Two-Loop Bhabha Scattering and Luminosity Determination at e^+e^- Colliders

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- *High-Energy Large-Angle Scattering*
  (Luminosity spectrum at ILC)

  required accuracy 5 pm *available accuracy* **?**

Born approximation

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\Omega} = \frac{\alpha^2}{s}\left(\frac{1-x+x^2}{x}\right)^2 + \mathcal{O}(m_e^2/s), \qquad x = \frac{1-\cos\theta}{2}$$

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- Phenomenologycally interesting:
  - *High energy region* $s,\ t,\ u \gg m_e^2$
  - *Small angle Bhabha scattering* $t \ll s,\ x \sim 0$
  - *Large angle Bhabha scattering* $t \sim s,\ x \sim 1$

Radiative corrections

Observable $=$ Virtual corrections $+$ Bremsstrahlung

1. *Split real radiation into "soft" and "hard" by* $\mathcal{E}_{cut} \ll m_e$
2. *Compute the virtual+soft real part analytically*
3. *Compute the hard real part with actual experimental cuts by means of Monte Carlo*

Structure of the virtual + soft corrections

- First order

$$\frac{d\sigma^{(1)}}{d\sigma^{(0)}} = \delta_1^{(1)} \ln\left(\frac{s}{m_e^2}\right) + \delta_0^{(1)} + \mathcal{O}(m_e^2/s)$$

$$\delta_1^{(1)} = 4\ln\left(\frac{\mathcal{E}_{cut}}{\mathcal{E}}\right) + \ldots, \qquad \mathcal{E} = \sqrt{s}/2$$

- Second order

$$\frac{d\sigma^{(2)}}{d\sigma^{(0)}} = \delta_2^{(2)} \ln^2\left(\frac{s}{m_e^2}\right) + \delta_1^{(2)} \ln\left(\frac{s}{m_e^2}\right) + \delta_0^{(2)} + \mathcal{O}(m_e^2/s)$$

$$\delta_2^{(2)} = 8\ln^2\left(\frac{\mathcal{E}_{cut}}{\mathcal{E}}\right) + 12\ln\left(\frac{\mathcal{E}_{cut}}{\mathcal{E}}\right) + \ldots$$

$$\delta_1^{(2)} = -16\left[1 + \ln\left(\frac{1-x}{x}\right)\right] \ln^2\left(\frac{\mathcal{E}_{cut}}{\mathcal{E}}\right) + \ldots$$

Review of two-loop calculations

- Small angle scattering ➪ limit $x \to 0$
  - *Only form factor corrections survive*

    V.S. Fadin, E.A. Kuraev, L.N. Lipatov, N.P. Merenkov, L. Trentadue
  - *Dominant part is included into BHLUMI MC*

    S. Jadach, W. Placzek, E. Richter-Wąs, B.F.L. Ward, Z. Wąs

Review of two-loop calculations

- Large angle scattering ➪ arbitrary x
  - *Full massless result for virtual correction*

    Z. Bern, L. Dixon, A. Ghinculov
  - *Logarithmic corrections, leading order in* m_e^2/s

    E.W. Glover, J.B. Tausk, J.J. van der Bij
  - $m_e \neq 0$, *fermion loop insertions*

    R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J.J. van der Bij
  - *Photonic corrections, leading order in* m_e^2/s

    A. Penin

Infrared matching

❶ For a given amplitude $\mathcal{A}$ construct an auxilary amplitude $\bar{\mathcal{A}}$ with the same structure of IR singularities

❷ Compute the matching term for $\lambda,\ m_e = 0$

$$\delta\mathcal{A} = \left[\mathcal{A}(\epsilon) - \bar{\mathcal{A}}(\epsilon)\right]_{\epsilon\to 0}$$

❸ Compute the auxilary amplitude $\bar{\mathcal{A}}$ for $\lambda,\ m_e \to 0$

❹ The amplitude $\mathcal{A}$ in the limit $\lambda,\ m_e \to 0$ is given by

$$\mathcal{A}(\lambda, m_e)\Big|_{\lambda,\ m_e\to 0} = \bar{\mathcal{A}}(\lambda, m_e)\Big|_{\lambda,\ m_e\to 0} + \delta\mathcal{A}$$

How to construct $\bar{\mathcal{A}}$?

- Factorization/exponentiation of IR singularities
- Nonrenormalization of IR exponent in quenched QED

(D.R. Yennie, S.C. Frautschi, H. Suura; A. Mueller; J. Collins; A.Sen; ...)

