Wake-Field in the e-Cloud

Levi Schächter

Department of Electrical Engineering

Technion-Israel Institute of Technology

Haifa 32000, ISRAEL

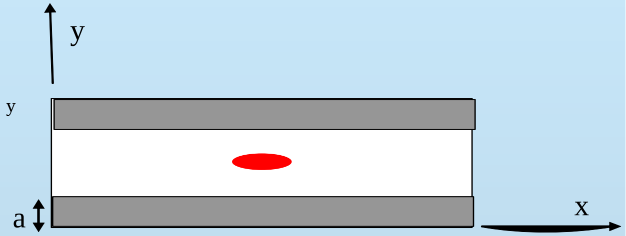
Outline

- Brief description of the model
- Eigen frequencies
- Power generation
- Vertical dynamics
- Theory & experiment

Brief Description of the Model

A train of N_b bunches propagates in a rectangular beam-pipe $D_x \times D_y$

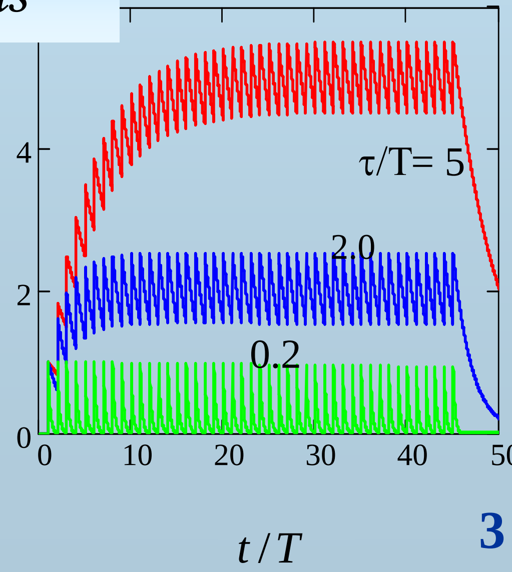
$$\rho(x,y,z,t) = -q_e \sum_{\nu=1}^{N_b} \delta\left(x - \frac{D_x}{2}\right)\delta\left(y - \frac{D_y}{2}\right)\delta\left[z - V\left(t - T_\nu\right)\right]$$



Assuming a strong vertical magnetic field the dielectric tensor describing the e-cloud in the frequency domain is

$$\underline{\underline{\varepsilon}}(y) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \omega_p^2(y)/\omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

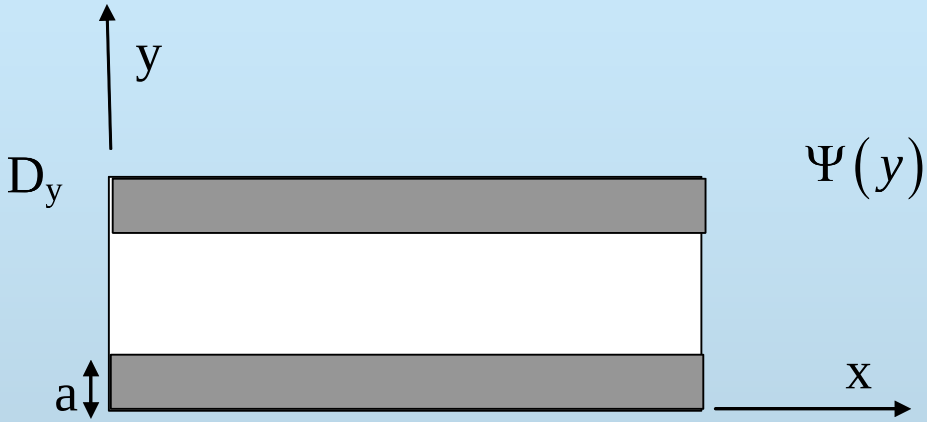
We consider the average (in time) cloud density.



Eigen-frequencies

Boundary conditions & wake

$$E_y(x,y,z) \propto \Psi(y)\sin\left(\pi n \frac{x}{D_x}\right)\exp\left(-j\frac{\omega}{V}z\right)$$



$$\Psi(y) = \begin{cases} A\cosh(\Gamma y) & 0 < y < a \\ B\sinh\left[\frac{\pi n}{D_x}\left(y - \frac{D_y}{2}\right)\right] & a < y < D_y/2 \end{cases}$$

$$\Gamma^2 = \left(\frac{\pi^2 n^2}{D_x^2} + \frac{\omega_p^2}{c^2}\right)\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{-1}, \quad \Gamma a = j\xi$$

$$\kappa_n^2 \equiv \left(\frac{\pi n a}{D_x}\right)^2 + \left(\frac{\omega_p}{c}a\right)^2$$

$$\xi_0 \equiv \frac{\pi n a}{D_x}\coth(\psi_n), \quad \psi_n = \frac{\pi n a}{D_x}\left(\frac{D_y}{2a} - 1\right)$$

Dispersion Relation

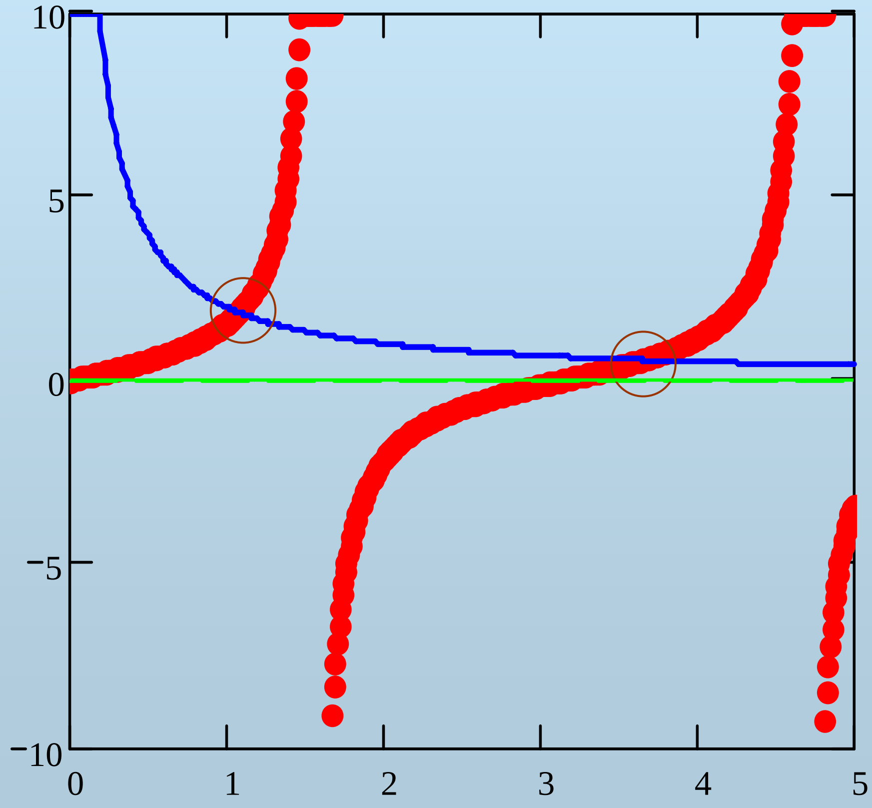
$$\tan(\xi) = \frac{\xi_0}{\xi}$$

Eigen-frequencies

$$\xi_i : \quad \tan(\xi) = \frac{\xi_0}{\xi}$$

$$\omega_i^2 = \omega_p^2 \frac{\xi_i^2}{\xi_i^2 + \kappa_n^2}$$

- *For each n there is an infinite set of solutions* (ξ_i) *each corresponding to one eigen-frequency.*
- *Eigen-frequency dependent on the plasma frequency.*

