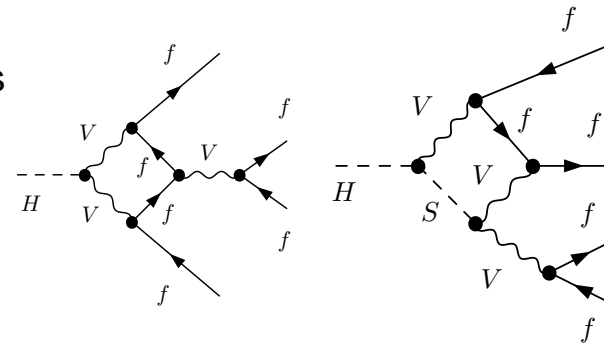
Electroweak  $\mathcal{O}(\alpha)$  corrections:

typical one-loop diagrams: # diagrams =  $\mathcal{O}(200-400)$

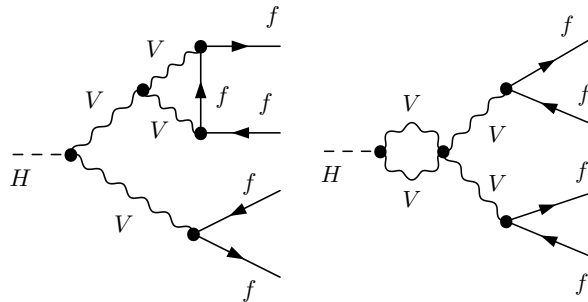
pentagons



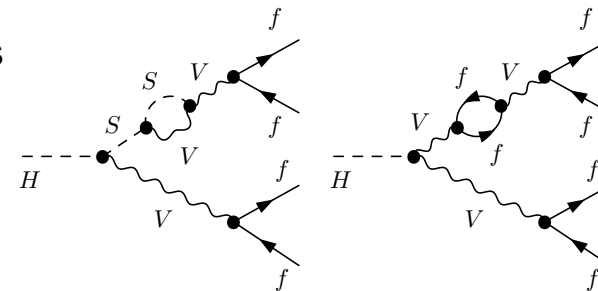
boxes



vertices



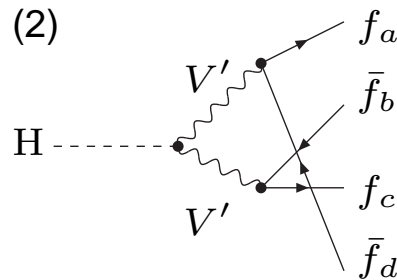
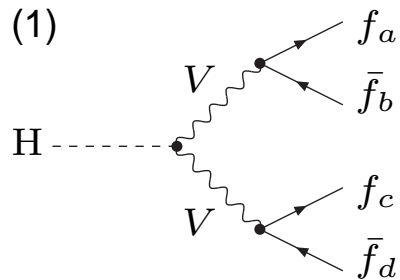
self-energies



+ photon bremsstrahlung (final-state radiation only)

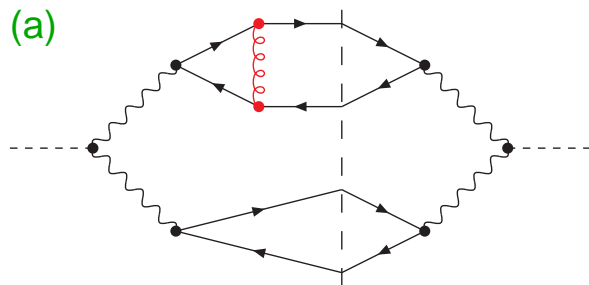
# QCD corrections to semileptonic or hadronic final states:

Possible Born diagrams:

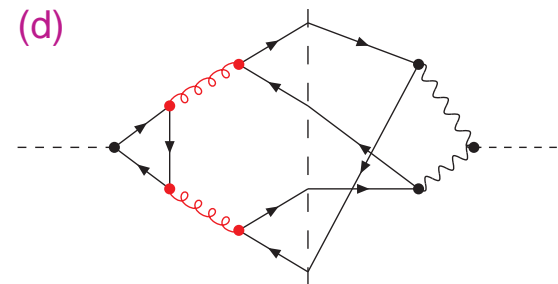
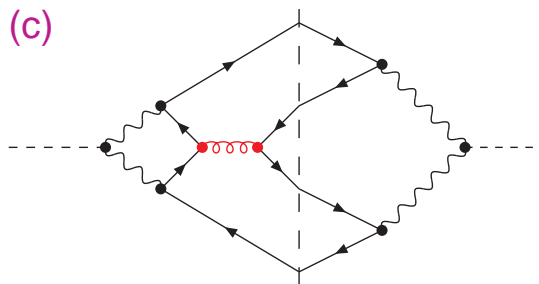
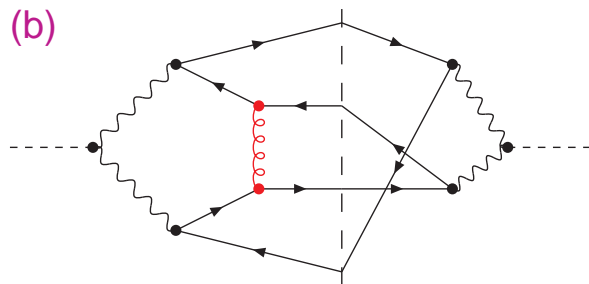


diagrams (2) only for  $q\bar{q}q\bar{q}$  and  $q\bar{q}q'\bar{q}'$  final states  
 ( $q'$  = weak-isospin partner of  $q$ )

Classification of QCD corrections into four categories: (typical diagrams shown)



(a) = correction to W/Z decays



(b,c,d) = corrections to interferences (only for  $q\bar{q}q\bar{q}$  and  $q\bar{q}q'\bar{q}'$  final states)



## Comments on the calculation of corrections

- Main complications in the loop calculation:

- ◇ gauge-invariant treatment of  $W$  and  $Z$  resonances

- ↪ “complex-mass scheme” Denner, S.D., Roth, Wieders '05

- ◇ numerical instabilities in Passarino–Veltman reduction of tensor integrals

- ↪ new reduction methods developed Denner, S.D. '05

New concepts already used in  $\mathcal{O}(\alpha)$  correction to  $e^+e^- \rightarrow 4f$

Denner, S.D., Roth, Wieders '05

- Features of PROPHECY4F:

- ◇  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\alpha_s)$  calculation to all channels  $H \rightarrow WW/ZZ \rightarrow 4f$

- ◇ improved Born approximation for simplified evaluation

- ◇ final-state radiation beyond  $\mathcal{O}(\alpha)$  via structure functions

- ◇ multi-channel Monte Carlo integration (checked by VEGAS)

- Berends, Kleiss, Pittau '94; Kleiss, Pittau '94

- ◇ still to be done: unweighted events, interface to parton showers



## Numerical evaluation of one-loop integrals

### Passarino–Veltman reduction of tensor to scalar integrals

- ↪ inverse Gram determinants of external momenta
- ↪ **serious numerical instabilities where  $\det(G) \rightarrow 0$**   
(at phase-space boundary but not only !)