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➥ The auxilary amplitude

$$\bar{\mathcal{A}}^{(2)} = \frac{1}{2}\left(\mathcal{A}^{(1)}\right)^2 + 2\left[\mathcal{F}^{(2)} - \frac{1}{2}\left(\mathcal{F}^{(1)}\right)^2\right]$$

Result *(page 1 of 2)*

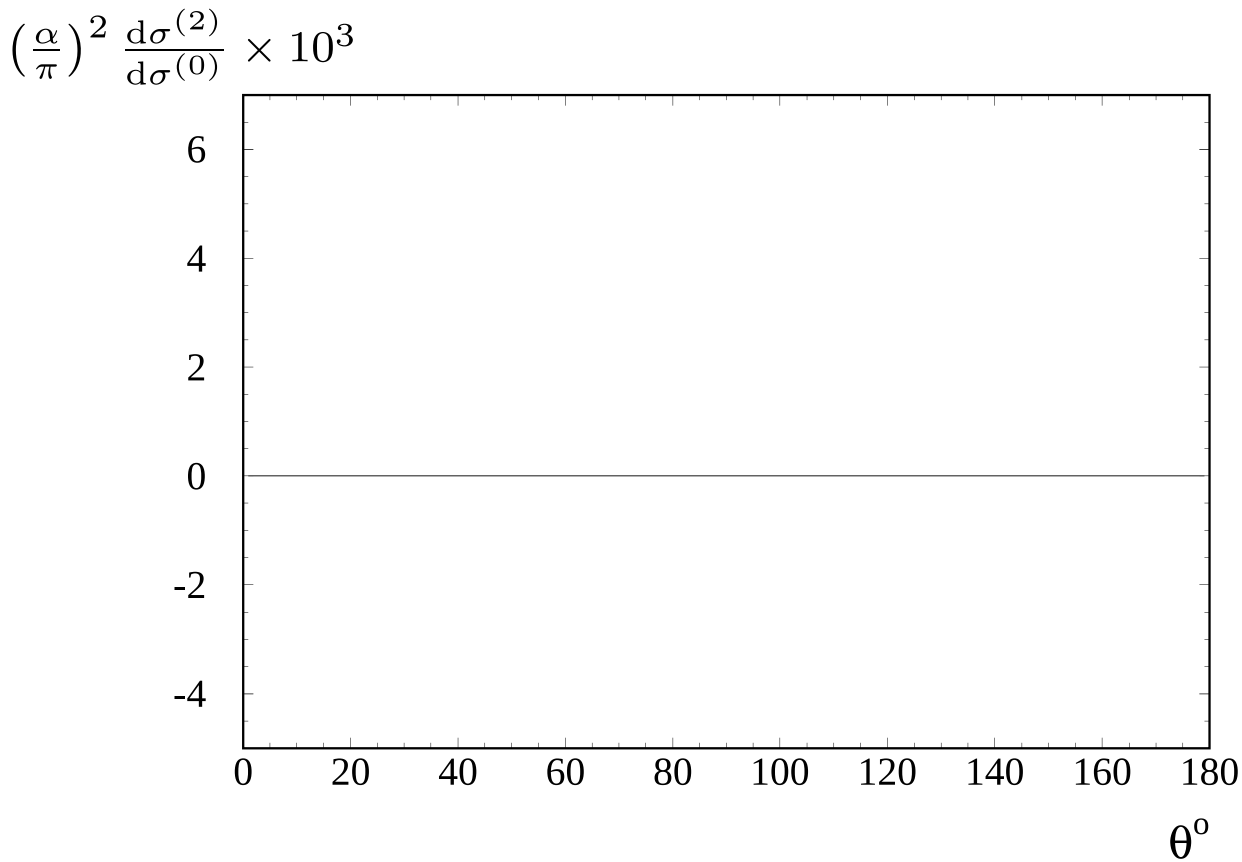
$$
\begin{aligned}
\delta_0^{(2)} = {} & 8\mathcal{L}_\varepsilon^2 + \left(1-x+x^2\right)^{-2} \Big[\Big(\Big(\frac{4}{3} - \frac{8}{3}x - x^2 + \frac{10}{3}x^3 - \frac{8}{3}x^4\Big) \pi^2 + \left(-12+16x-18x^2+6x^3\right) \ln(x) \\
& + \left(2x+2x^3\right) \ln(1-x) + \left(-3x+x^2+3x^3-4x^4\right) \ln^2(x) + \left(-8+16x-14x^2+4x^3\right) \ln(x) \\
& \times \ln(1-x) + \left(4-10x+14x^2-10x^3+4x^4\right) \ln^2(1-x) + \left(1-x+x^2\right)^2 (16+8\mathrm{Li}_2(x) \\
& -8\mathrm{Li}_2(1-x)) \Big] \mathcal{L}_\varepsilon + \frac{27}{2} - 2\pi^2 \ln(2) + \left(1-x+x^2\right)^{-2} \Big(\Big(\frac{83}{24} - \frac{125}{24}x + \frac{13}{4}x^2 + \frac{19}{24}x^3 - \frac{25}{24}x^4\Big) \\
& \times\pi^2 + \Big(-9 + \frac{43}{2}x - 34x^2 + 22x^3 - 9x^4\Big) \zeta(3) + \Big(-\frac{11}{90} - \frac{5}{24}x + \frac{29}{180}x^2 + \frac{23}{180}x^3 - \frac{49}{480}x^4\Big) \pi^4 \\
& + \Big[-\frac{93}{8} + \frac{231}{16}x - \frac{279}{16}x^2 + \frac{93}{16}x^3 + \Big(-\frac{3}{2} + \frac{13}{4}x - \frac{7}{12}x^2 - \frac{11}{8}x^3\Big) \pi^2 + \Big(12 - 12x + 8x^2 \\
& -x^3\Big) \zeta(3)\Big] \ln(x) + \Big[\frac{9}{2} - \frac{43}{8}x + \frac{17}{8}x^2 + \frac{29}{8}x^3 - \frac{9}{2}x^4 + \Big(\frac{x}{4} + \frac{x^2}{2} + \frac{5}{24}x^3 + \frac{19}{48}x^4\Big) \pi^2\Big] \ln^2(x) \\
