New Progress of the Nonlinear Collimation System for CLIC

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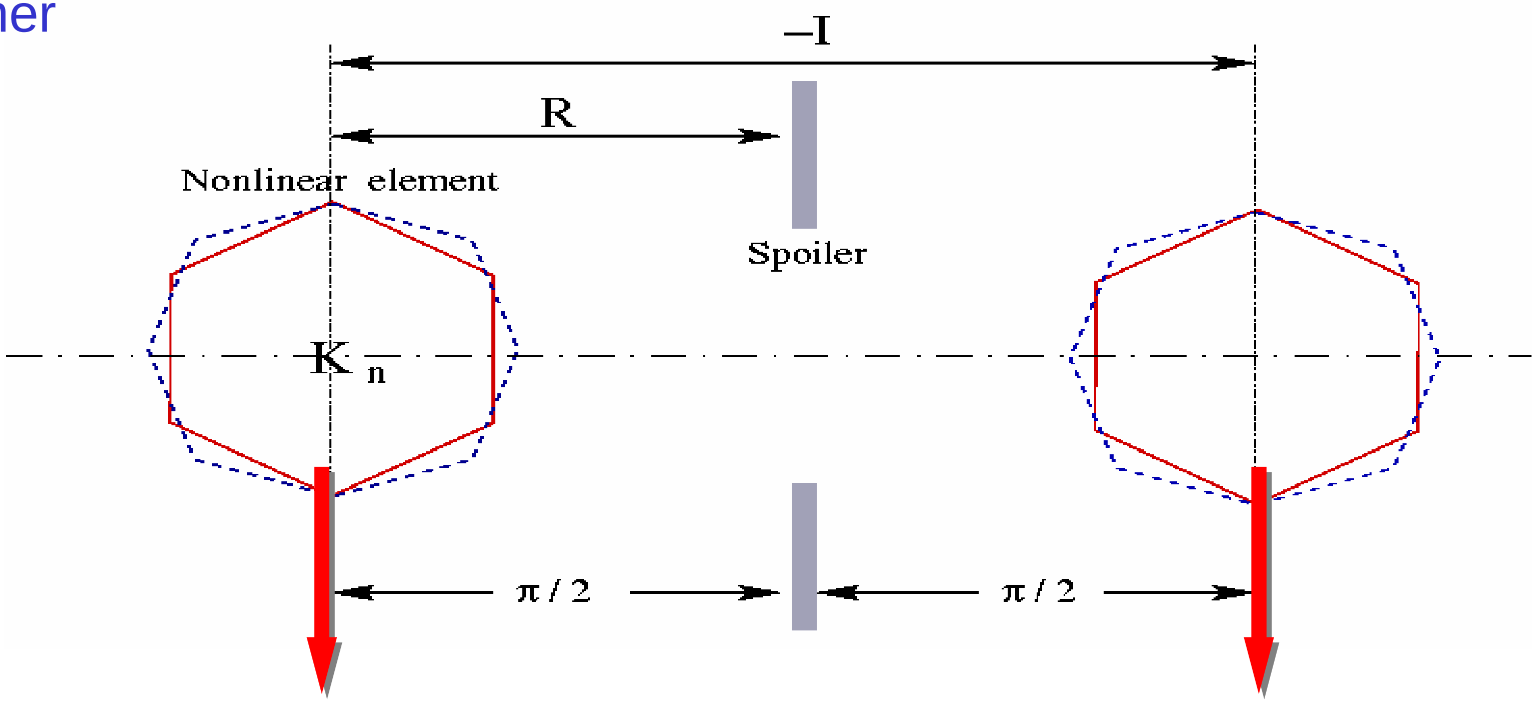
Basic Scheme:

The purpose of the 1st nonlinear element is to blow up beam sizes and particle amplitudes, so that the collimator jaw can be placed further away form the nominal beam orbit (reducing the wake fields) and the beam density is decreased (for collimation survival). A 2nd nonlinear element downstream of the spoiler, and π from the 1st, cancels all the aberrations induces by the former



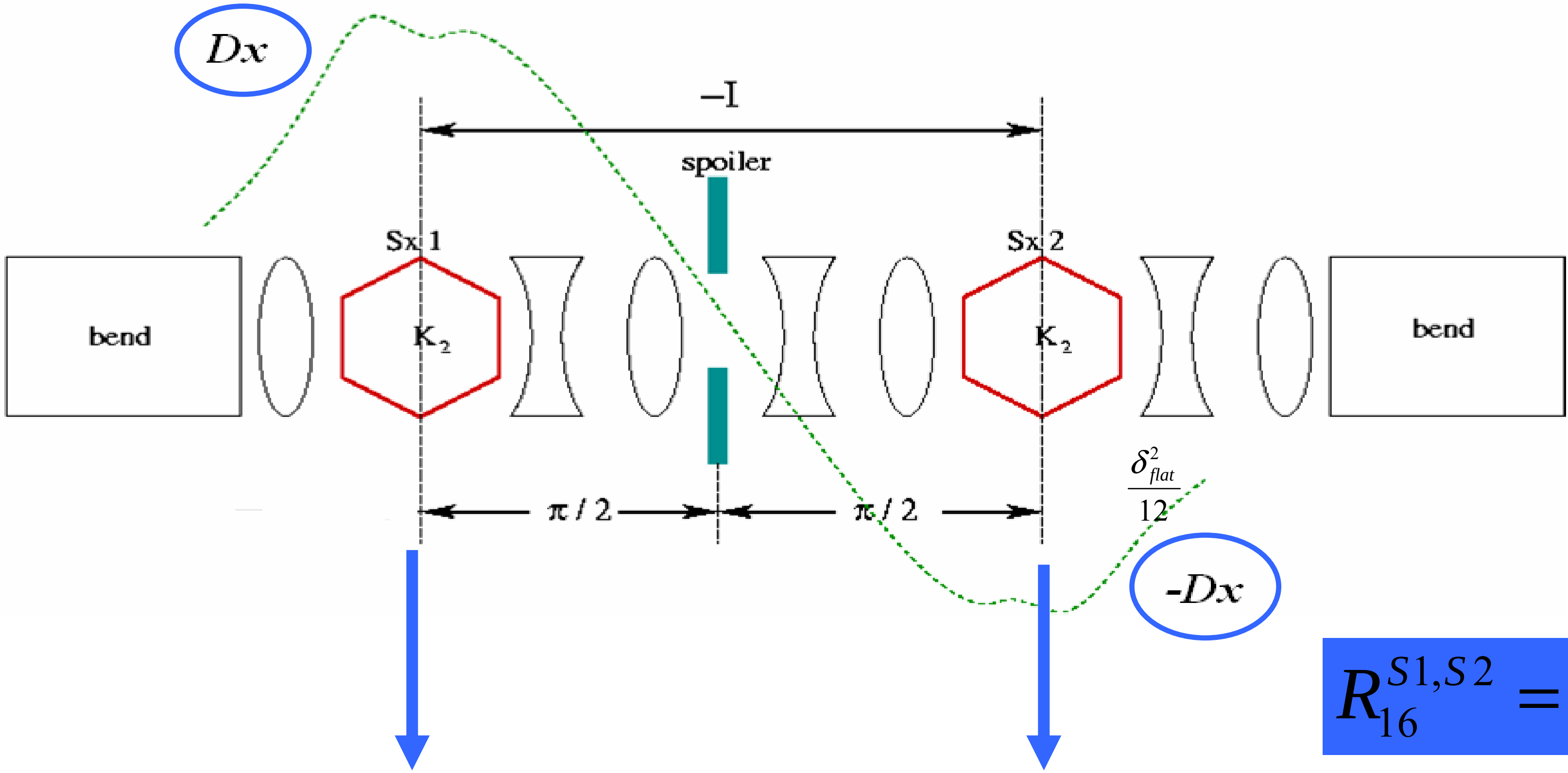
increase beam size at the spoilers

cancel aberrations

The changes respect to the previous:

- collimation only in energy
- maximize the overall fraction of the system occupied by bends and decreased the bending angle until SR became reasonably small
- but no bends between the skews to cancel the geometric aberration ($R_{16}{}^{s1s2} = 0$) avoiding the luminosity degradation
- keep β-functions as regular as possible to avoid the need of chromatic correction

The nonlinear system for CLIC:

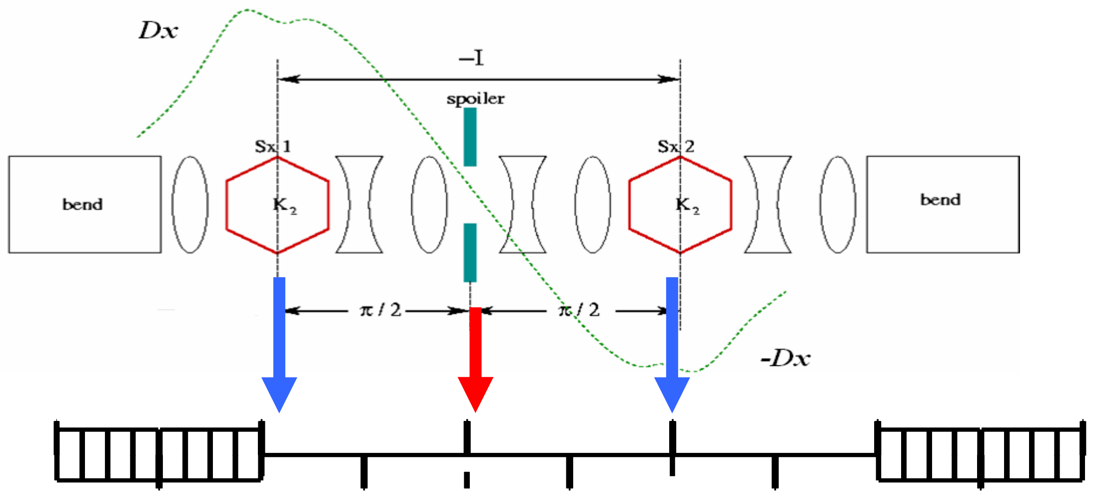


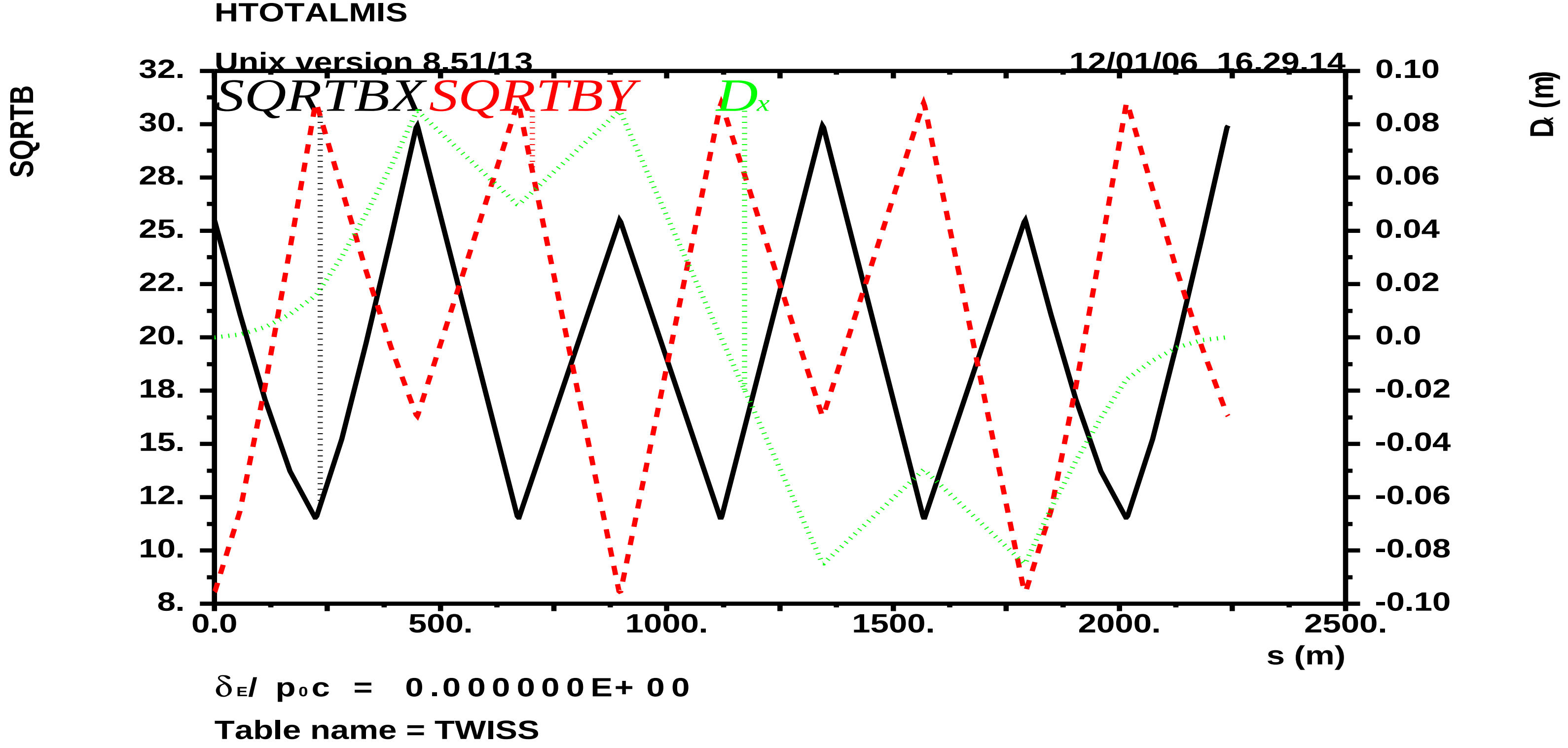
$R_{16}^{S1,S2} = 0$

Increase σ_y at spoiler

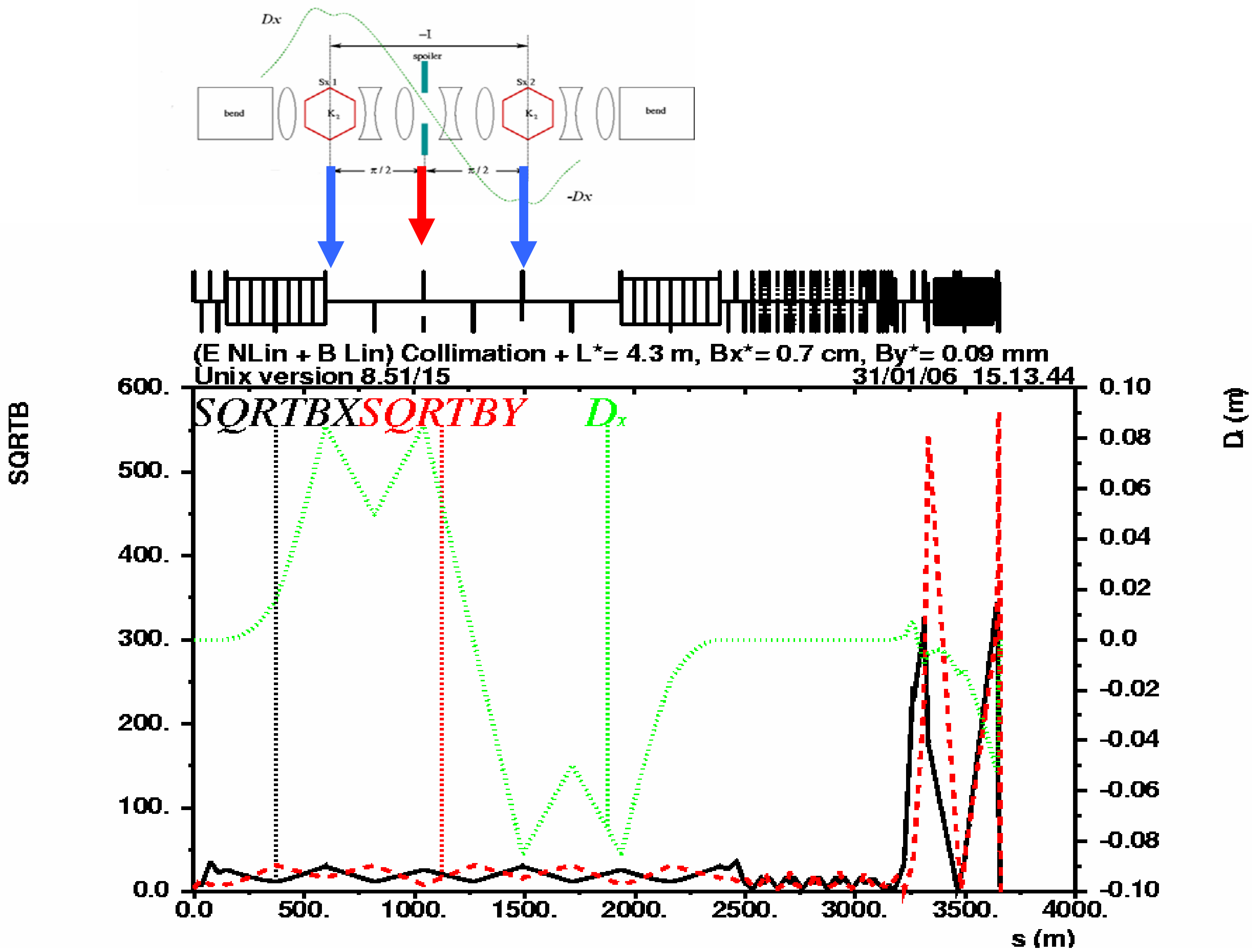
Cancellation of geometric aberrations

The optics solution:





The optics solution in the BDS:

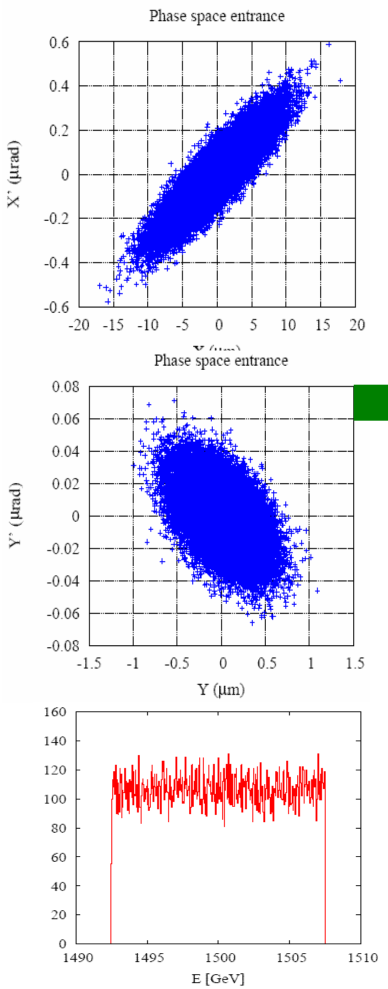


Performance from analytical studies:

E	1.5	Tev
σ_ε	2.8 x 10^{-3}	
ε_x	0.23	pm
ε_y	6.8	fm
δ_{flat}	0.01	
L_t	2536	m
l_d	220	m
θ_b	0.00014	rad
K_s	20.9	m^{-2}
β_x^{s}	896.1	m
β_y^{s}	266.0	m
$\Delta\mu_{x,y}^{s,sp}$	0.25/0.25	2π
$\Delta\mu_{x,y}^{si,sf}$	0.5/0.5	2π
$R_{12}^{s,sp}$	763.2	m
$R_{34}^{s,sp}$	131.5	m
I_5	1.64 x 10^{-21}	
σ_r^{sp}	134.27	μm
Δ	0.013	
a_x^{sp}	1.103	mm
a_y^{sp}	1.669	mm

Tracking studies:

Entrance:



Importance of the benchmarking of codes

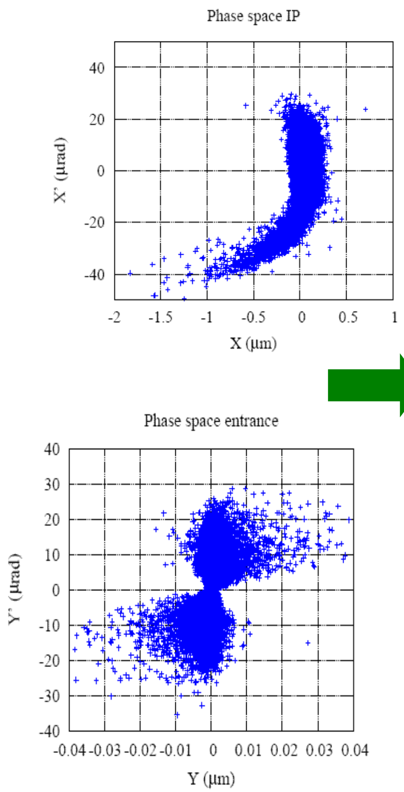
Multiparticle tracking

Optics lattice

MAD
Placet
SAD
…

transport
Lie

IP:

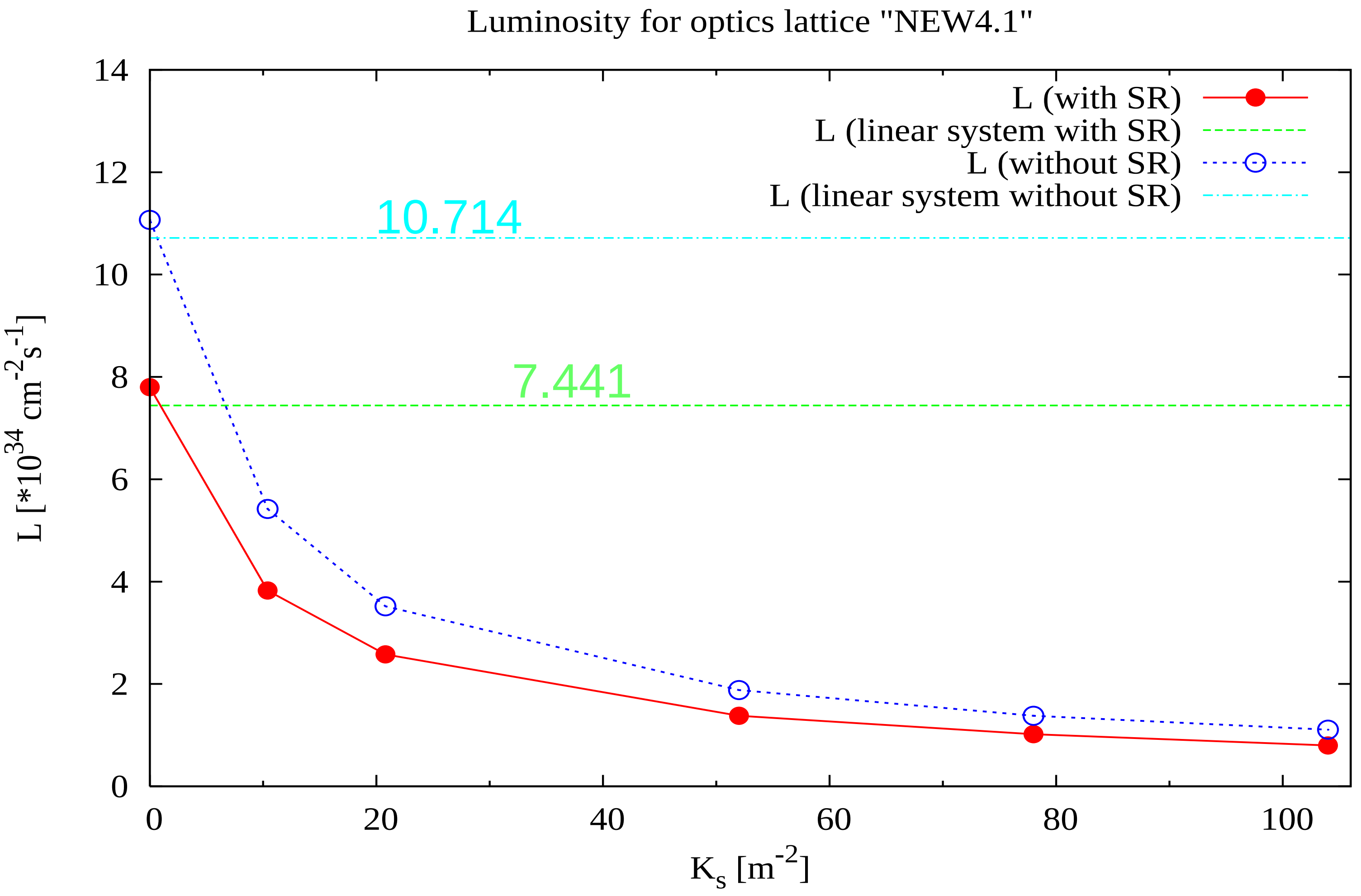


Guinea-Pig

Beam-beam interaction

performance

Luminosity vs skew sextupole strength:



Luminosity optimization:

Optimization of the beam sizes

⬇

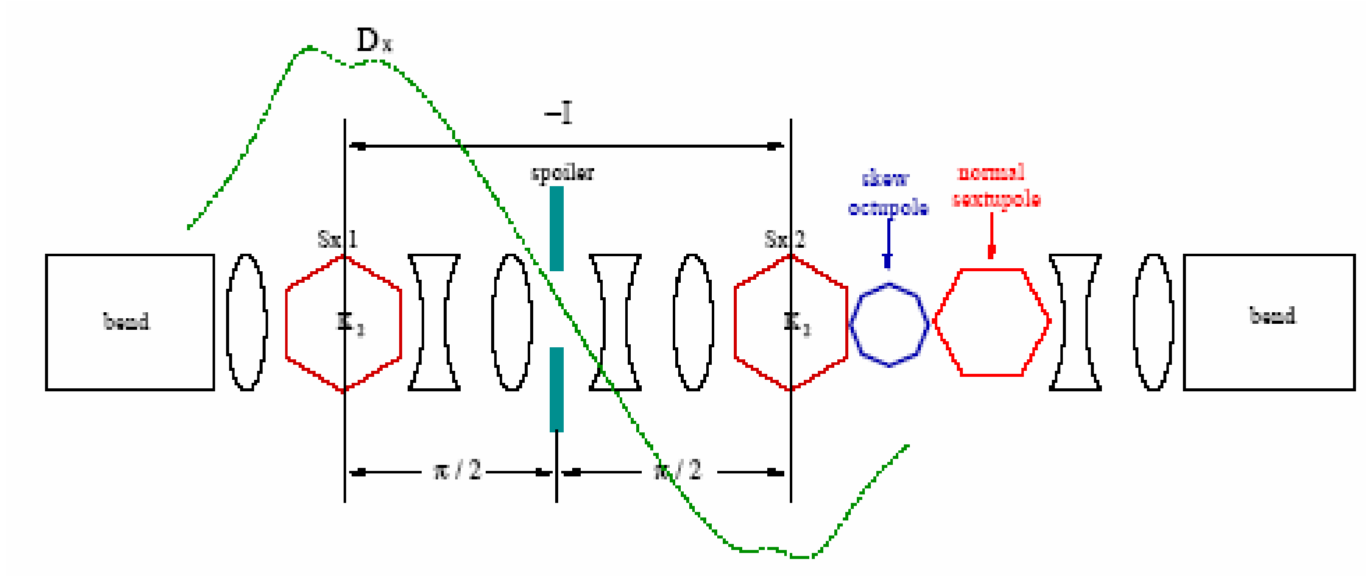
MAPCLASS (Python code)

- Generate/reads MADx input/output
- compute beam sigmas from map coefficients
- use SVD or Simplex algorithm to optimize the sigmas

[R. Tomás,"MAPCLASS: a code to optimize high order aberrations" CLIC Note (2006)]

Luminosity optimization:

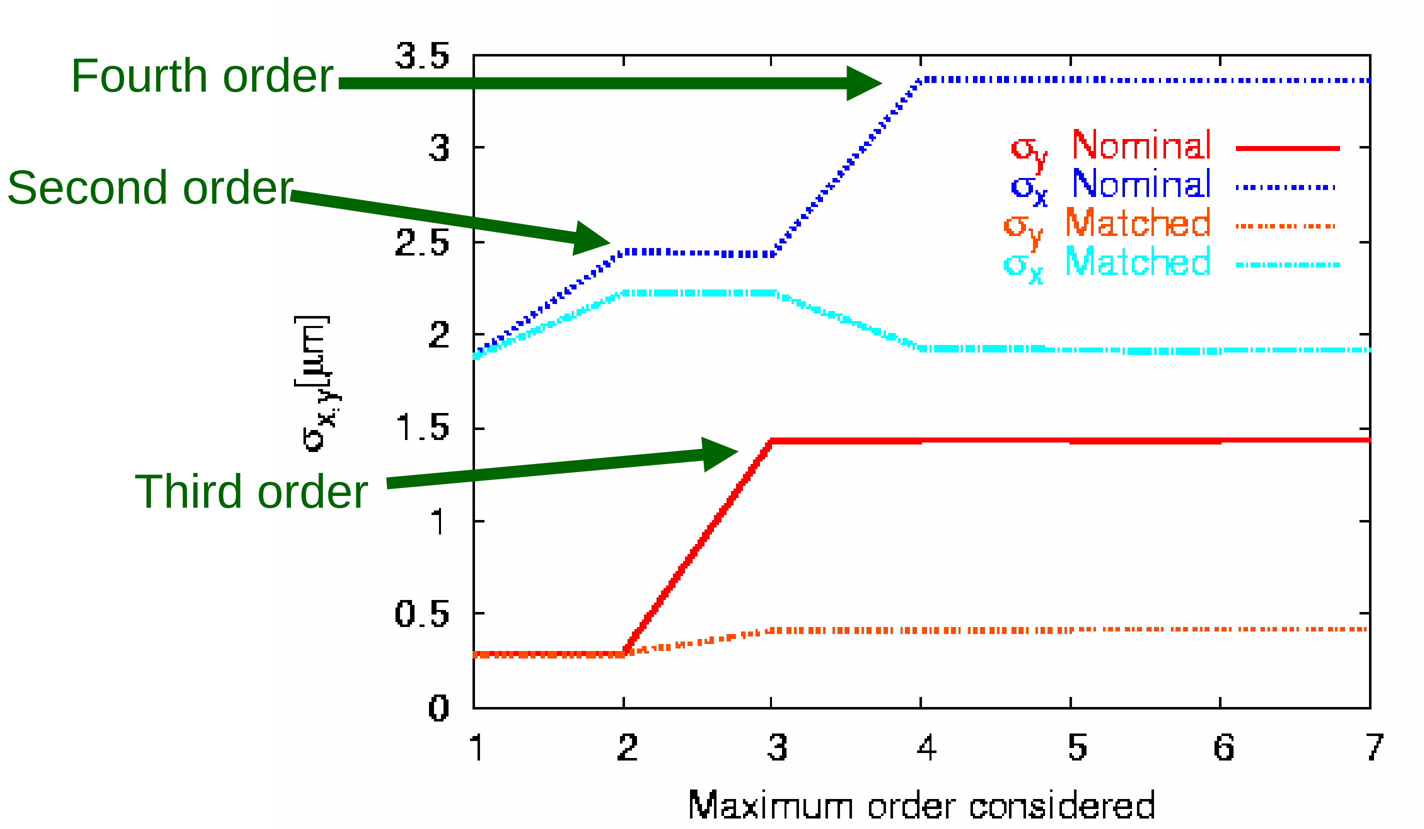
Two additional multipoles for local cancellation of the higher order aberrations



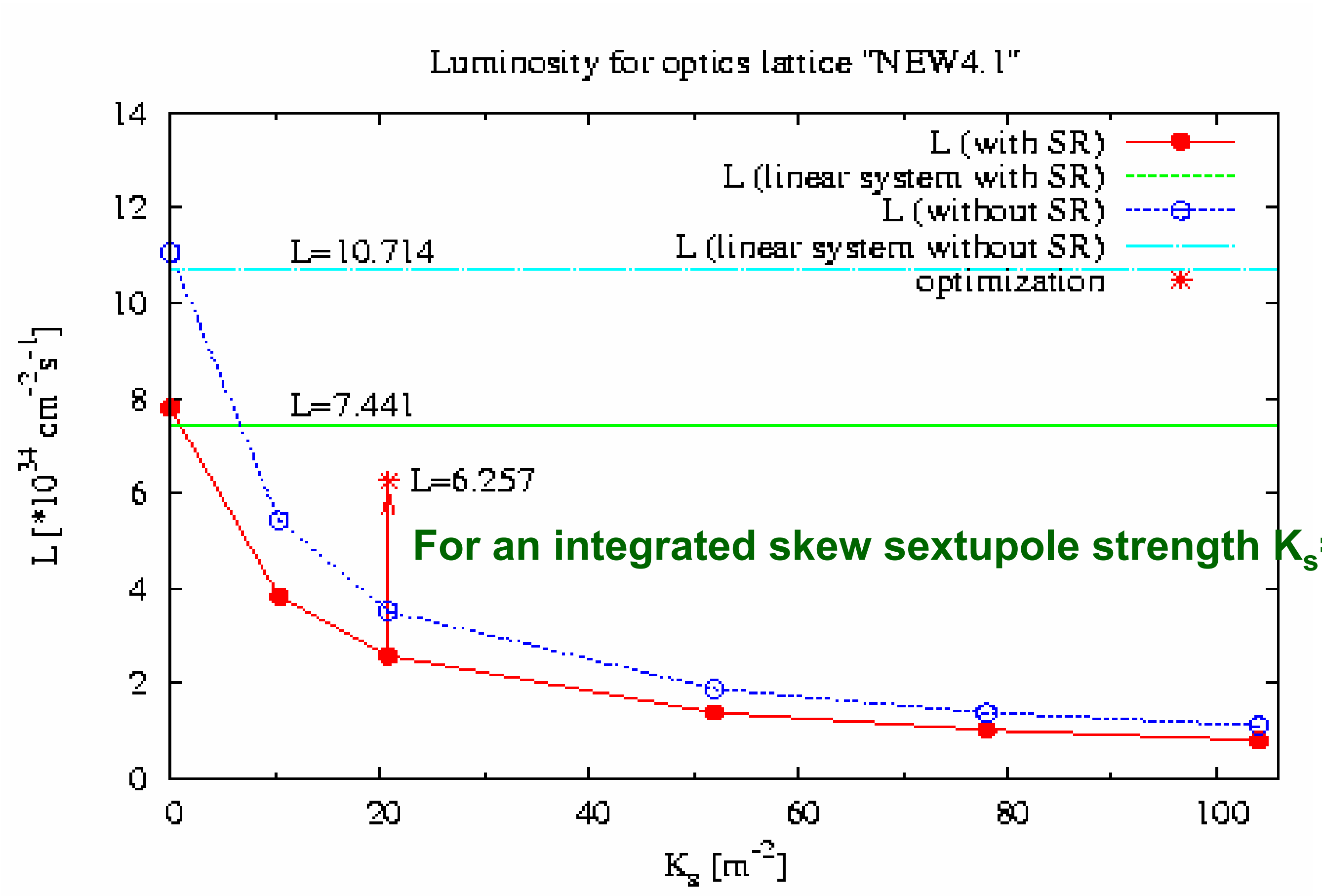
- Skew sextupoles: K_2L =20.86 m^{-2}
- Skew octupole: K_3L= -5464.93 m^{-3}
- Normal sextupole: K_2L = -0.8 m^{-2}