Our solutions: [Denner, S.D., NPB734 \(2006\) 62 \[hep-ph/0509141\]](#)

- **1- and 2-point integrals** → stable direct calculation
- **3- and 4-point integrals** → two hybrid methods
  - Passarino–Veltman  $\oplus$  seminumerical method  $\oplus$  analytical special cases
  - Passarino–Veltman  $\oplus$  expansions in small Gram and other kin. determinants
- **5-(and 6-)point integrals**
  - ↪ stable reduction to lower-point integrals without Gram determinants

⇒ **Techniques ready for further applications**

(dim. regularization for IR singularities possible; complex masses supported)

### Practical experience

↪ **Power + reliability of techniques can only be assessed via non-trivial applications !**





## Checks:

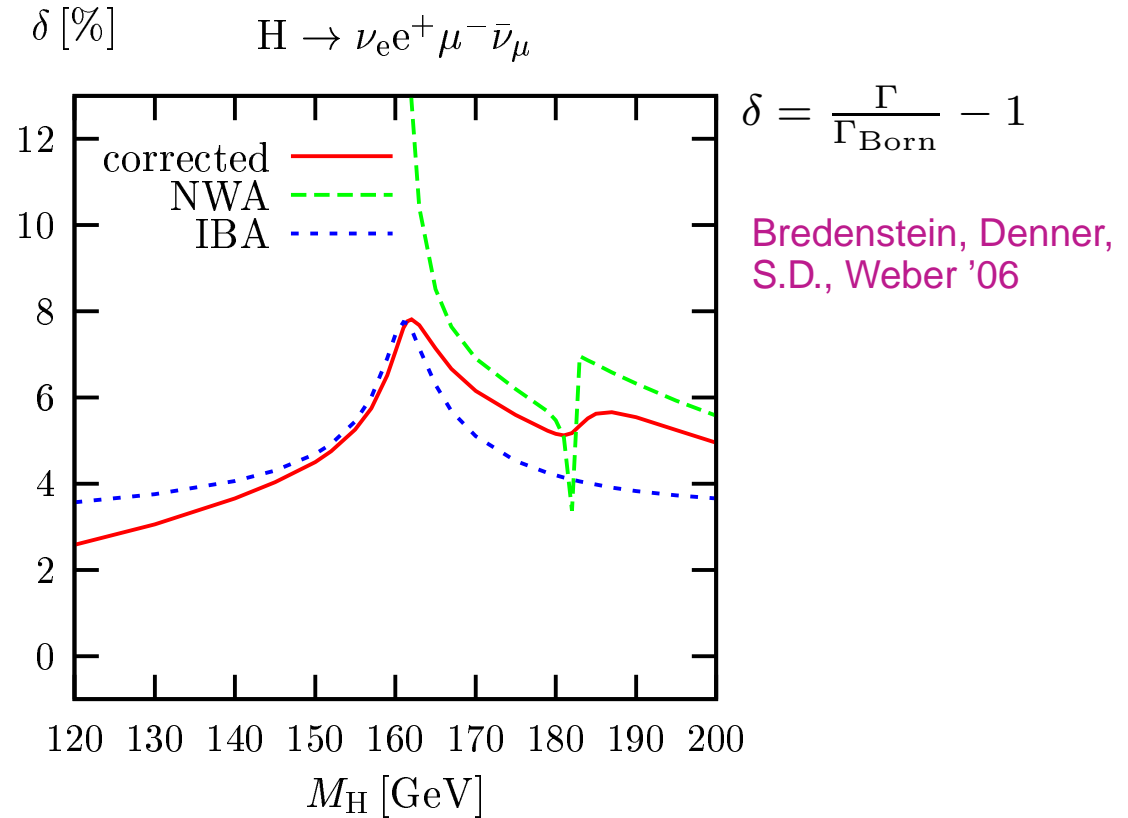
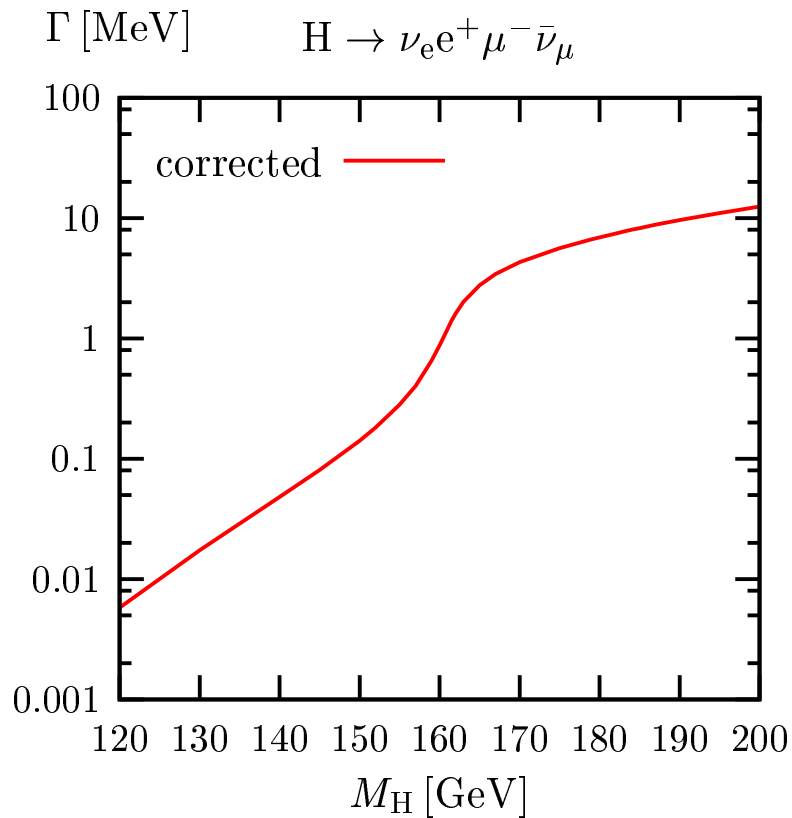
- **UV structure** of virtual corrections
  - ↪ independence of reference mass  $\mu$  of dimensional regularization
- **IR structure** of virtual + soft-photon corrections
  - ↪ independence of  $\ln m_\gamma$  ( $m_\gamma =$  infinitesimal photon mass)
- **mass singularities** of virtual + related collinear photonic corrections
  - ↪ independence of  $\ln m_{f_i}$  ( $m_{f_i} =$  small masses of external fermions)
- **gauge invariance** of amplitudes with  $\Gamma_W, \Gamma_Z \neq 0$ 
  - ↪ identical results in 't Hooft–Feynman and background-field gauge  
Denner, S.D., Weiglein '94
- **real corrections**
  - ↪ squared amplitudes compared with MADGRAPH  
Stelzer, Long '94
- **combination of virtual and real corrections**
  - ↪ identical results with two-cutoff slicing and dipole subtraction  
Catani, Seymour '96; S.D. '00; Bredenstein, S.D., Roth '05
- **two completely independent calculations of all ingredients !**



### 3 Selection of numerical results

#### 3.1 Leptonic final states

Partial decay width for  $H \rightarrow WW \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$   $G_\mu$ -scheme