& + \Big(\frac{67}{24}x - \frac{5}{4}x^2 - \frac{2}{3}x^3\Big) \ln^3(x) + \Big(\frac{7}{48}x + \frac{5}{96}x^2 - \frac{x^3}{12} + \frac{43}{96}x^4\Big) \ln^4(x) + \Big\{ 3x + 3x^3 + \Big(\frac{7}{6}x \\
& - \frac{73}{24}x^2 + \frac{15}{8}x^3\Big) \pi^2 + \left(-6+6x-x^2-4x^3\right) \zeta(3) + \Big[-8 + \frac{21}{2}x - \frac{45}{4}x^2 + x^4 + \Big(1 - \frac{x}{6} + \frac{x^2}{12} \\
& - \frac{x^3}{3} - \frac{x^4}{8}\Big) \pi^2\Big] \ln(x) + \Big(6 - 11x + \frac{35}{4}x^2 - \frac{15}{8}x^3\Big) \ln^2(x) + \Big(\frac{2}{3} + \frac{x}{12} - \frac{x^3}{3} + \frac{5}{24}x^4\Big) \ln^3(x)\Big\} \\
& \times \ln(1-x) + \Big[\frac{7}{2} - 6x + \frac{45}{4}x^2 - 6x^3 + \frac{7}{2}x^4 + \Big(-\frac{17}{24} + \frac{7}{6}x - \frac{25}{24}x^2 - \frac{13}{48}x^4\Big) \pi^2 + \Big(-3 + \frac{23}{4}x \\
& - \frac{23}{4}x^2 + \frac{9}{8}x^3\Big) \ln(x) + \Big(\frac{7}{2} - \frac{41}{8}x + \frac{31}{8}x^2 + \frac{3}{8}x^3 - \frac{13}{16}x^4\Big) \ln^2(x)\Big] \ln^2(1-x) + \Big[\frac{3}{8}x + \frac{1}{6}x^2 \\
& + \frac{3}{8}x^3 + \Big(-4 + \frac{29}{6}x - \frac{49}{12}x^2 + \frac{5}{6}x^3 + \frac{7}{8}x^4\Big) \ln(x)\Big] \ln^3(1-x) + \Big(\frac{1}{32} - \frac{3}{4}x + \frac{71}{48}x^2 - \frac{29}{24}x^3 \\
& + \frac{9}{32}x^4\Big) \ln^4(1-x) + \Big\{ 8 - 16x + 24x^2 - 16x^3 + 8x^4 + \Big(\frac{7}{3} - 3x + \frac{3}{4}x^2 + \frac{5}{6}x^3 - \frac{2}{3}x^4\Big) \pi^2
\end{aligned}
$$