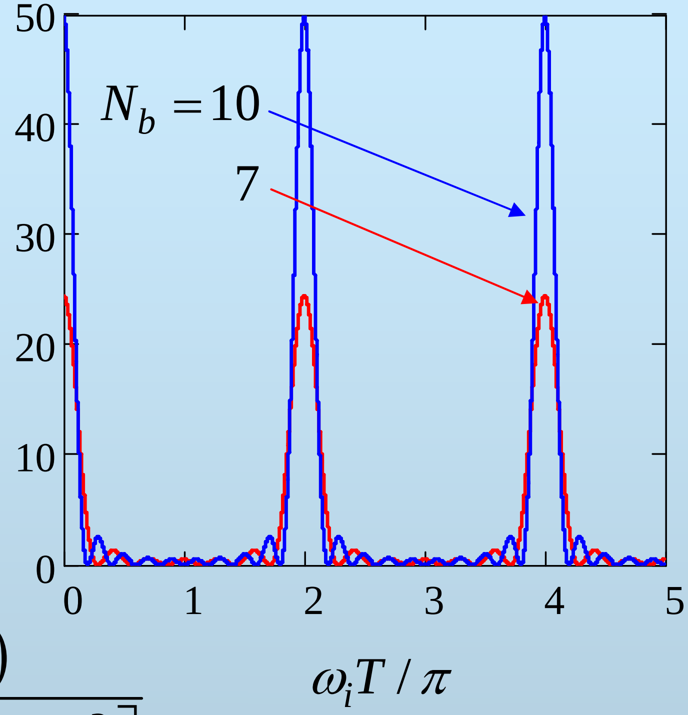


Global Power Generated

$$\bar{P} = \frac{-P}{\eta_0\left(\frac{q_e c}{D_x}\right)^2} = \frac{-\int dx\int dy\int dz J_z E_z}{\eta_0\left(\frac{q_e c}{D_x}\right)^2}$$

$$= \sum_{\substack{n=0\\ i=1}}^{\infty} \frac{(2a/D_x)\xi_0^2(\omega_i D_x/c)^2 \sin^2(\pi n/2)}{\cosh^2(\psi_n)\left[\xi_i^2+\xi_0(1+\xi_0)\right]\left[\xi_i^2+(\pi n a/D_x)^2\right]}$$

$$\times\frac{N_b^2}{2}\frac{\operatorname{sinc}^2(\omega_i T N_b/2)}{\operatorname{sinc}^2(\omega_i T/2)}$$

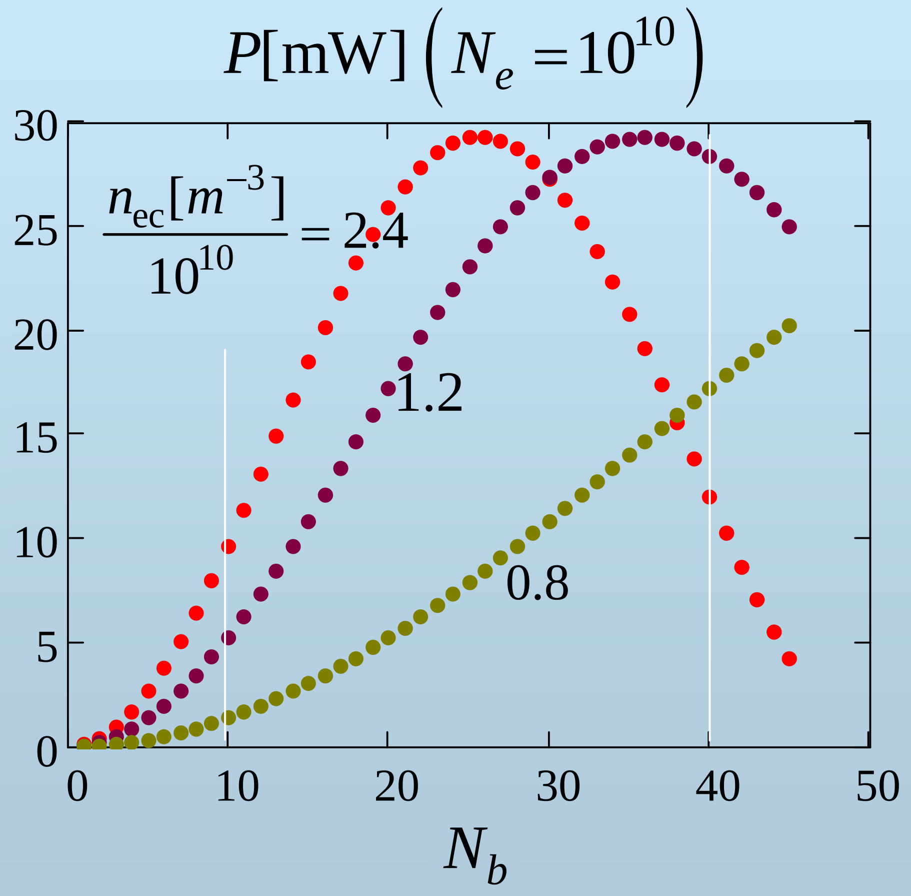


#of bunches effect

Global Power Generated

Dependence on # of bunches:

- *Power varies significantly with N_b*
- *Due to periodic character of the wake, trailing bunches may be affected differently.*
- *The effect is not monotonic with the cloud density.*