NWA = narrow-width approximation

IBA = improved Born approximation

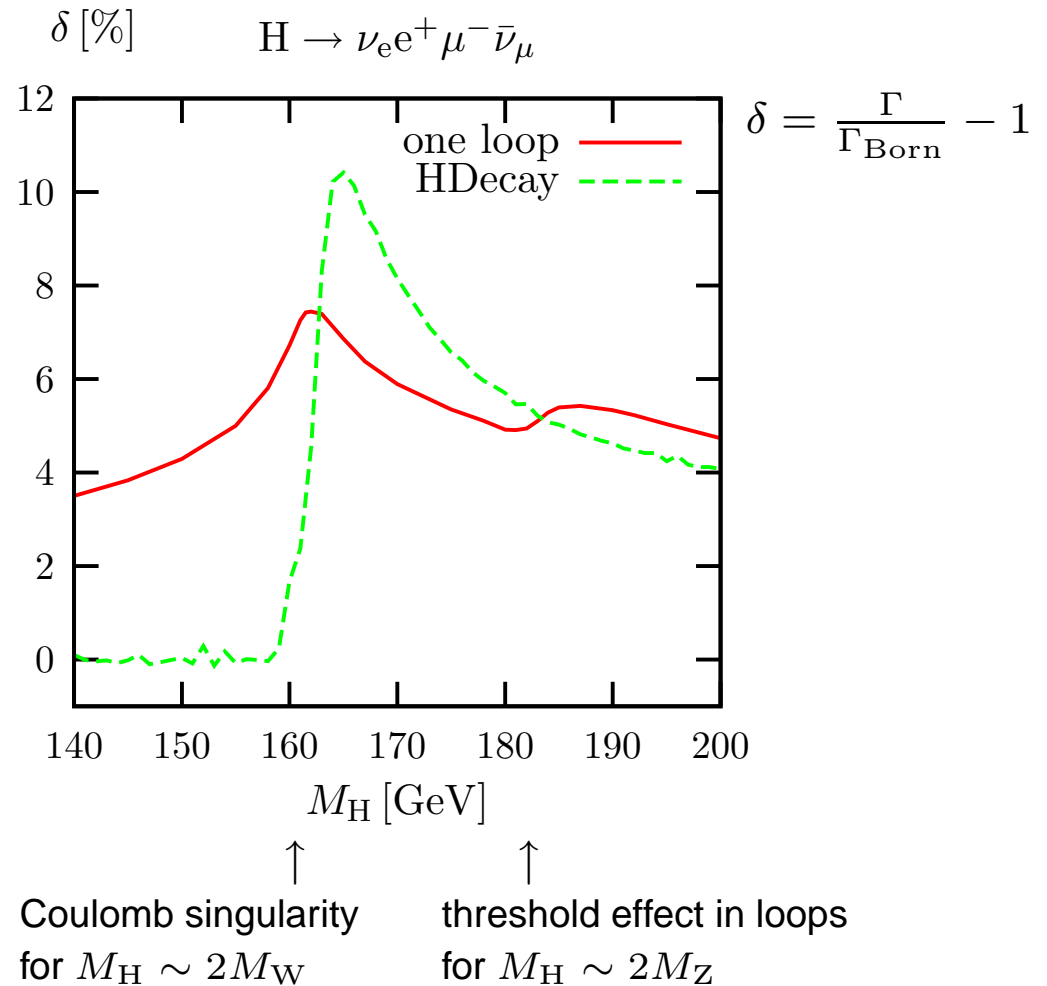
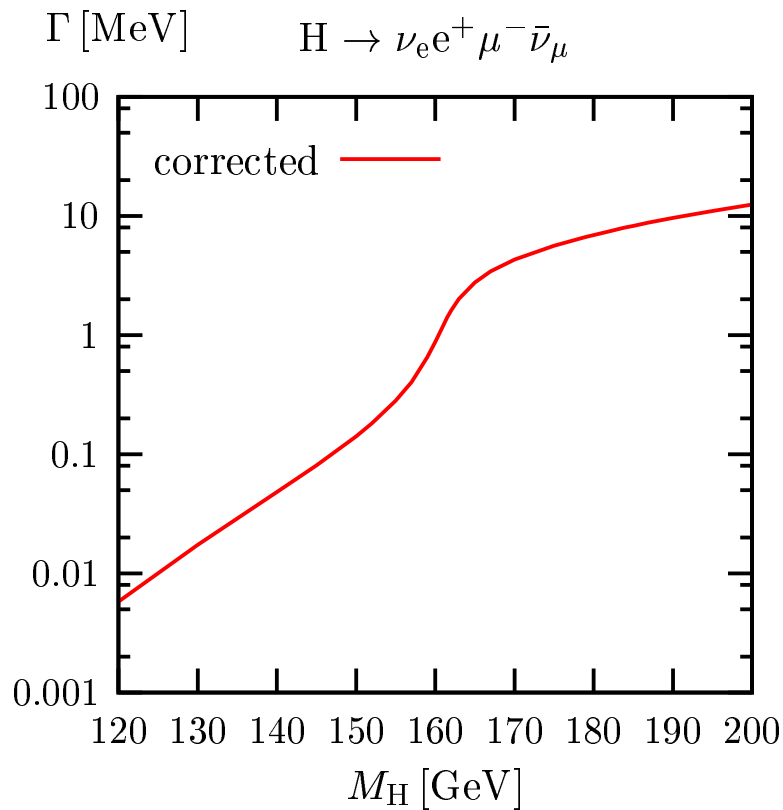
(Coulomb singularity, one fitting constant, leading effects for  $M_H, m_t \gg M_W$ )