Result *(page 2 of 2)*

$$+\left[-6+\frac{11}{2}x-4x^2+x^3+\left(2-\frac{11}{4}x+\frac{7}{4}x^2+\frac{x^3}{4}-x^4\right)\ln(x)\right]\ln(x)+\left[\frac{3}{2}x-\frac{x^2}{4}+x^3\right.$$
$$\left.+\left(-4+9x-\frac{15}{2}x^2+2x^3\right)\ln(x)+\left(-1-\frac{7}{2}x+\frac{25}{4}x^2-5x^3+2x^4\right)\ln(1-x)\right]\ln(1-x)+\left(2\right.$$
$$\left.\left.-4x+6x^2-4x^3+2x^4\right)\mathrm{Li}_2(x)\right\}\mathrm{Li}_2\left(x\right)+\left\{-8+16x-24x^2+16x^3-8x^4+\left[-\frac{2}{3}+\frac{4}{3}x\right.\right.$$
$$\left.+\frac{x^2}{2}-\frac{5}{3}x^3+\frac{2}{3}x^4\right]\pi^2+\left[6-8x+9x^2-3x^3+\left(\frac{3}{2}x-\frac{x^2}{2}-\frac{3}{2}x^3+2x^4\right)\ln(x)\right]\ln(x)+\left[-x\right.$$
$$\left.-\frac{x^2}{4}-\frac{x^3}{2}+\left(10-14x+9x^2\right)\ln(x)+\left(-8+11x-\frac{31}{4}x^2+\frac{x^3}{2}+x^4\right)\ln(1-x)\right]\ln(1-x)$$
$$\left.+\left(-4+8x-12x^2+8x^3-4x^4\right)\mathrm{Li}_2(x)+\left(2-4x+6x^2-4x^3+2x^4\right)\mathrm{Li}_2(1-x)\right\}\mathrm{Li}_2\left(1-x\right)$$
$$+\left[\frac{5}{2}x-5x^2+2x^3+\left(-4-x+x^2+2x^3-2x^4\right)\ln(x)+(6-6x+x^2+4x^3)\ln(1-x)\right]\mathrm{Li}_3\left(x\right)$$
$$+\left[\frac{x}{2}-\frac{x^3}{2}+(-6+5x+3x^2-5x^3)\ln(x)+\left(6-10x+10x^3-6x^4\right)\ln(1-x)\right]\mathrm{Li}_3\left(1-x\right)$$
$$+\left(-2+\frac{17}{2}x-\frac{17}{2}x^3+2x^4\right)\mathrm{Li}_4\left(x\right)+\left(7x-\frac{9}{2}x^2-4x^3+6x^4\right)\mathrm{Li}_4\left(1-x\right)+\left(-6+4x\right.$$
$$\left.\left.+\frac{9}{2}x^2-7x^3\right)\mathrm{Li}_4\left(-\frac{x}{1-x}\right)\right),$$

$$\mathcal{L}_\varepsilon=\left[1-\ln\left(x/(1-x)\right)\right]\ln\left(\mathcal{E}_{cut}/\mathcal{E}\right).$$

Two-loop photonic corrections to LA Bhabha scattering



$\sqrt{s} = 1$ GeV, $\quad \ln\left(\frac{\mathcal{E}_{cut}}{\mathcal{E}}\right) = 0$