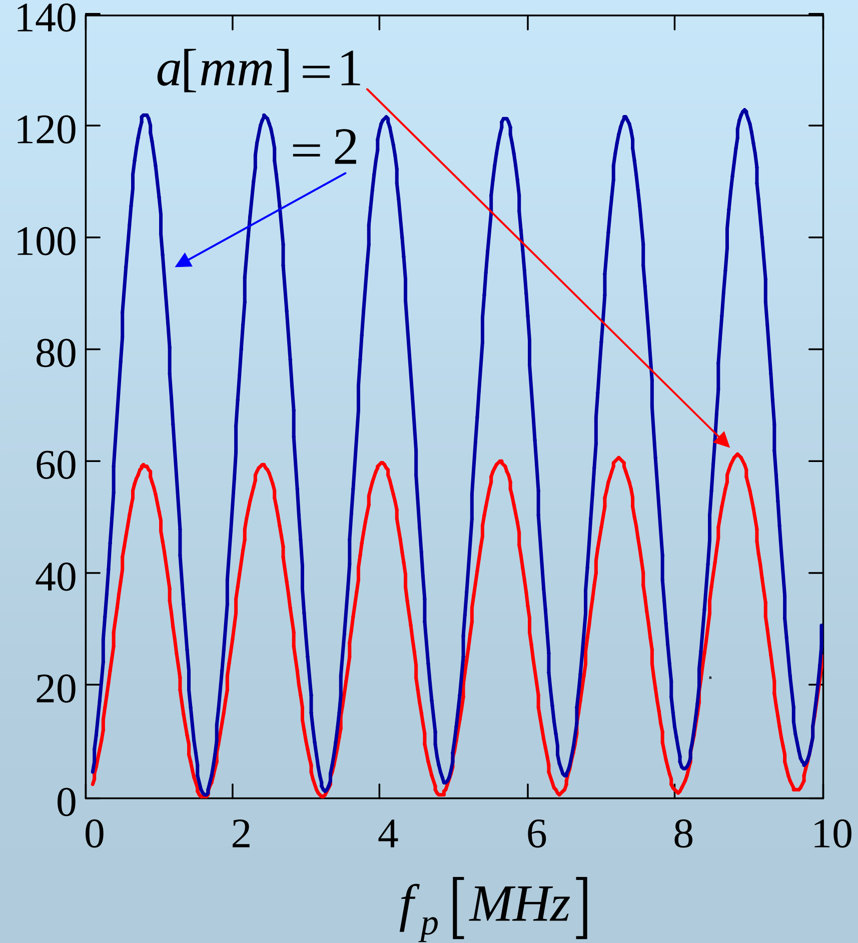
Global Power Generated

- *Dependence on the plasma-frequency*

- *Periodicity determined by* $1/TN_b \sim 1.6[MHz]$

- *Bunch close to the edge generates more power.*

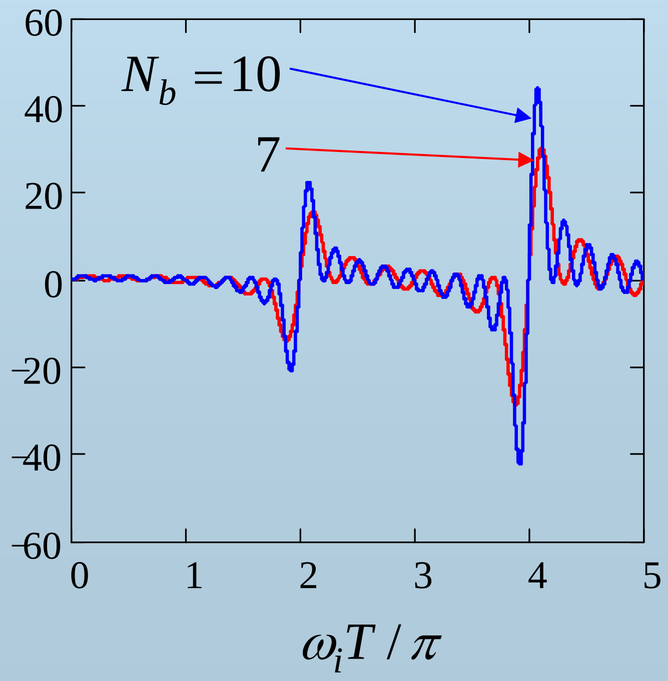
$$P[mW]\left(N_e = 10^{10}, N_b = 45\right)$$



Vertical Dynamics – Single Bunch

$$\frac{F_{y,\nu}(\delta y)}{\frac{e^2 N_e}{4\pi\varepsilon_0 D_x^2}\, 32\pi\left(\frac{D_x}{cT}\right)} = \sum_{\substack{n=0\\ i=1}}^{\infty} \frac{\xi_0 \sin^2\left(\pi n \frac{1}{2}\right)\frac{\sinh\left[\pi\frac{n}{D_x}\delta y\right]}{\sinh 2\psi_n}}{\left[\xi_0(1+\xi_0)+\xi_i^2\right]\left[\xi_i^2+\left(\frac{\pi n a}{D_x}\right)^2\right]}$$

$$\times \left\{\left(\frac{1}{2}\omega_i T\right)\frac{\sin\left(\frac{1}{2}\omega_i T\nu\right)}{\sin\left(\frac{1}{2}\omega_i T\right)}\sin\left[\frac{1}{2}\omega_i T(\nu-1)\right]\right\}$$

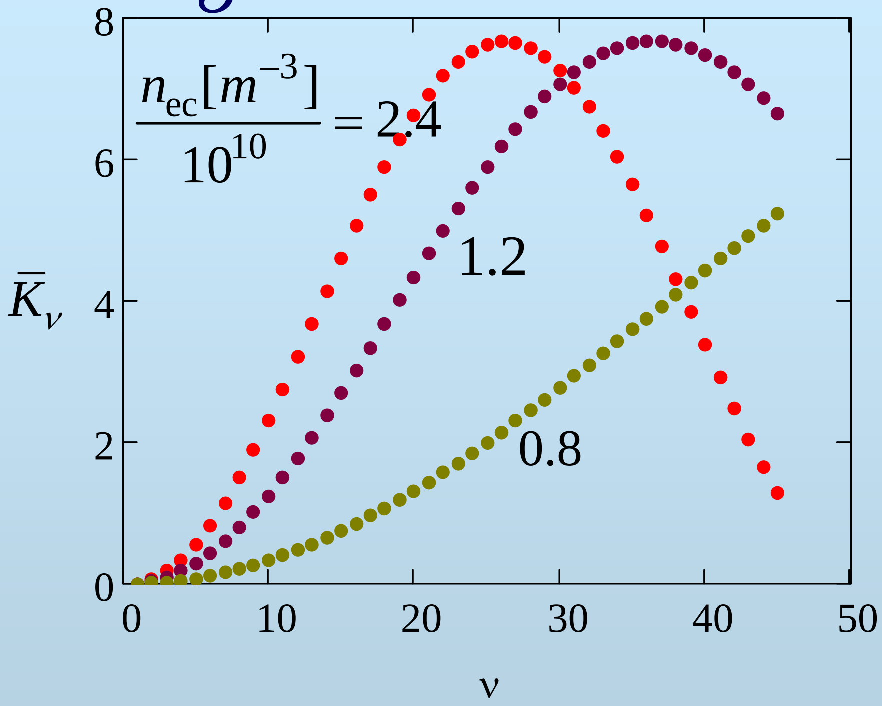


- *Force is linear in the displacement*
- *Dominant peaks around* $\omega_i T = 2\pi M$