↑  
Coulomb singularity  
for  $M_H \sim 2M_W$

↑  
threshold effect in loops  
for  $M_H \sim 2M_Z$

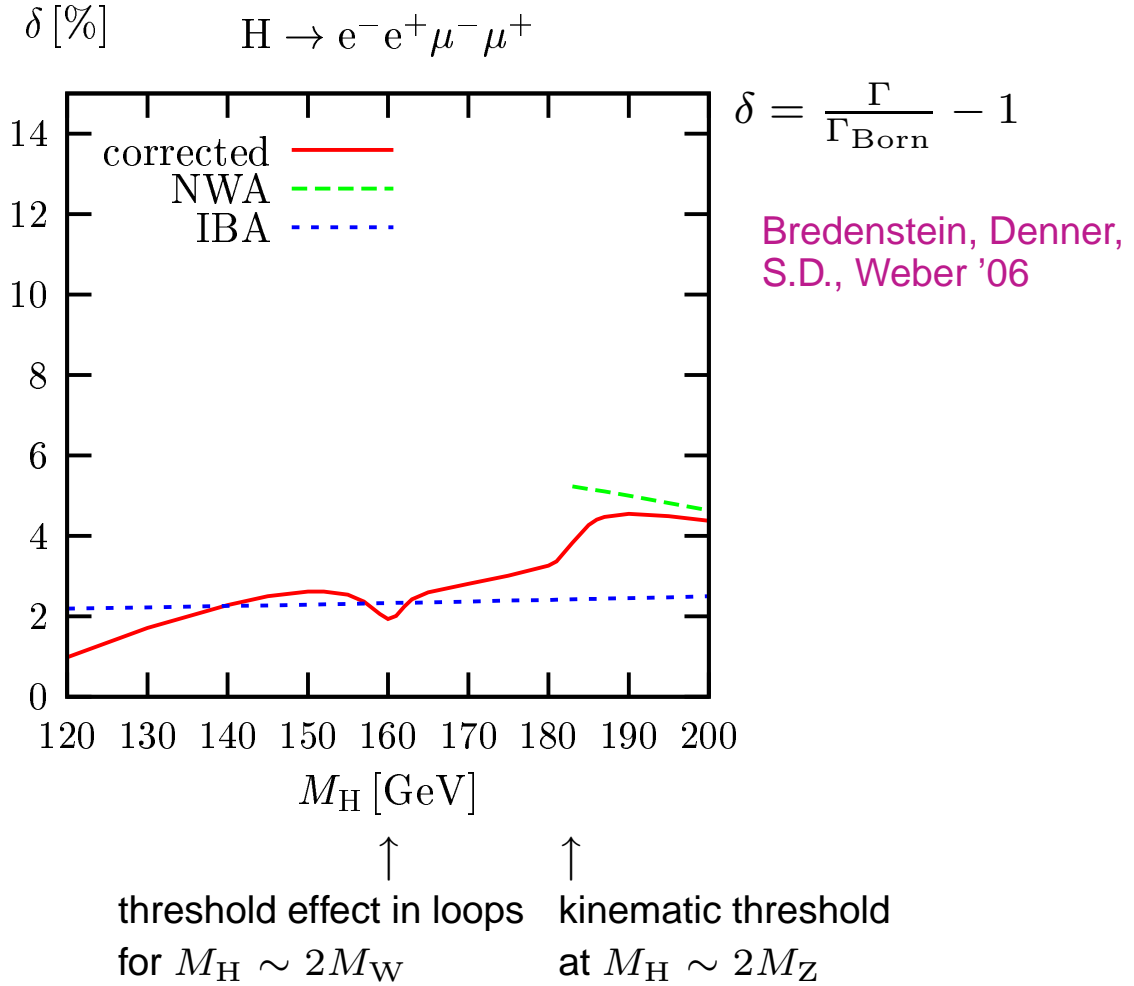
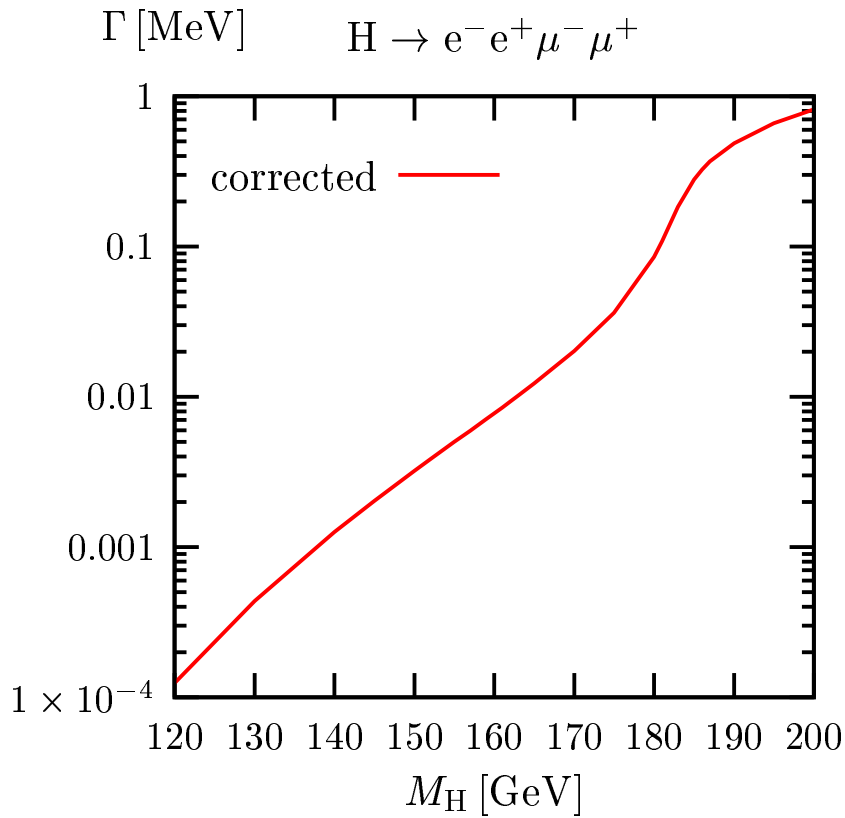


## Comparison with HDECAY

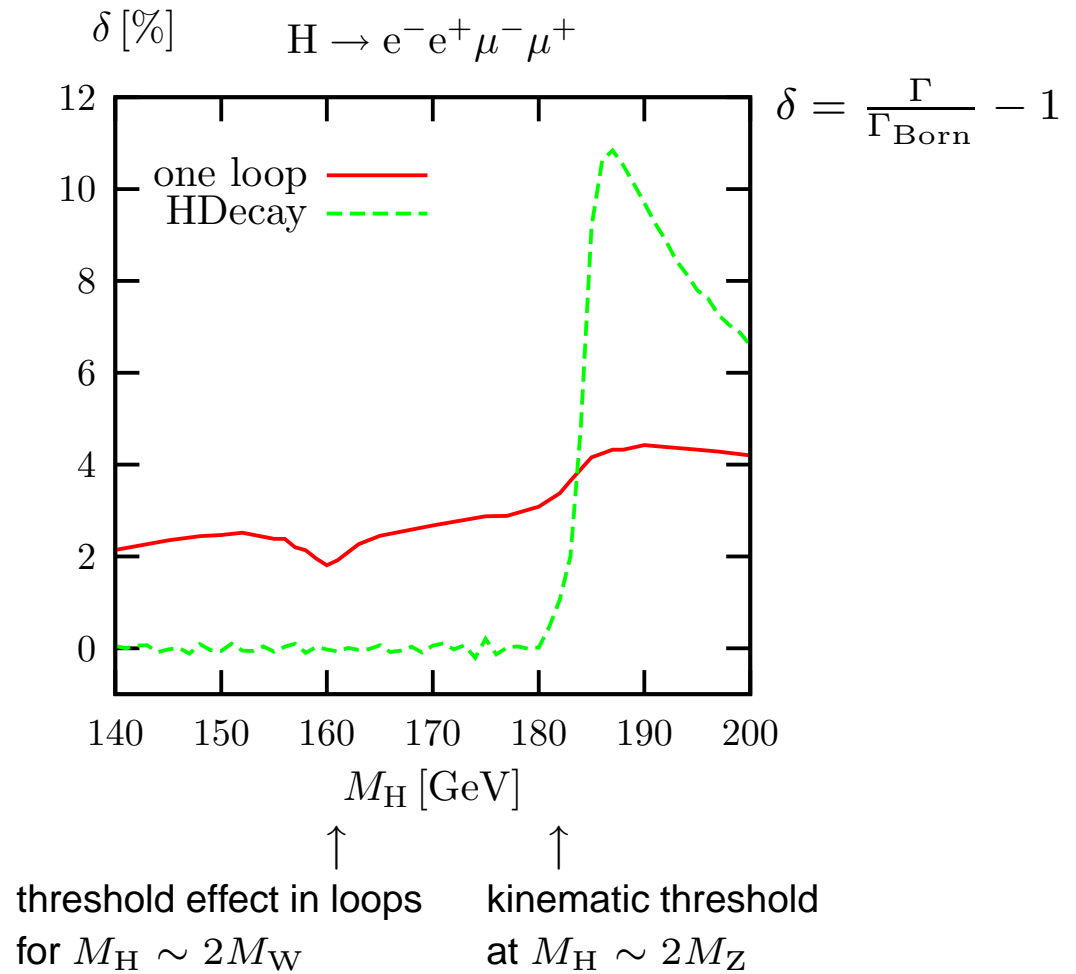
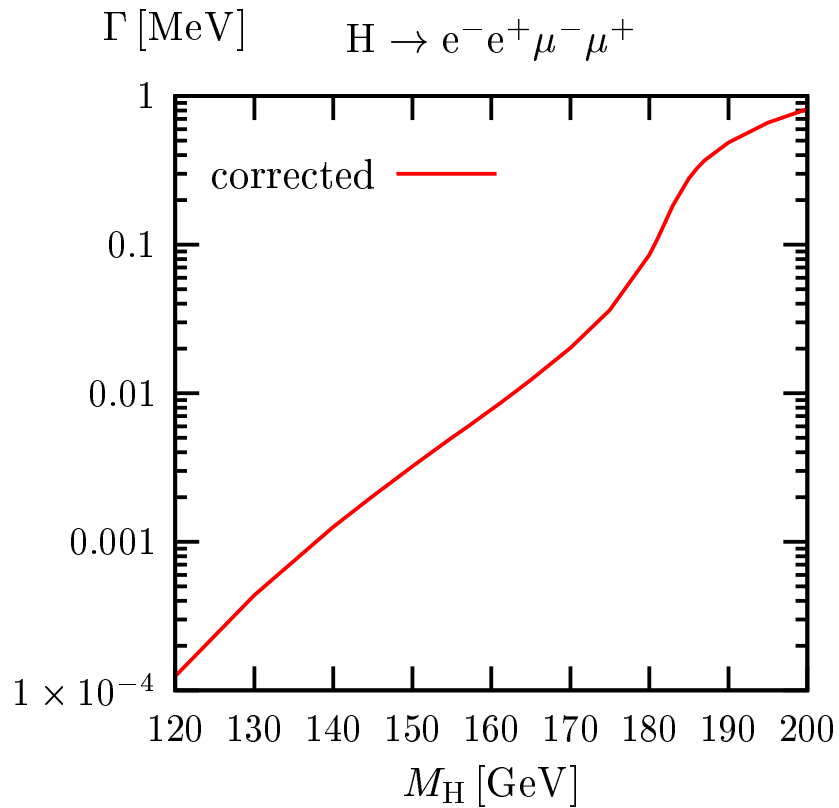