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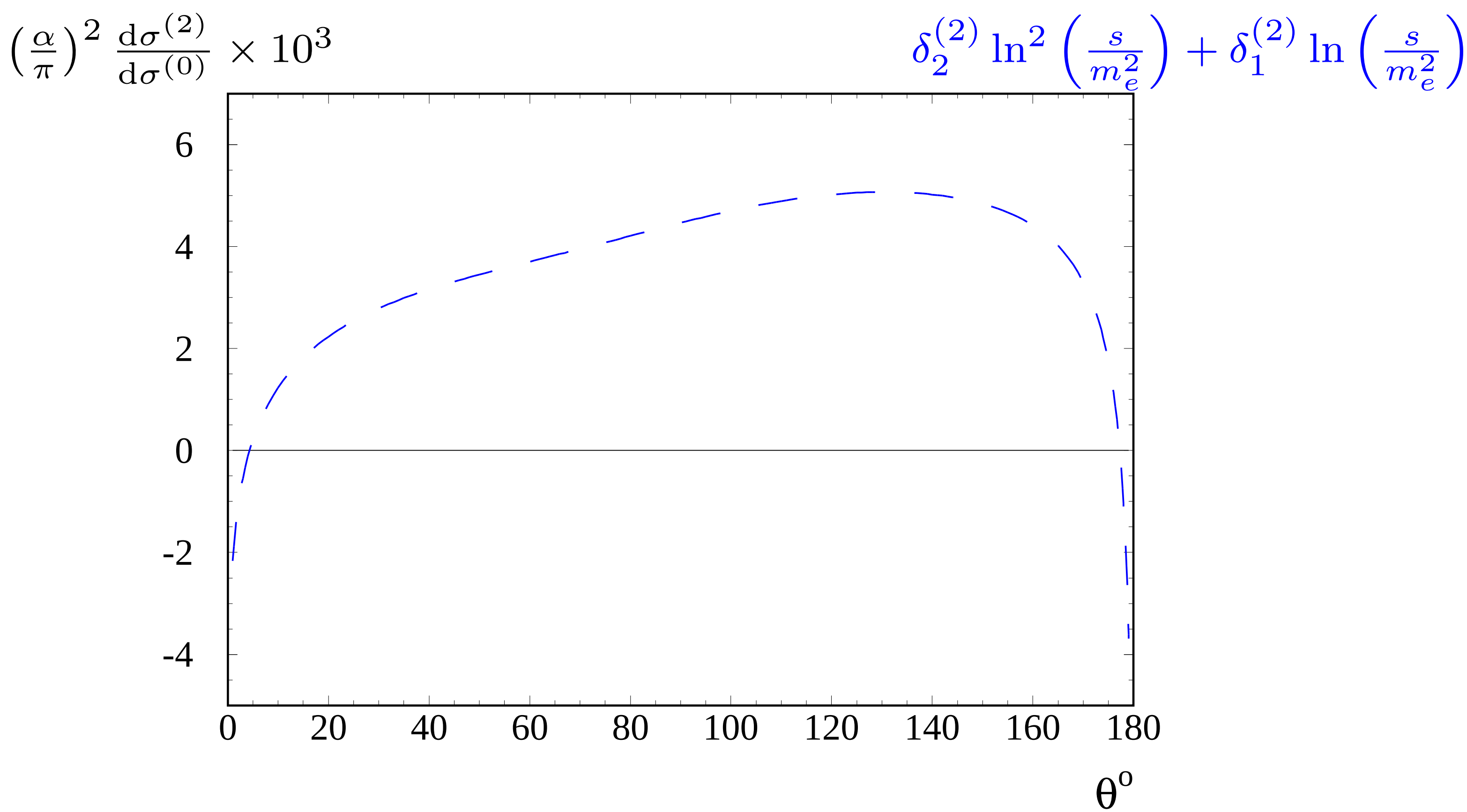

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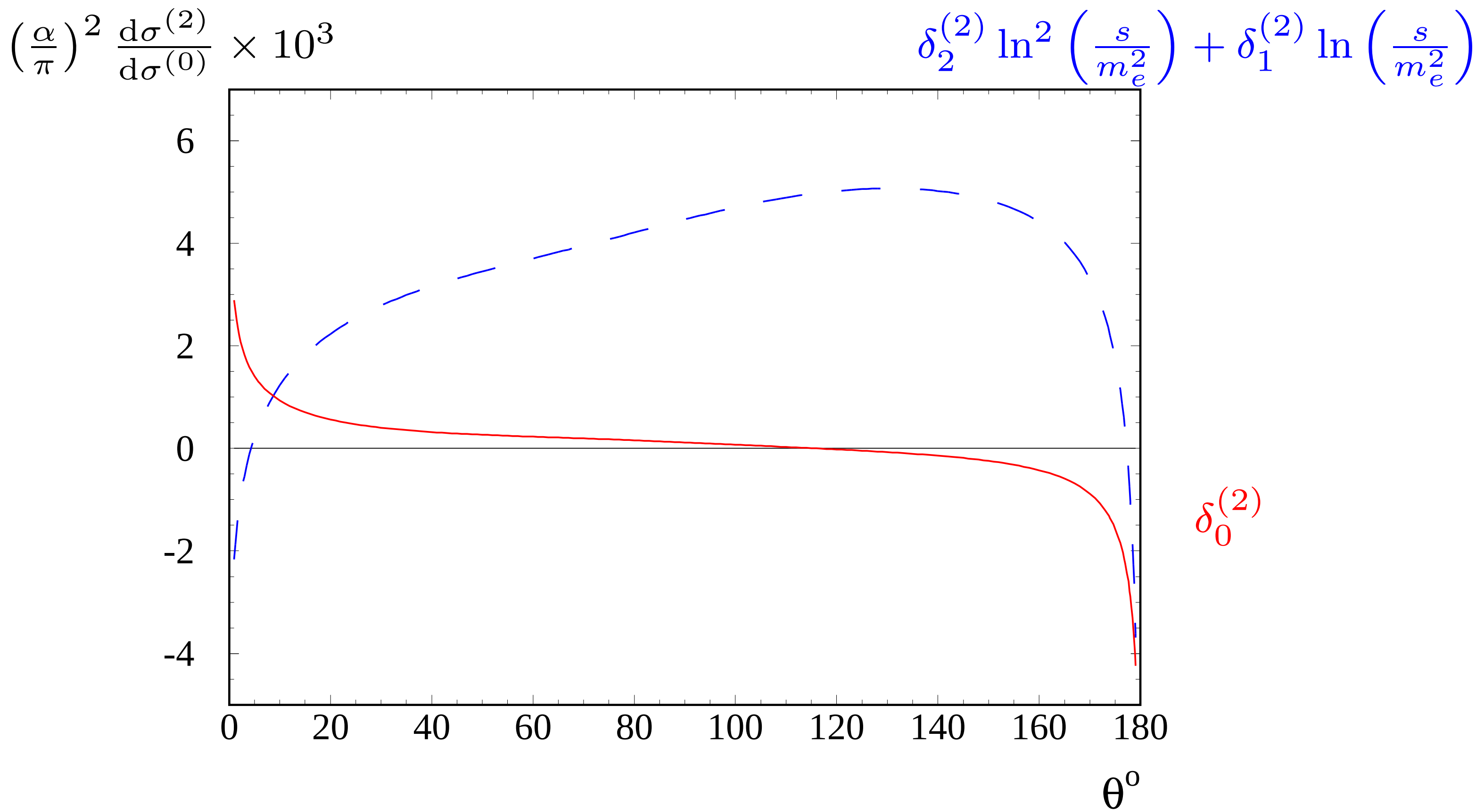