Vertical Dynamics – Single Bunch

Vertical Kick on individual bunch ("Spring Coefficient"):

- *Peak independent of the cloud density.*
- *Peak-location along the train, dependent on n_{ec}*



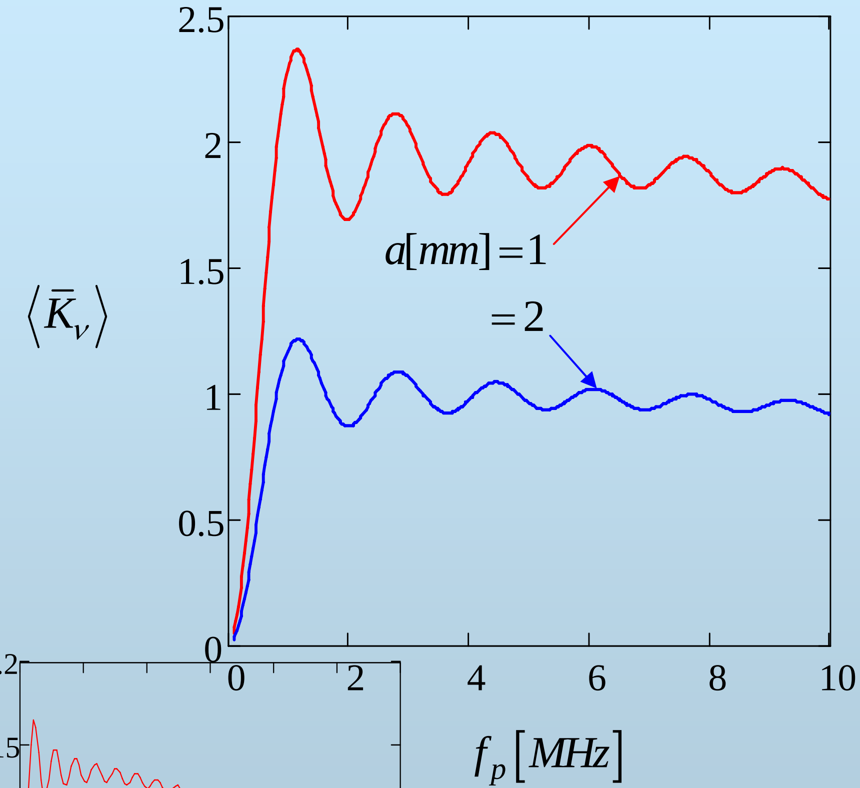
$$\bar{K}_\nu = \left[\frac{F_{y,\nu}(u)}{u}\right]_{u=\delta y=0} \left(\frac{e^2 N_e}{4\pi\varepsilon_0 D_x^2}\frac{32\pi^2}{cT}\right)^{-1}$$

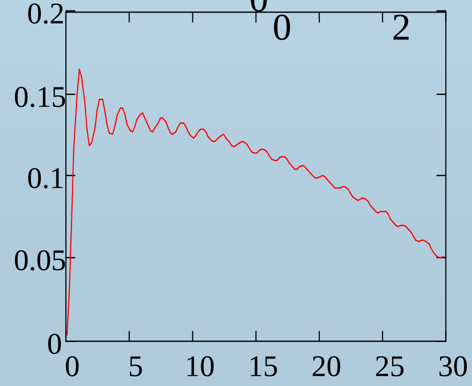
$$= \sum_{\substack{n=0\\ i=1}}^{\infty} \frac{n\,\xi_0 \sin^2(\pi n/2)/\sinh 2\psi_n}{\left[\xi_0(1+\xi_0)+\xi_i^2\right]\left[\xi_i^2+(\pi n a/D_x)^2\right]} \left\{(\omega_i T/2)\frac{\sin(\omega_i T\nu/2)}{\sin(\omega_i T/2)}\sin\left[\omega_i T(\nu-1)/2\right]\right\}$$

Vertical Dynamics

Average Vertical Kick

- *Uniformly distributed cloud ($a=D_x/2$) generates no propagating waves.*
- *Average kick is weaker for thicker cloud*





- *Beyond a peak value, the average kick varies slowly as a function of the plasma frequency – it eventually drops to zero*

Vertical Dynamics – Single Bunch

In the absence of the cloud and ignoring SC effect

$$\frac{d^2 b_\nu}{ds^2} + \frac{1}{\beta_y^2} b_\nu - \frac{\varepsilon_y^2}{b_\nu^3} \simeq 0 \Rightarrow \underbrace{\bar{b}_\nu \simeq \sqrt{\beta_y \varepsilon_y}}_{Steady\ State}$$

Kapchinskij & Vladimirskij

With the cloud

$$\frac{d^2 b_\nu}{ds^2} + \frac{1}{\beta_y^2} b_\nu - \frac{\varepsilon_y^2}{b_\nu^3} \simeq \bar{K}_\nu \left(\frac{e^2 N_e}{4\pi\varepsilon_0 D_x^2} \frac{32\pi^2}{cT} \right) \frac{1}{mc^2\gamma} b_\nu \cong \frac{\Omega_\nu^2}{c^2} b_\nu \Rightarrow \boxed{\frac{d^2 b_\nu}{ds^2} + \left(\frac{1}{\beta_y^2} - \frac{\Omega_\nu^2}{c^2} \right) b_\nu - \frac{\varepsilon_y^2}{b_\nu^3} \simeq 0}$$