Note: peak structure in HDECAY is an artefact of the on-shell approximation above threshold.

# Partial decay width for $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$ $G_\mu$ -scheme



# Comparison with HDECAY



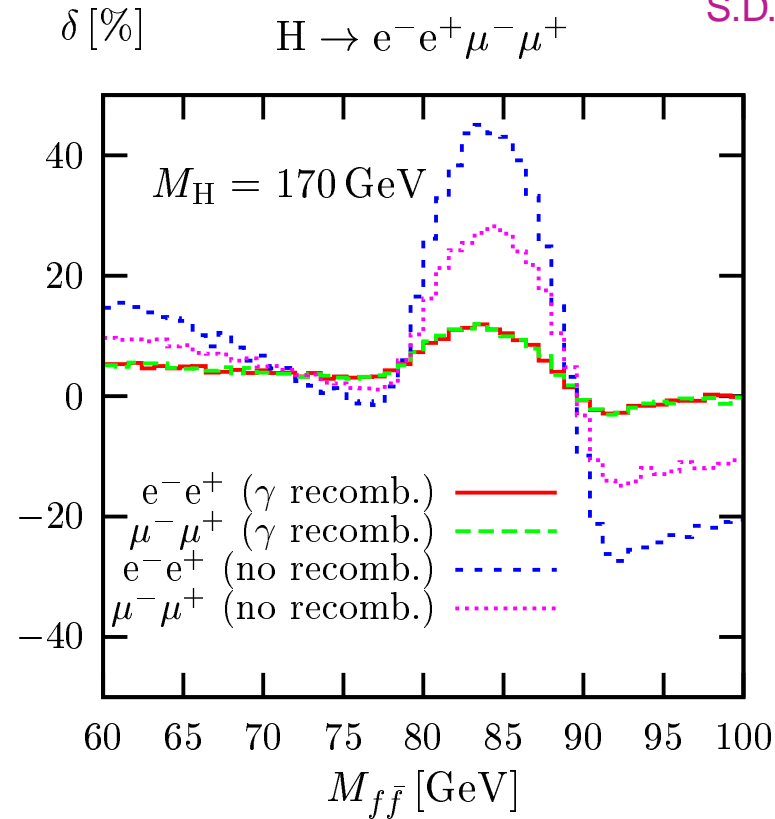
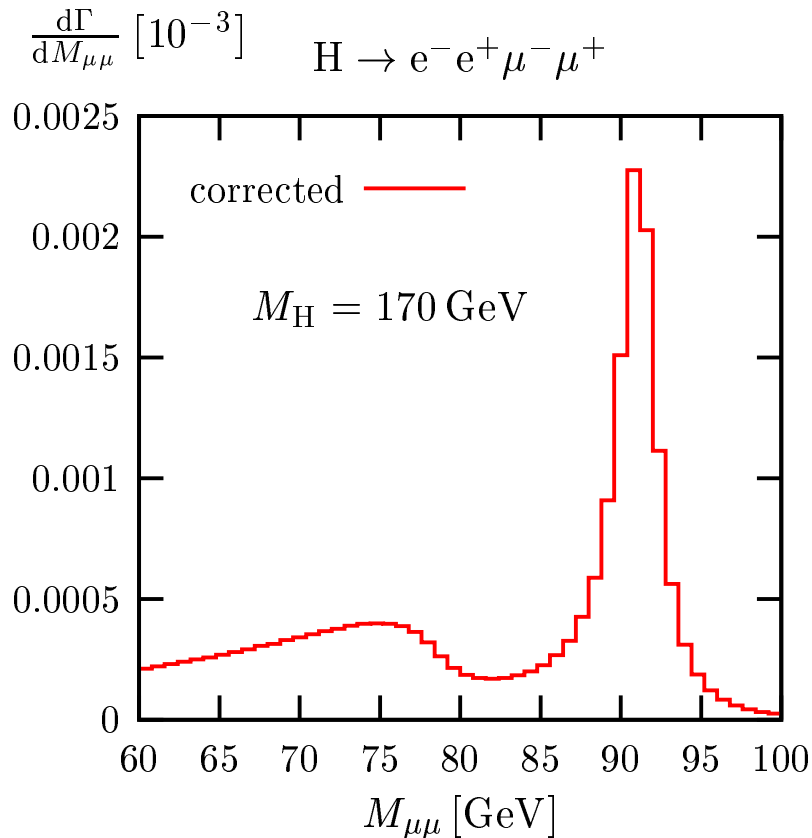
Note: peak structure in HDECAY is an artefact of the on-shell approximation above threshold.