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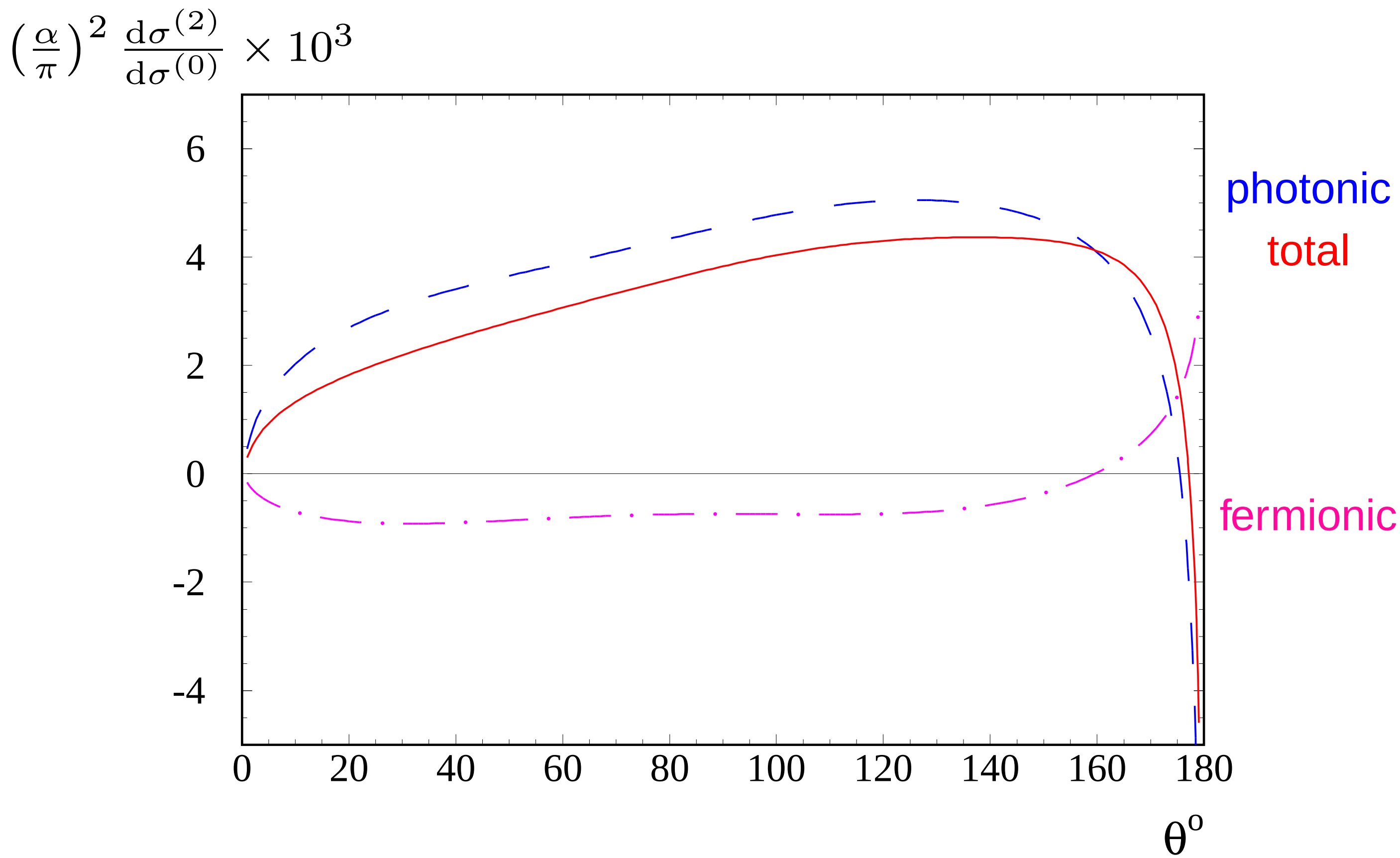