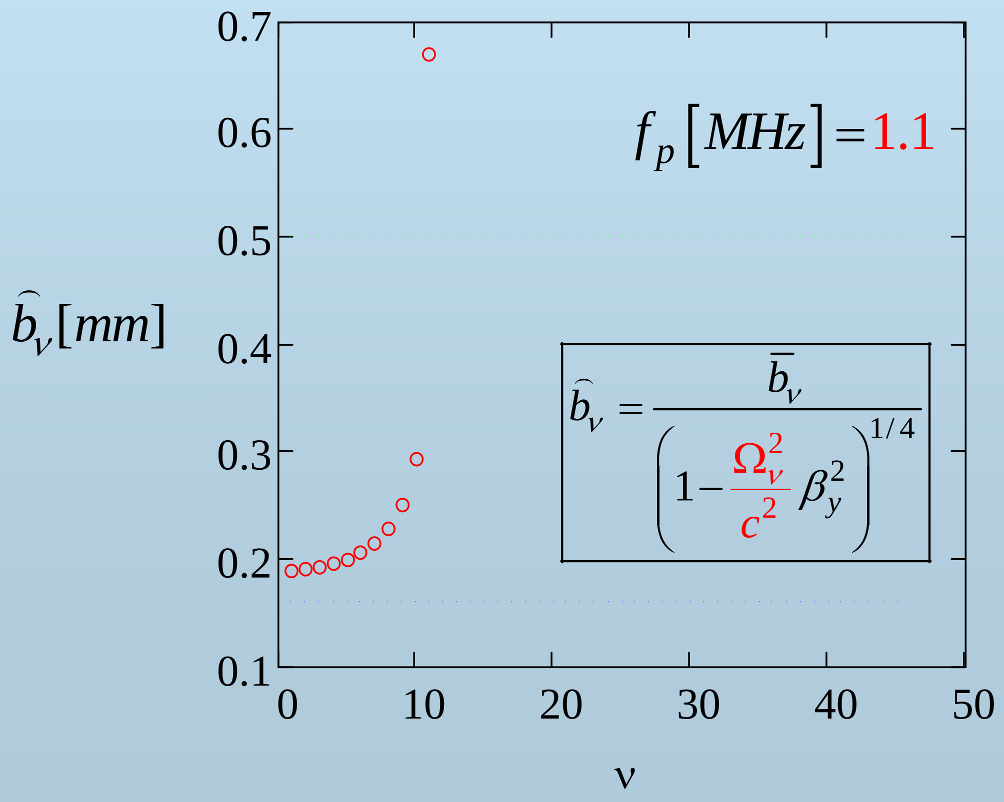
Thus in steady state

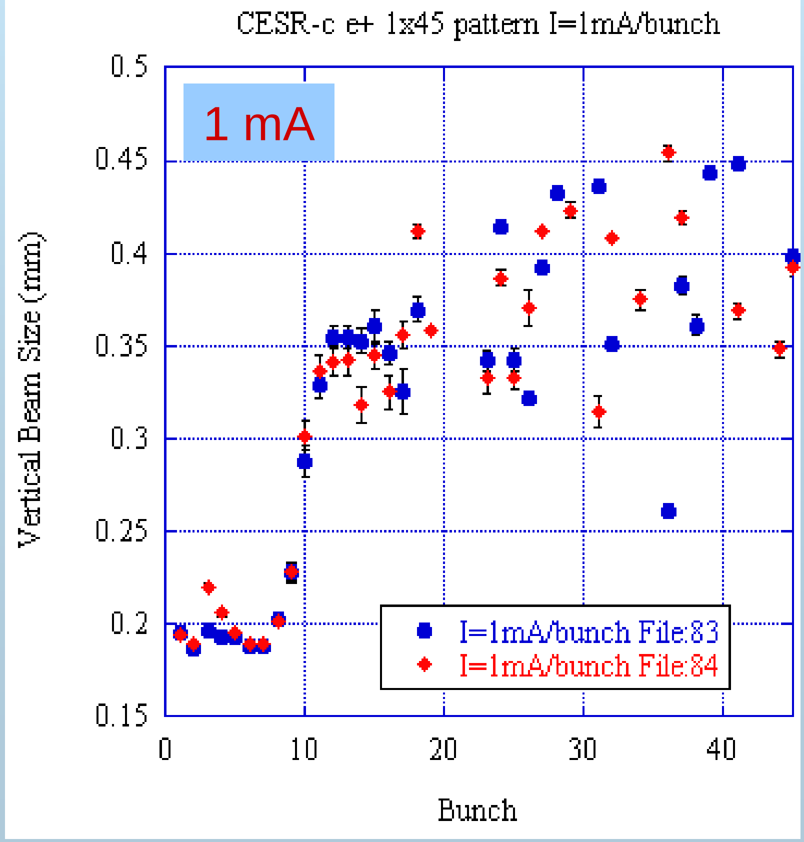
$$\boxed{\hat{b}_\nu = \frac{\bar{b}_\nu}{\left(1 - \frac{\Omega_\nu^2}{c^2} \beta_y^2 \right)^{1/4}}}$$