# Invariant-mass distribution for the Z boson in $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$

$G_\mu$ -scheme

Bredenstein, Denner,  
S.D., Weber '06



$\gamma$  recombination if  $M_{e\gamma/\mu\gamma} < 5 \text{ GeV}$

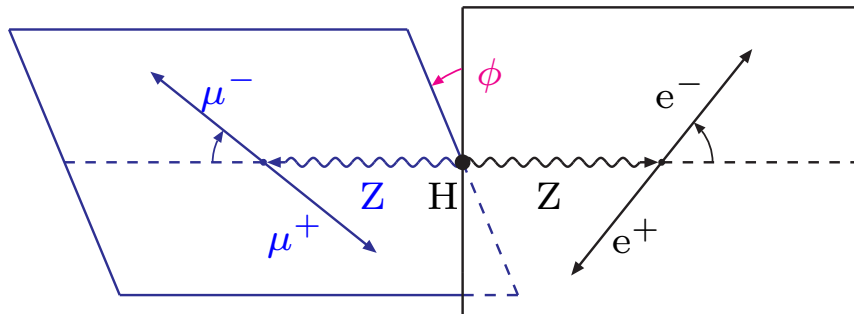
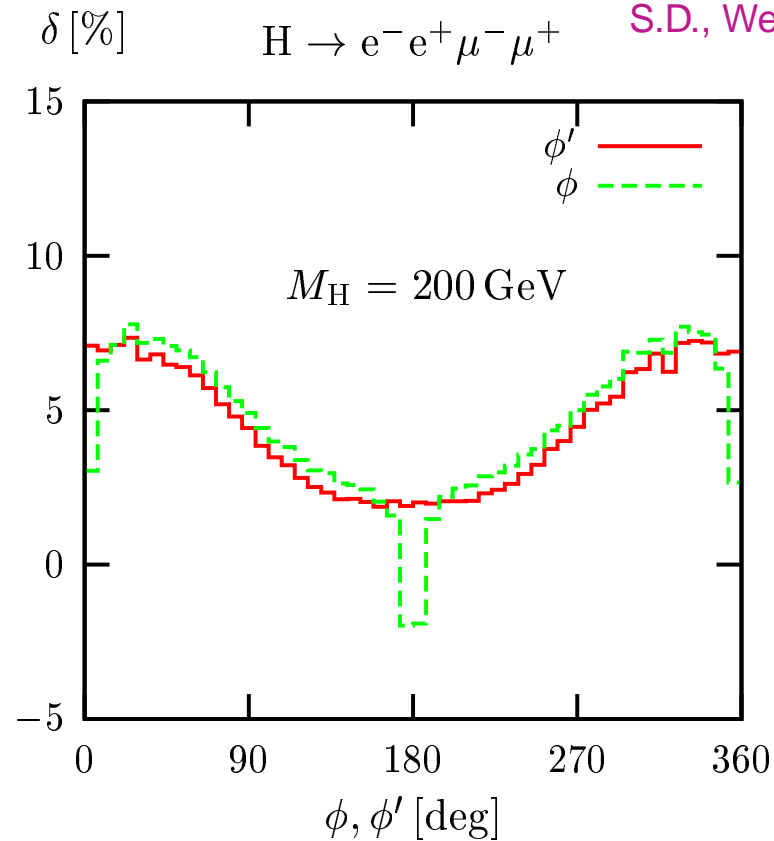
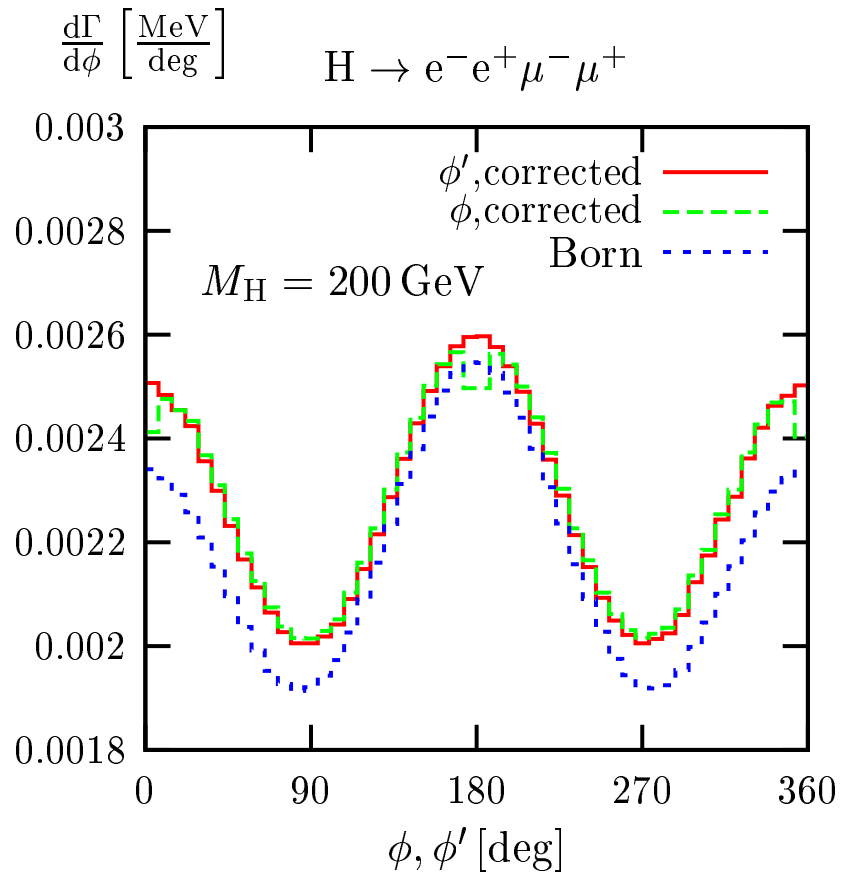
Large corrections from photon radiation in Z reconstruction



# Angle between decay planes for $H \rightarrow ZZ \rightarrow e^-e^+\mu^-\mu^+$

$G_\mu$ -scheme

Bredenstein, Denner,  
S.D., Weber '06



$$\cos \phi = \frac{(\mathbf{p}_{e^-e^+} \times \mathbf{p}_{e^-}) \cdot (-\mathbf{p}_{\mu^- \mu^+} \times \mathbf{p}_{\mu^-})}{|\mathbf{p}_{e^-e^+} \times \mathbf{p}_{e^-}| \cdot |-\mathbf{p}_{\mu^- \mu^+} \times \mathbf{p}_{\mu^-}|}$$