$$\sqrt{s} = 1 \text{ GeV}, \quad \ln\left(\frac{\mathcal{E}_{cut}}{\mathcal{E}}\right) = \ln\left(\frac{\mathcal{E}_{cut}^{e^+e^-}}{\mathcal{E}}\right) = 0$$

Electroweak corrections to LA Bhabha scattering

- Massive W, Z bosons ➪ exclusive reactions
- Sudakov double logs: $\ln^2(s/M^2_{Z,W})$ per loop
- ILC: $\ln^2(s/M^2_{Z,W}) \sim 25$
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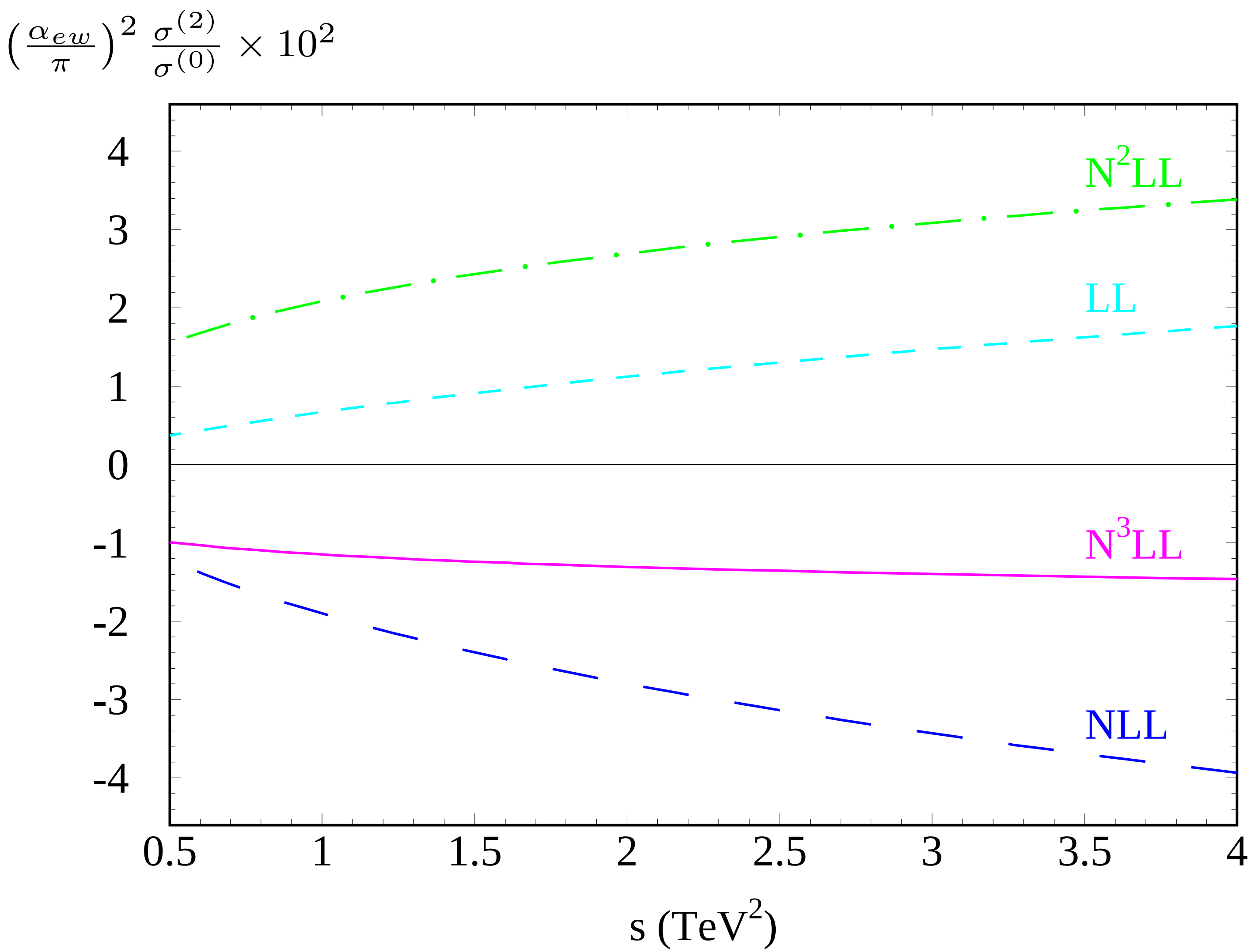
- *Huge cancellations!*