Diverges if $\Omega_\nu \simeq c / \beta_y$

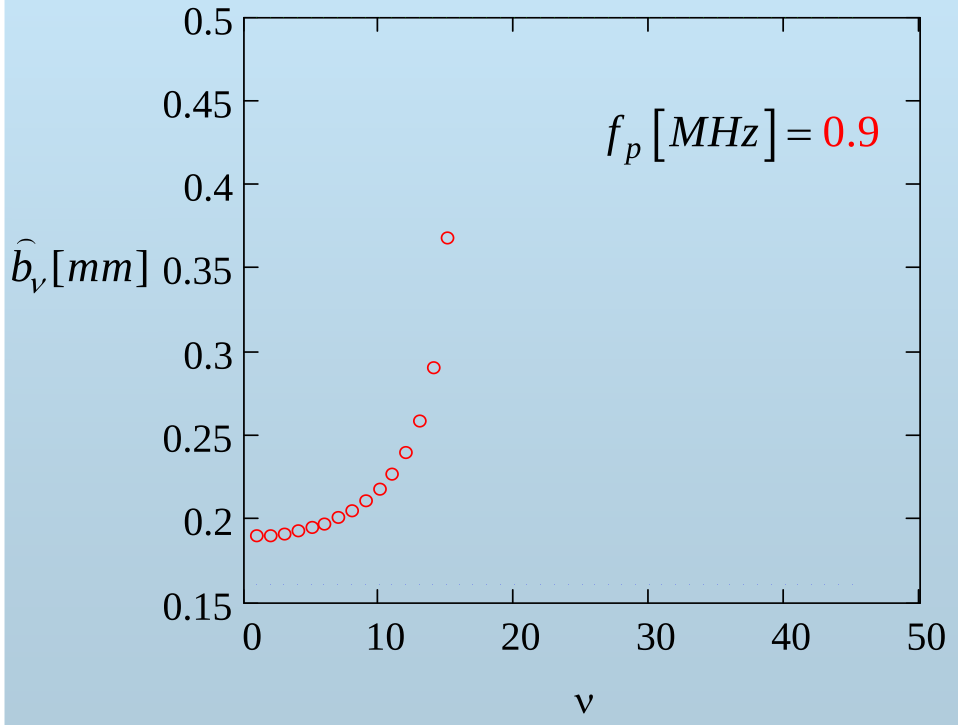
Experiment & Model

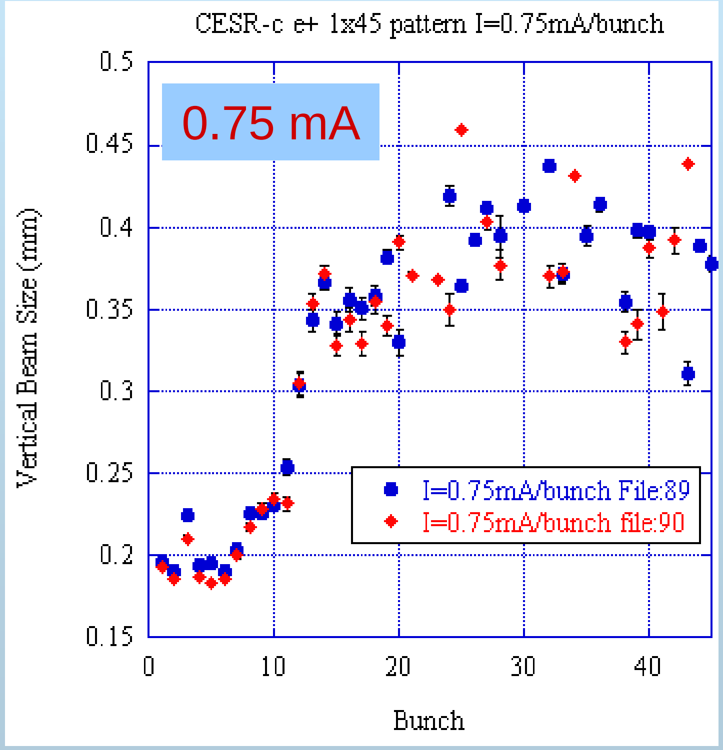
Qualitative comparison: if the transverse eigen-frequency becomes comparable with the corresponding betatron frequency, then the transverse motion becomes unstable. Need to take into account the horizontal motion as well.