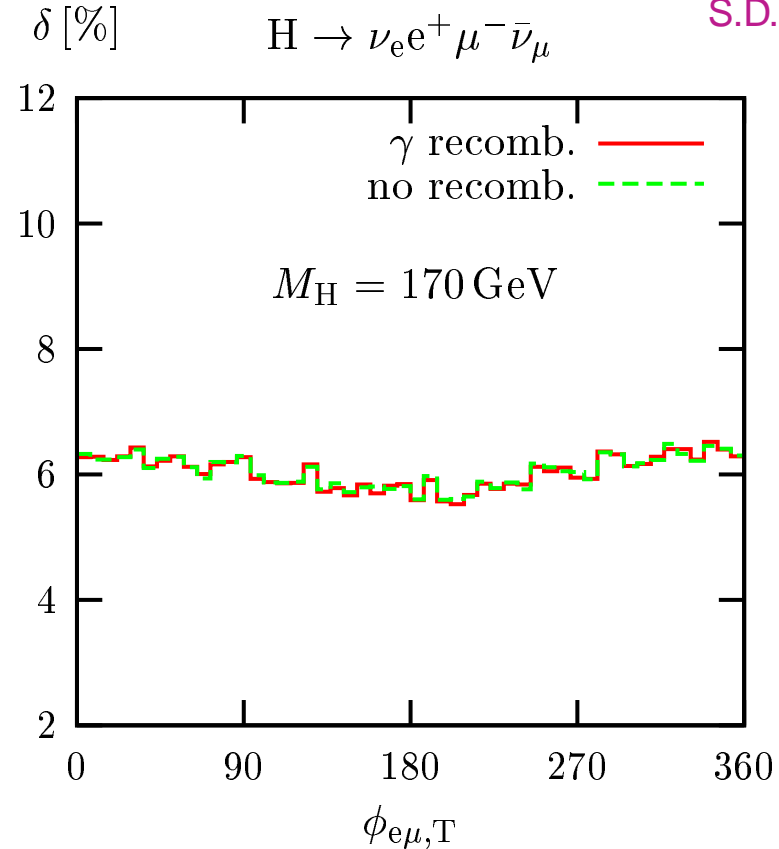
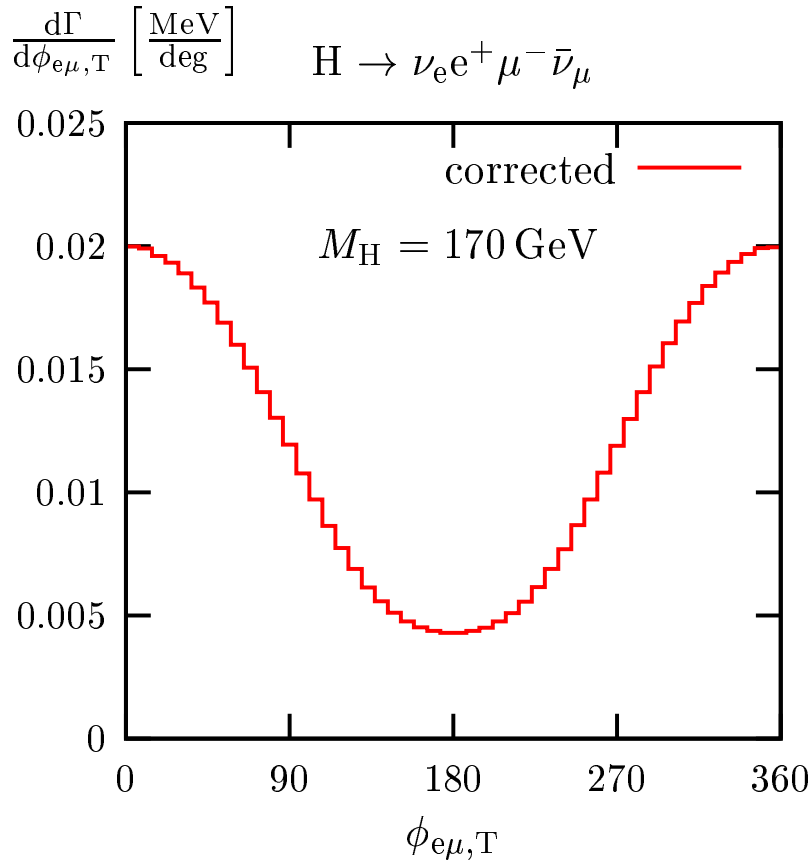
$$\cos \phi' = \frac{(\mathbf{p}_{e^-e^+} \times \mathbf{p}_{e^-}) \cdot (\mathbf{p}_{e^-e^+} \times \mathbf{p}_{\mu^-})}{|\mathbf{p}_{e^-e^+} \times \mathbf{p}_{e^-}| \cdot |\mathbf{p}_{e^-e^+} \times \mathbf{p}_{\mu^-}|}$$



# Distribution in the transverse angle between $e^+$ and $\mu^-$ in $H \rightarrow WW \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$