Two-loop electroweak logs for $e^+e^- \to \mu^+\mu^-$

(B. Feucht/Jantzen, J.H. Kühn, A.A. Penin, V.A. Smirnov)



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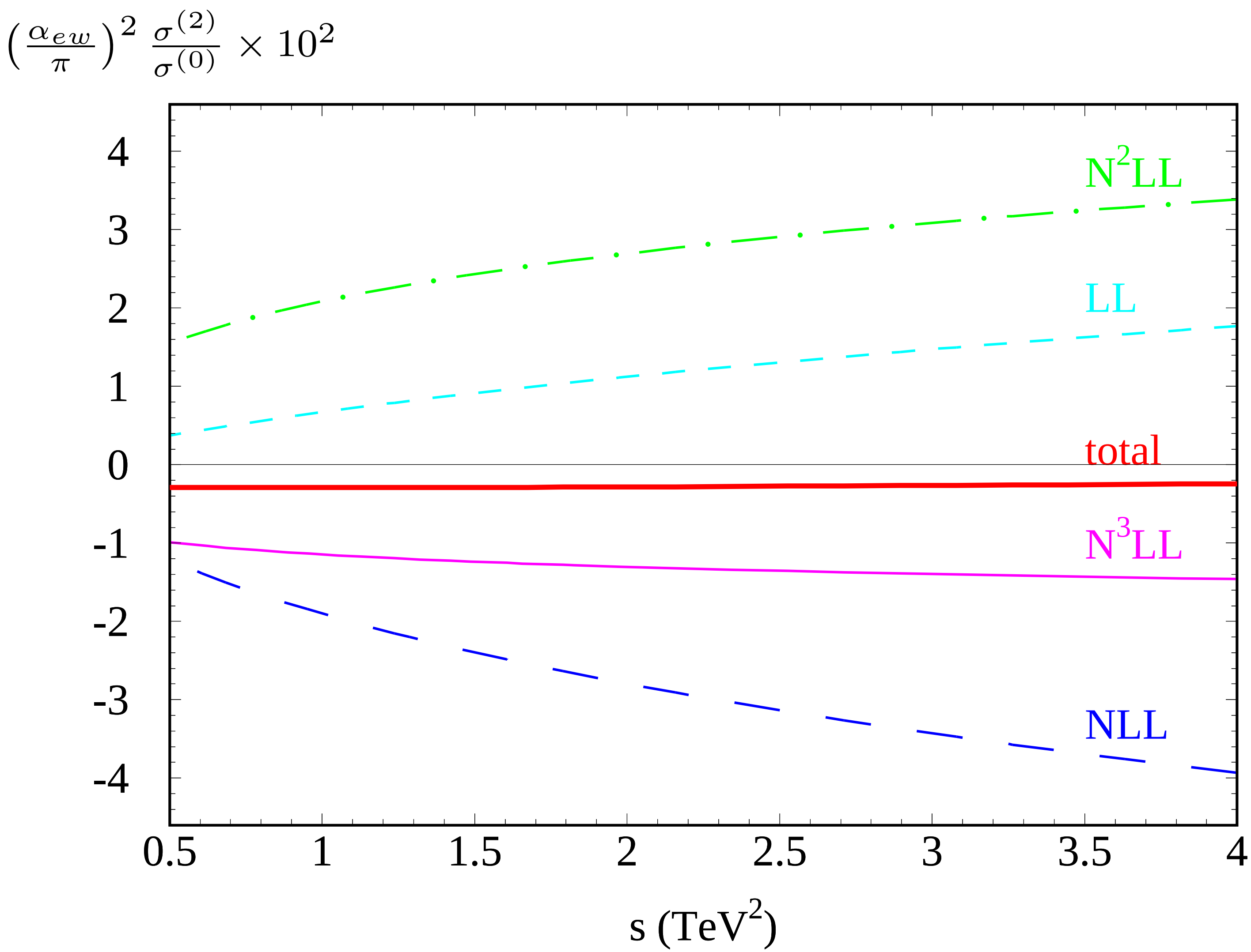

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- *Complete NNLO accuracy still not available!*

Problems to solve

- MC event generators
  - *Consistent inclusion of two-loop corrections*
  - *One-loop correction to single emission*
  - *Pair emission*
- Theory
  - *VP by heavy fermions in one-loop corrections*