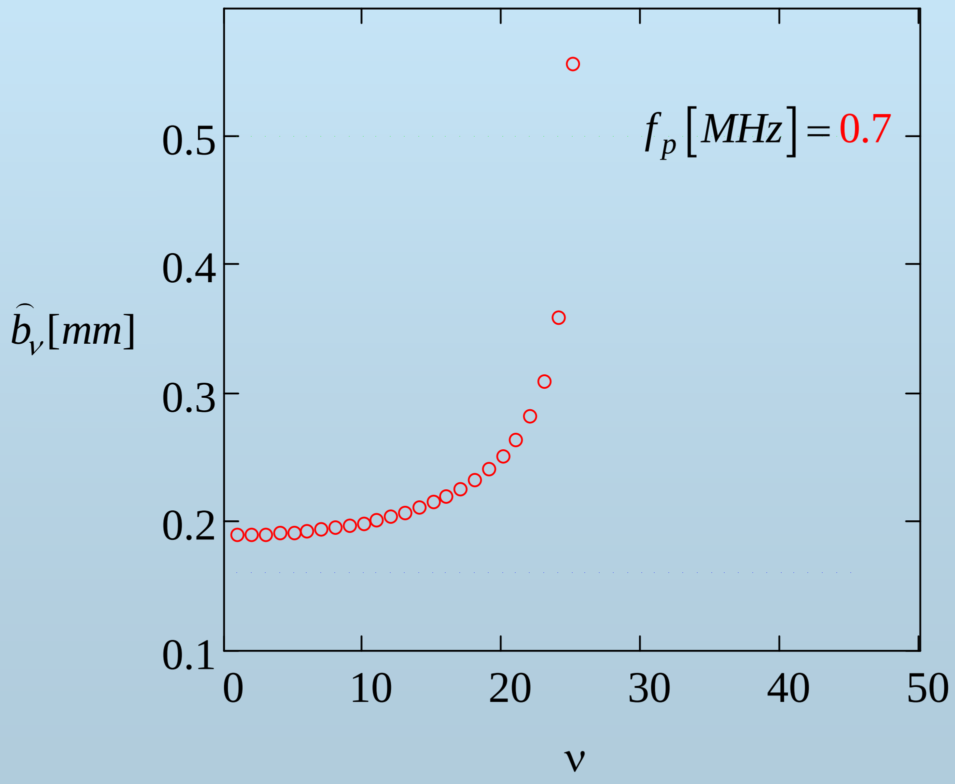


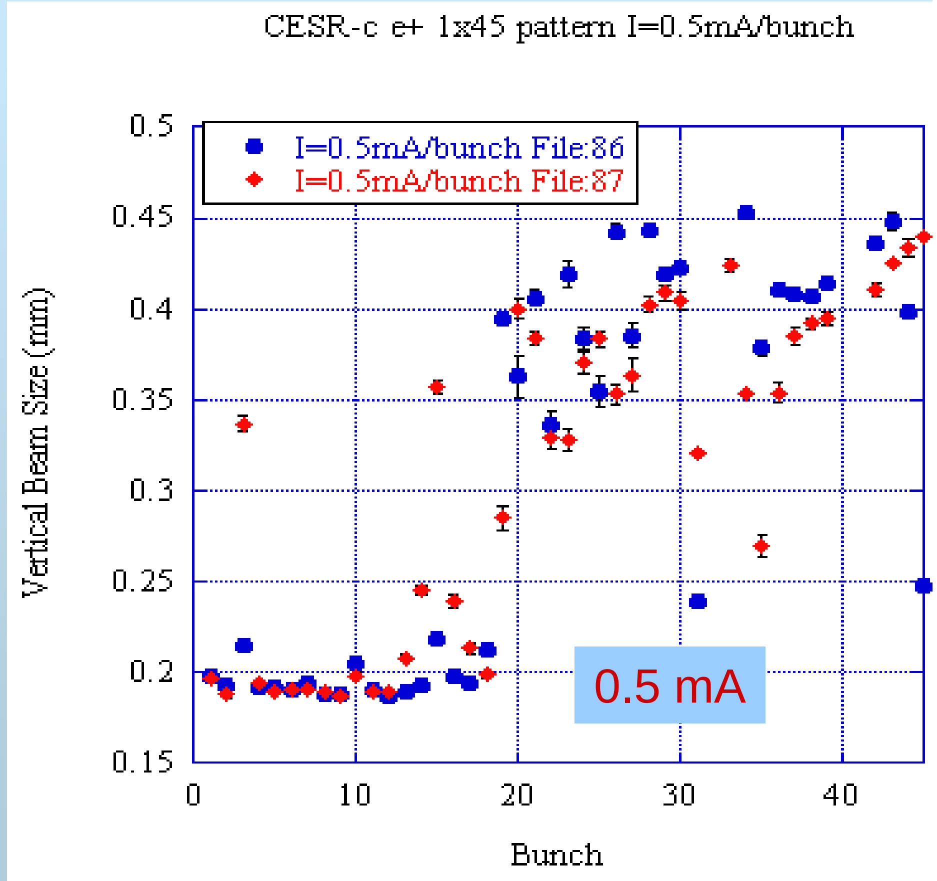
Experiment & Model



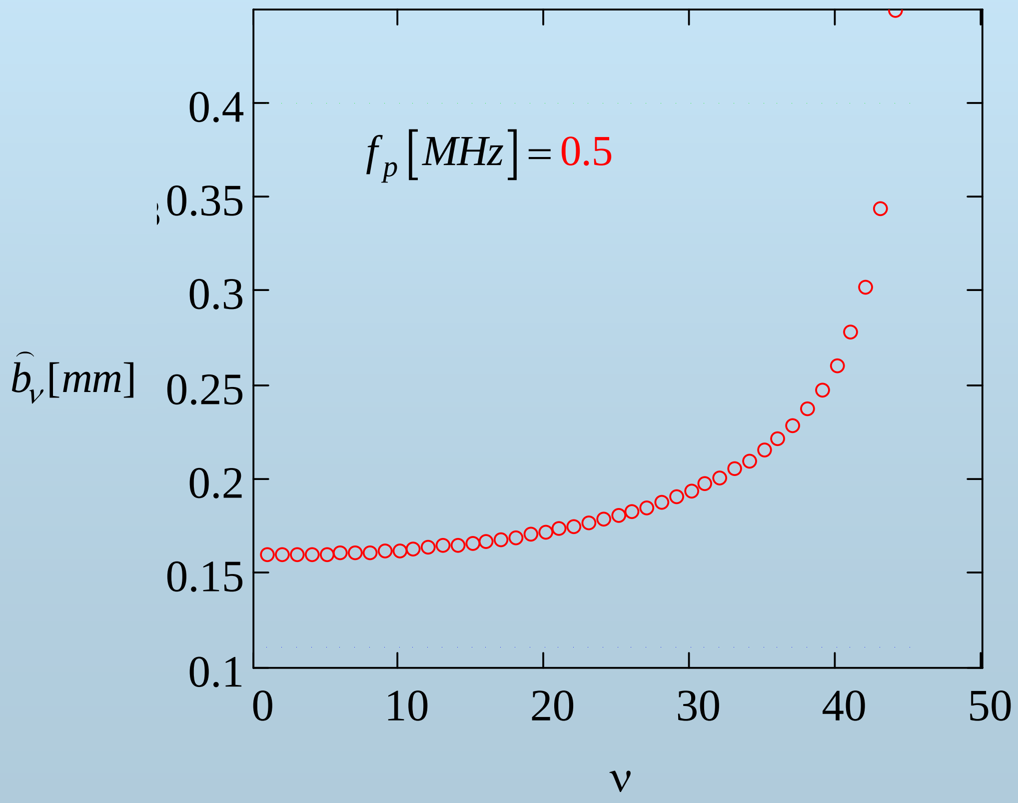


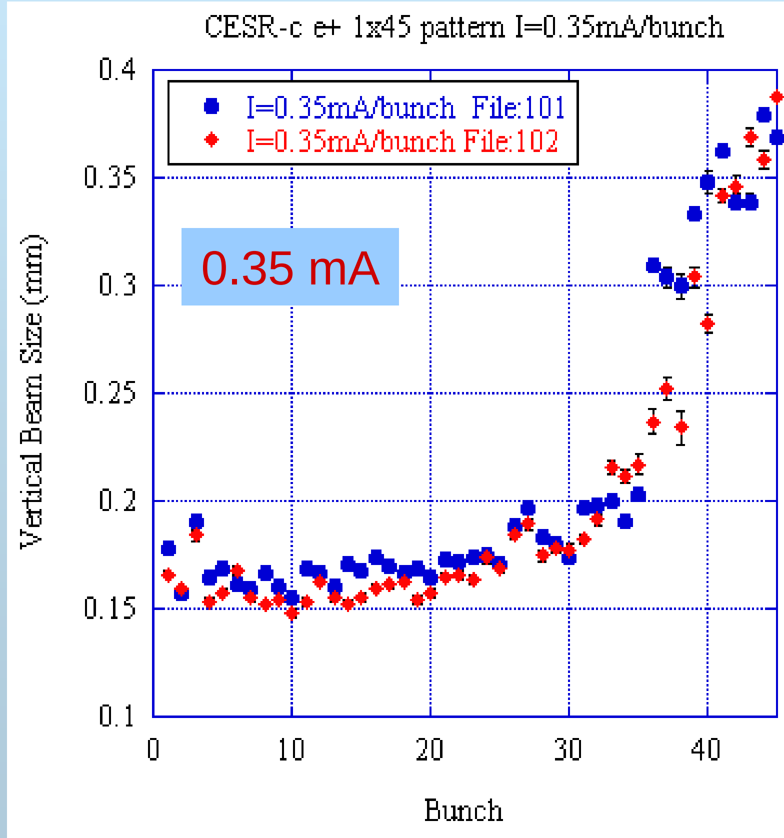
Experiment & Model



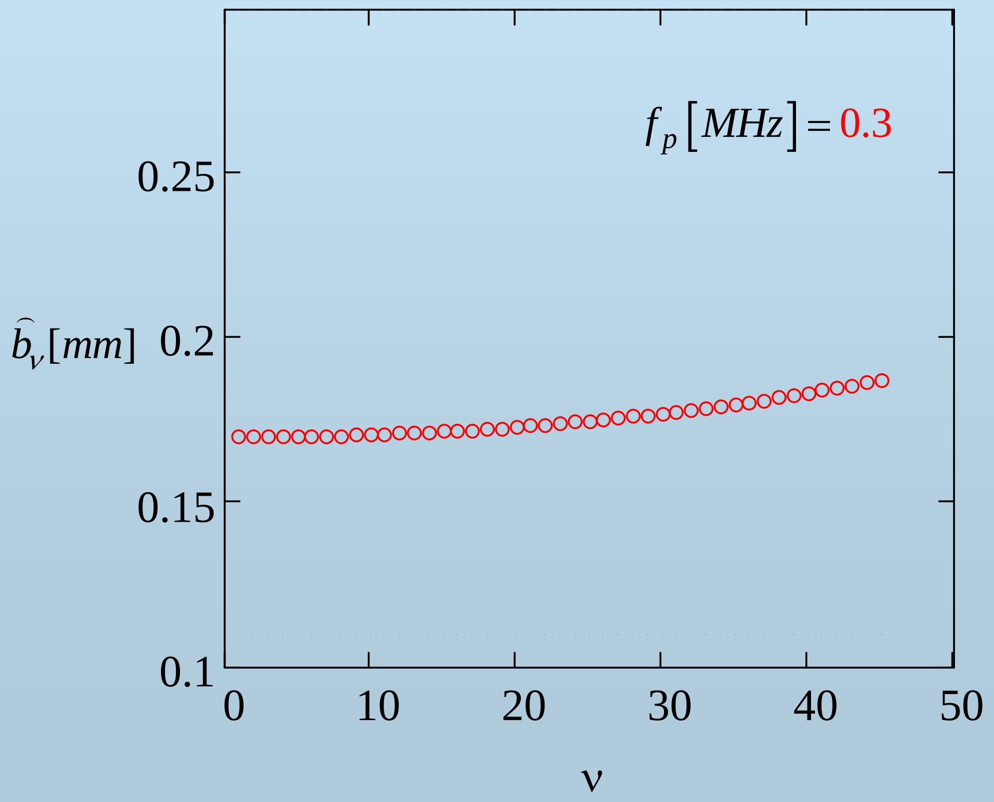


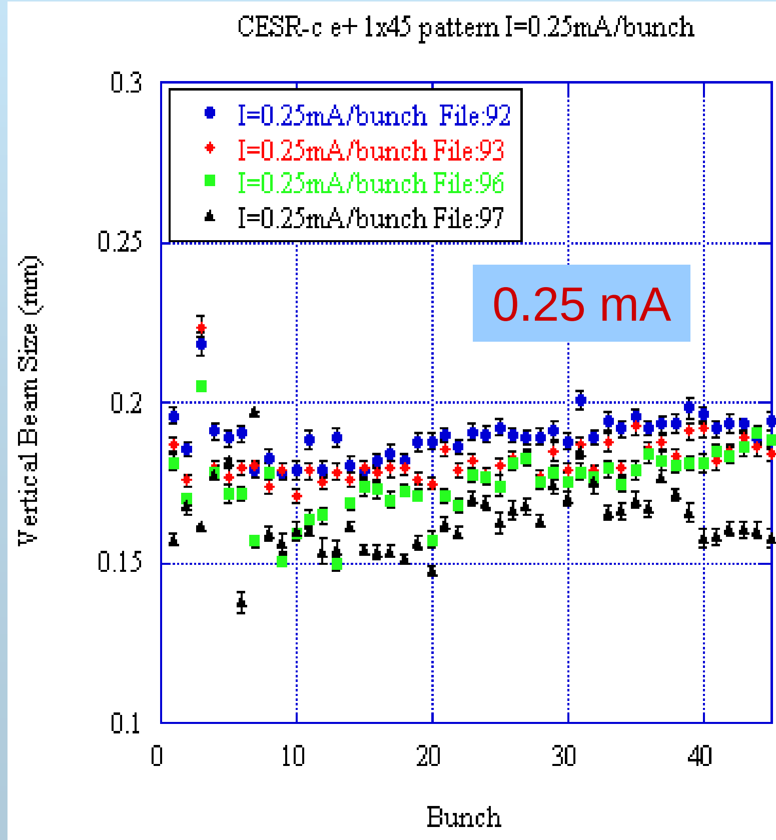
Experiment & Model





Experiment & Model





Summary of preliminary studies

- *Wake-field in plasma reveals clear signature of microwave radiation (is it detectable?)*
- *Microwave radiation may be indicative of the transverse distribution of the cloud -- no radiation emitted if the cloud is uniform.*
- *Vertical dynamics of the bunches dramatically affected.*
  If the eigen-frequency of the bunch (Ω_v) *is comparable with betatron frequency, the transverse motion becomes unstable.*
- *Partial list of "open issues":*
  - *-- Incorporate in a code to realistically simulate the dyn.*