$G_\mu$ -scheme

Bredenstein, Denner,  
S.D., Weber '06



No significant distortion of shape by electroweak corrections

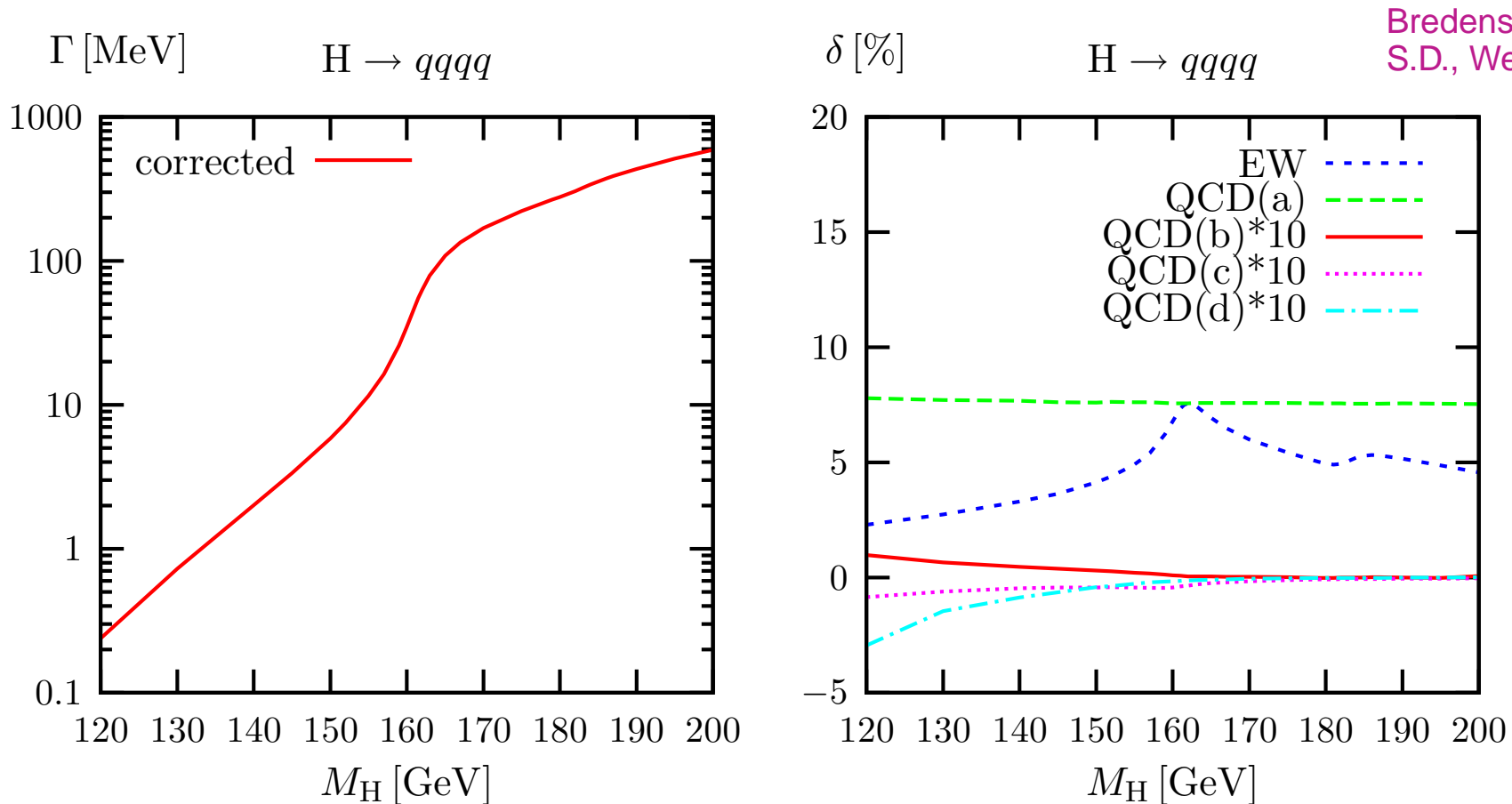




## 3.2 Semileptonic and hadronic final states

**EW corrections:** very similar for leptonic, semileptonic, and hadronic final states

**QCD corrections:** only type (a) significant (=corrections to W/Z decays)

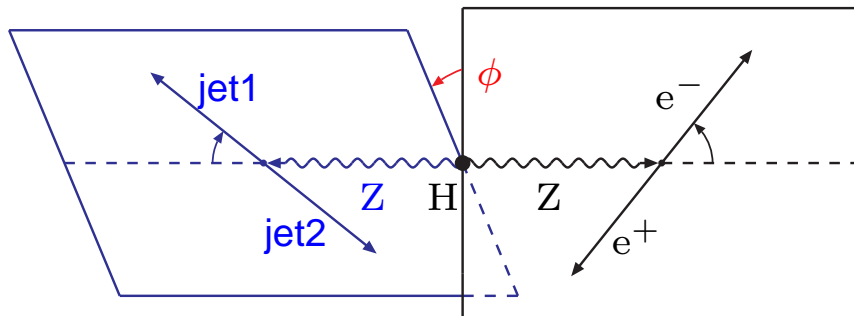
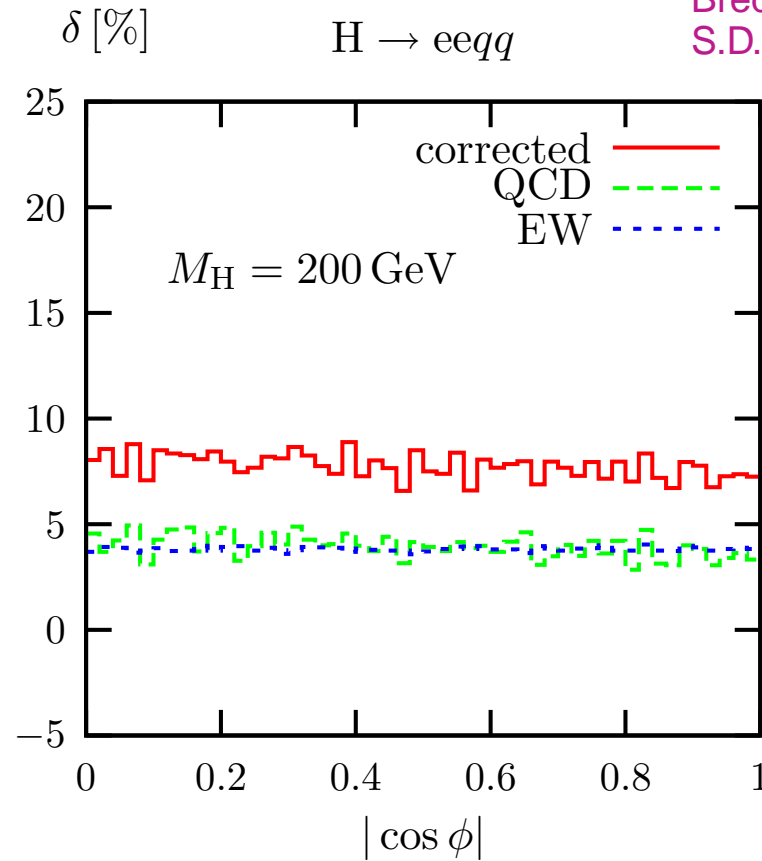
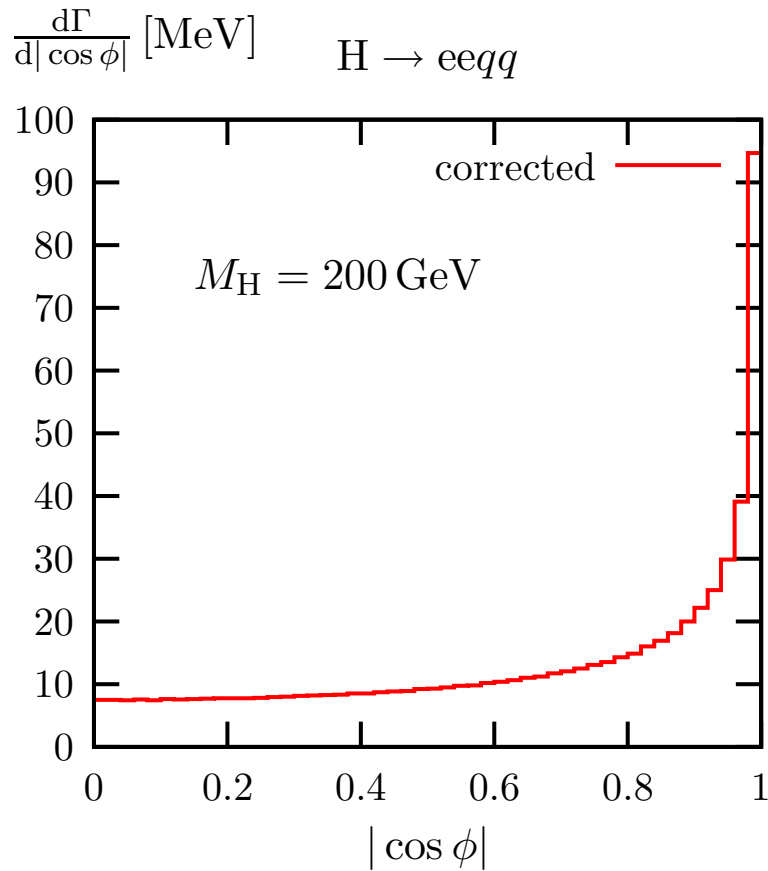


$$\delta_{\text{QCD}}^{\text{semileptonic}} \approx \frac{\alpha_s}{\pi} = 3.8\%, \quad \delta_{\text{QCD}}^{\text{hadronic}} \approx \frac{2\alpha_s}{\pi} = 7.6\%,$$



# Angle between decay planes for $H \rightarrow ZZ \rightarrow e^- e^+ qq$ $G_\mu$ -scheme

Bredenstein, Denner,  
S.D., Weber '06



$$\cos\phi = \frac{(\mathbf{p}_{2\text{jets}} \times \mathbf{p}_{e^-})(\mathbf{p}_{\text{jet1}} \times \mathbf{p}_{\text{jet2}})}{|\mathbf{p}_{2\text{jets}} \times \mathbf{p}_{e^-}| |\mathbf{p}_{\text{jet1}} \times \mathbf{p}_{\text{jet2}}|}$$



## 4 Conclusions

Higgs decays  $H \rightarrow WW/ZZ \rightarrow 4f$  are important for

- Higgs discovery at the LHC and precision Higgs studies at the ILC
- confirmation of Higgs quantum numbers (spin, CP) via differential distributions

**NEW:** PROPHECY4F – a generator for  $H \rightarrow WW/ZZ \rightarrow 4f$  including

- full  $\mathcal{O}(\alpha)$  EW and  $\mathcal{O}(\alpha_s)$  QCD corrections
  - ◊ W and Z resonances treated within the complex-mass scheme
  - ◊ tensor reduction numerically stabilized via seminumerical or expansion methods
- universal corrections beyond  $\mathcal{O}(\alpha)$  (FSR via structure functions, large- $M_H$  effects)

First results of PROPHECY4F on  $H \rightarrow WW/ZZ \rightarrow 4f$

- partial decay widths: EW corrections of  $\mathcal{O}(8\%)$  for  $M_H \lesssim 500$  GeV  
(reproduced by a simple improved Born approximation within  $\lesssim 2\%$  for  $M_H \lesssim 400$  GeV)
- angular distributions: EW corrections of  $\mathcal{O}(5-10\%)$  distort shapes
- invariant-mass distributions of W's and Z's:  
EW corrections of some 10% distort shapes (depend on inclusiveness of  $\gamma$  radiation)
- QCD corrections can be associated with W/Z decay (interference effects negligible)