  - *-- Instability associated with wake (include horizontal dyn.)*
  - *-- Temporal variations of the e-cloud (PE and/or Gas)*

Acknowledge discussions and data provided by the Cornell group

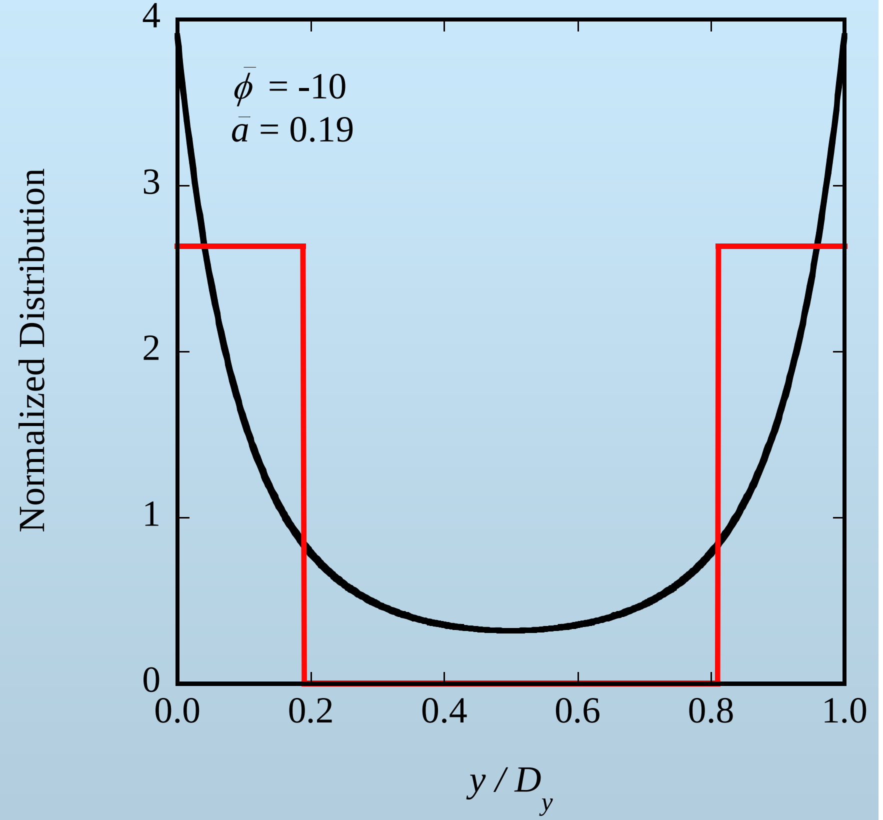
Brief Description of the Model

Approximating the cloud distribution to a step-wise function

$$f(u) = \frac{\exp\left[\bar{\phi} u (1-u)\right]}{\int_0^1 dv \exp\left[\bar{\phi} x (1-x)\right]}$$

$$f_{approx}(u) = \frac{1}{2\bar{a}} \begin{cases} 1 & 0 < u < \bar{a} \\ 0 & \bar{a} < u < 1-\bar{a} \\ 1 & 1-\bar{a} < u < 1 \end{cases}$$

$$u = y / D_y \qquad \bar{a} = a / D_y$$



For $\bar{\phi} < -8.5$ *and* $\alpha_1 = 1.4, \ \alpha_2 = 6.75$

$$\bar{a} \simeq \frac{\alpha_1}{\sqrt{\bar{\phi}^2 - \alpha_2^2}}, \quad \bar{\phi} = \frac{e\phi}{k_B T} = \frac{e\phi}{\langle E_{ec} \rangle}$$