Thin multipoles have been used for a first test !

beam sizes order by order:



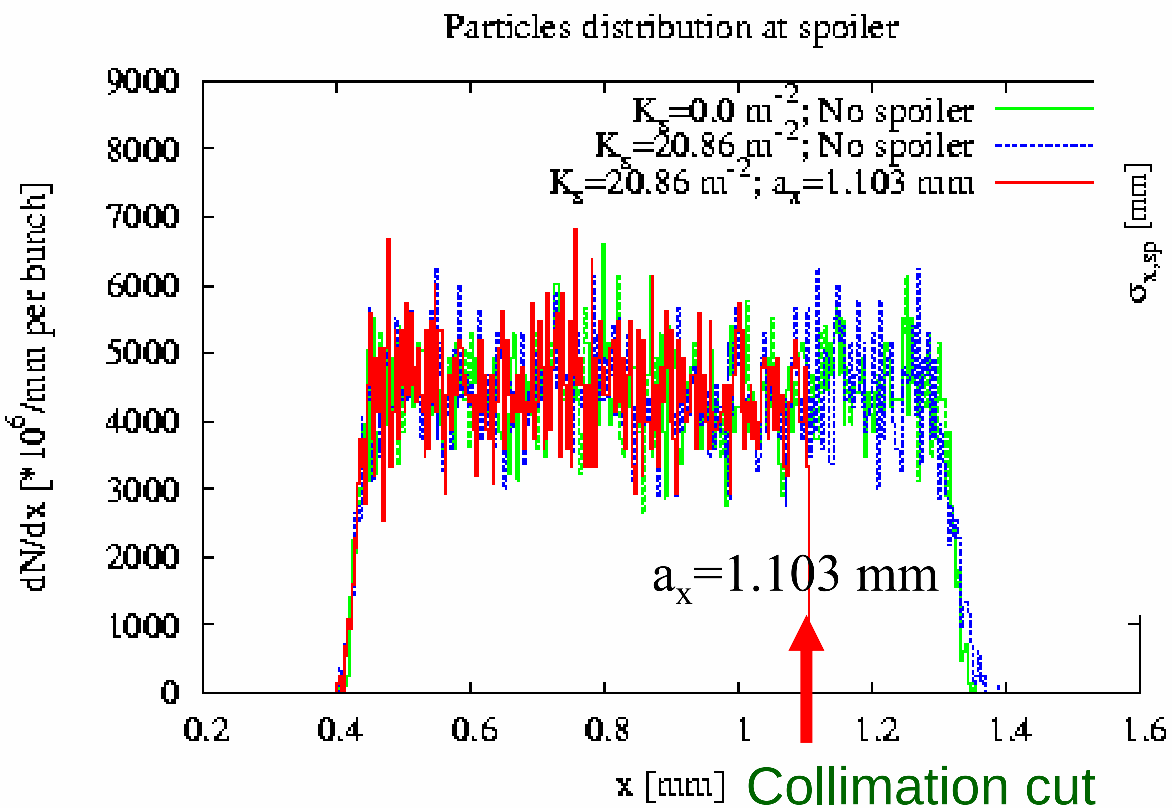
Luminosity optimization:



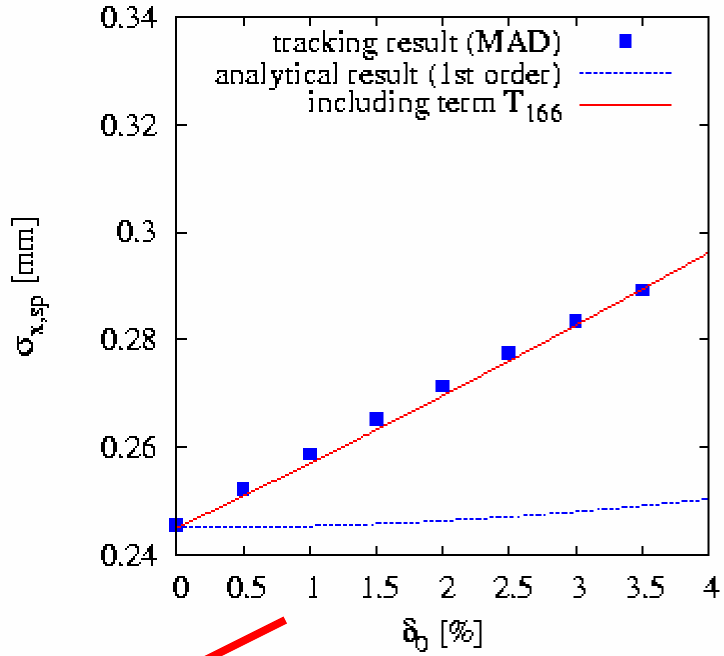
For an integrated skew sextupole strength K_s=20.86 m^{-2}

Particles distribution at the spoiler:

Horizontal plane:



Collimation cut



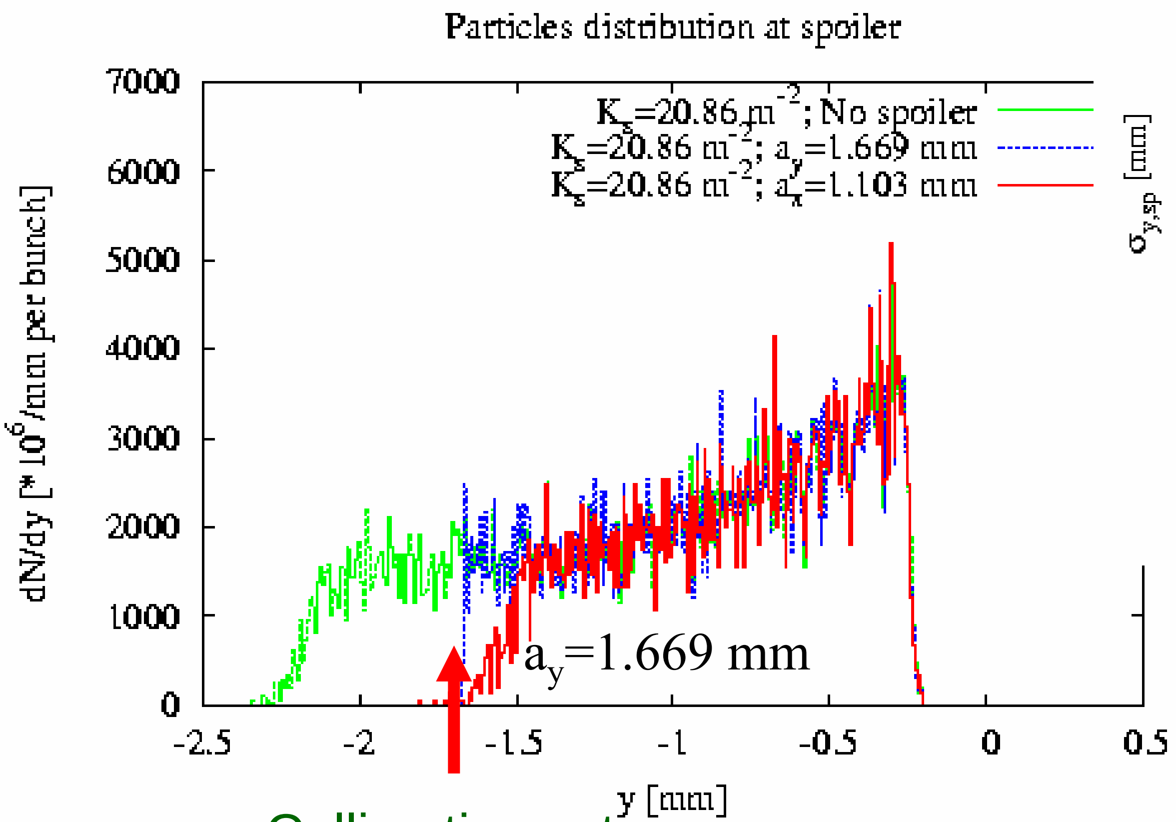
δ_0=1%

$$a_x = \Delta D_{x,spoiler}$$

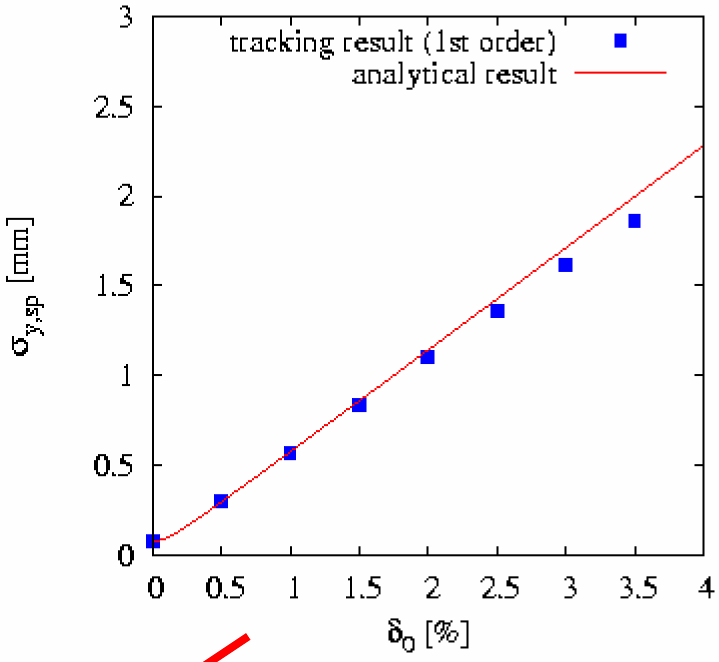
$$a_y = \frac{1}{2}\left|R_{34} K_s\right| D^2_{x,sext}\,\Delta^2$$

Particles distribution at the spoiler:

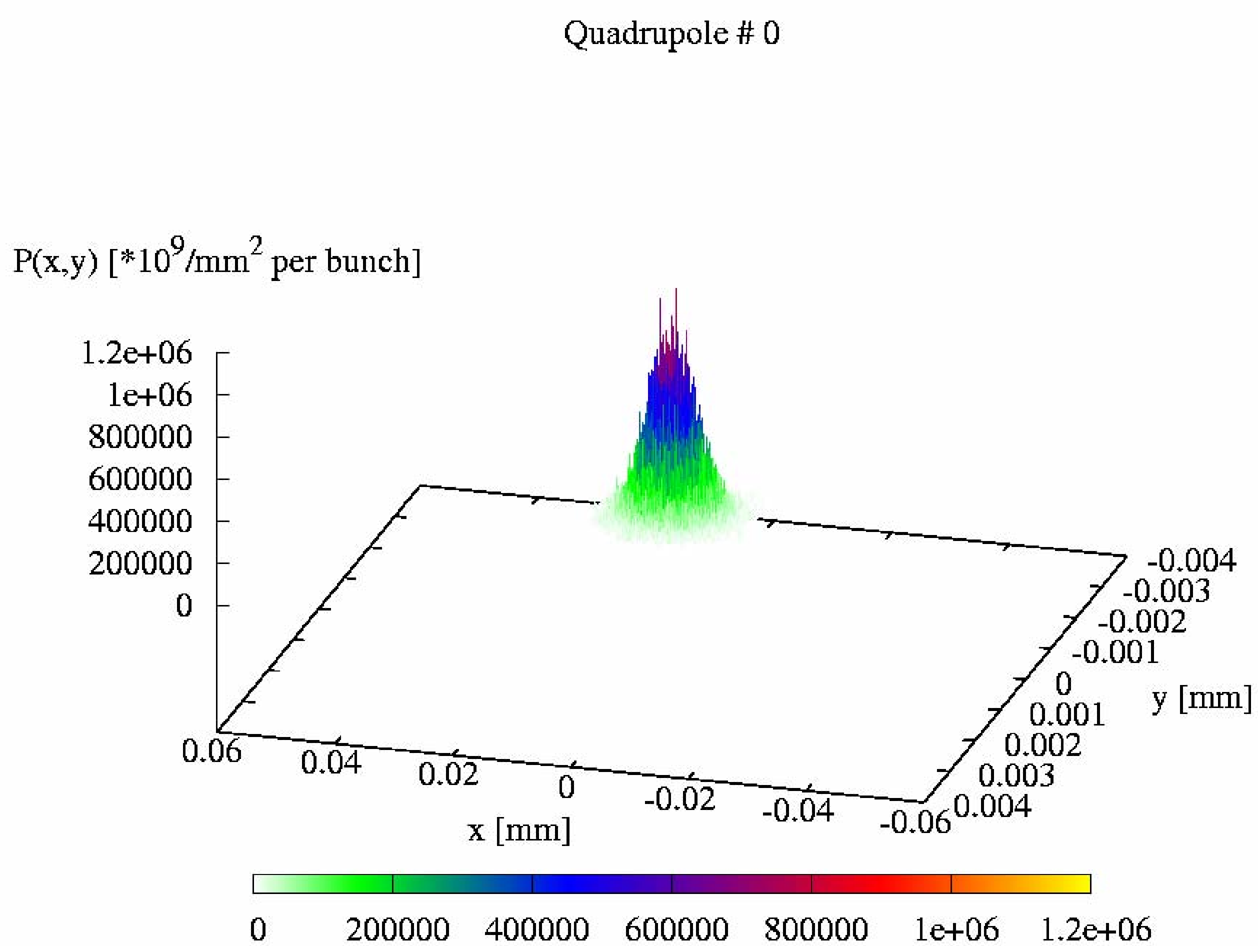
Vertical plane:

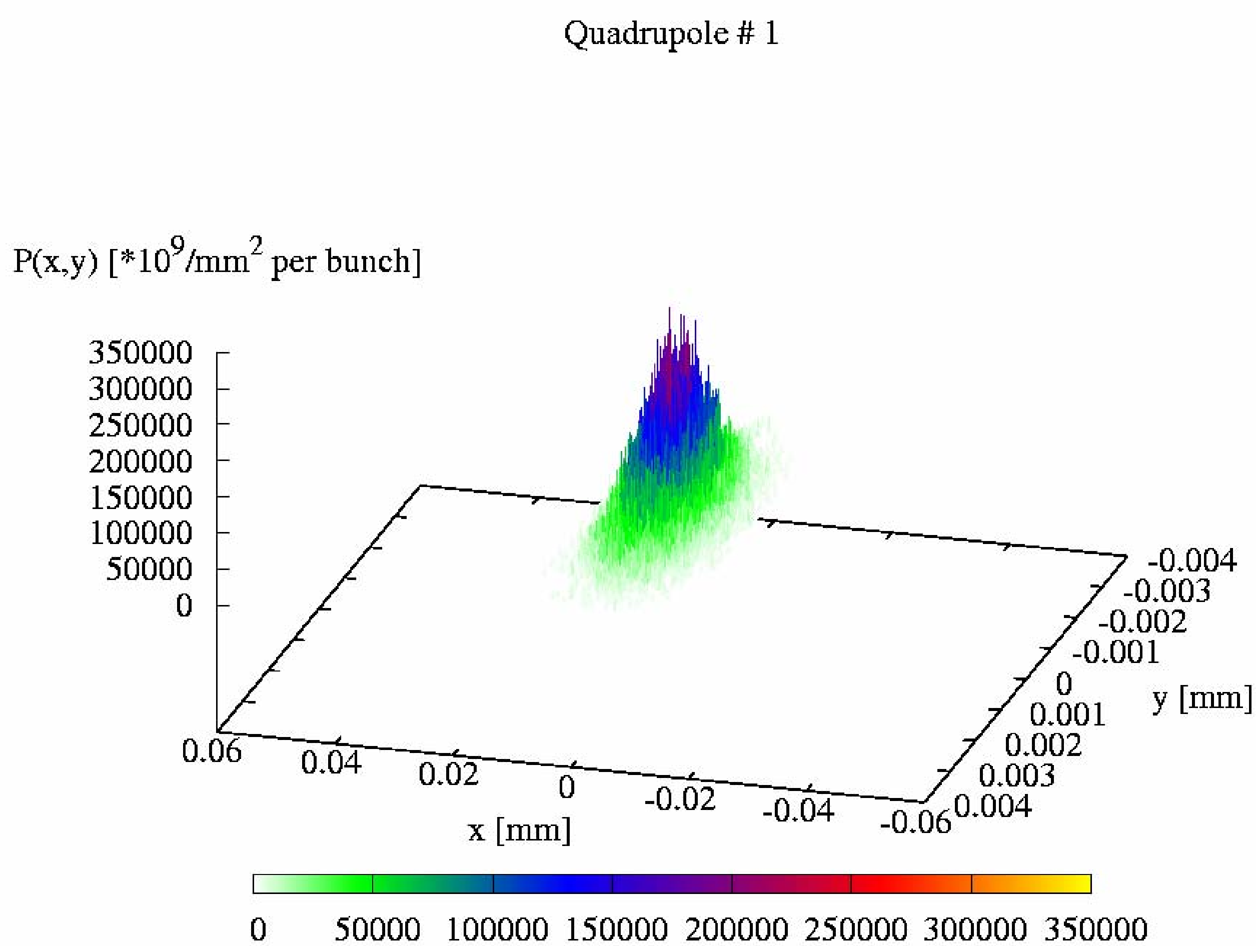


Collimation cut

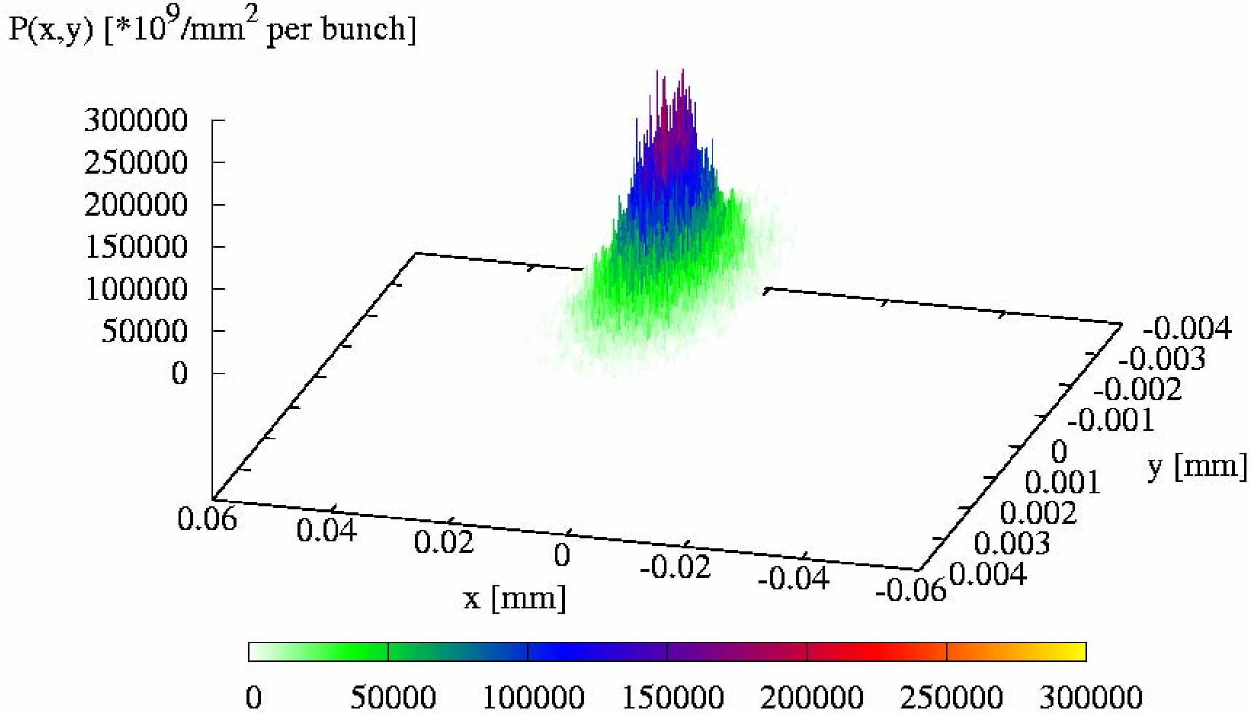


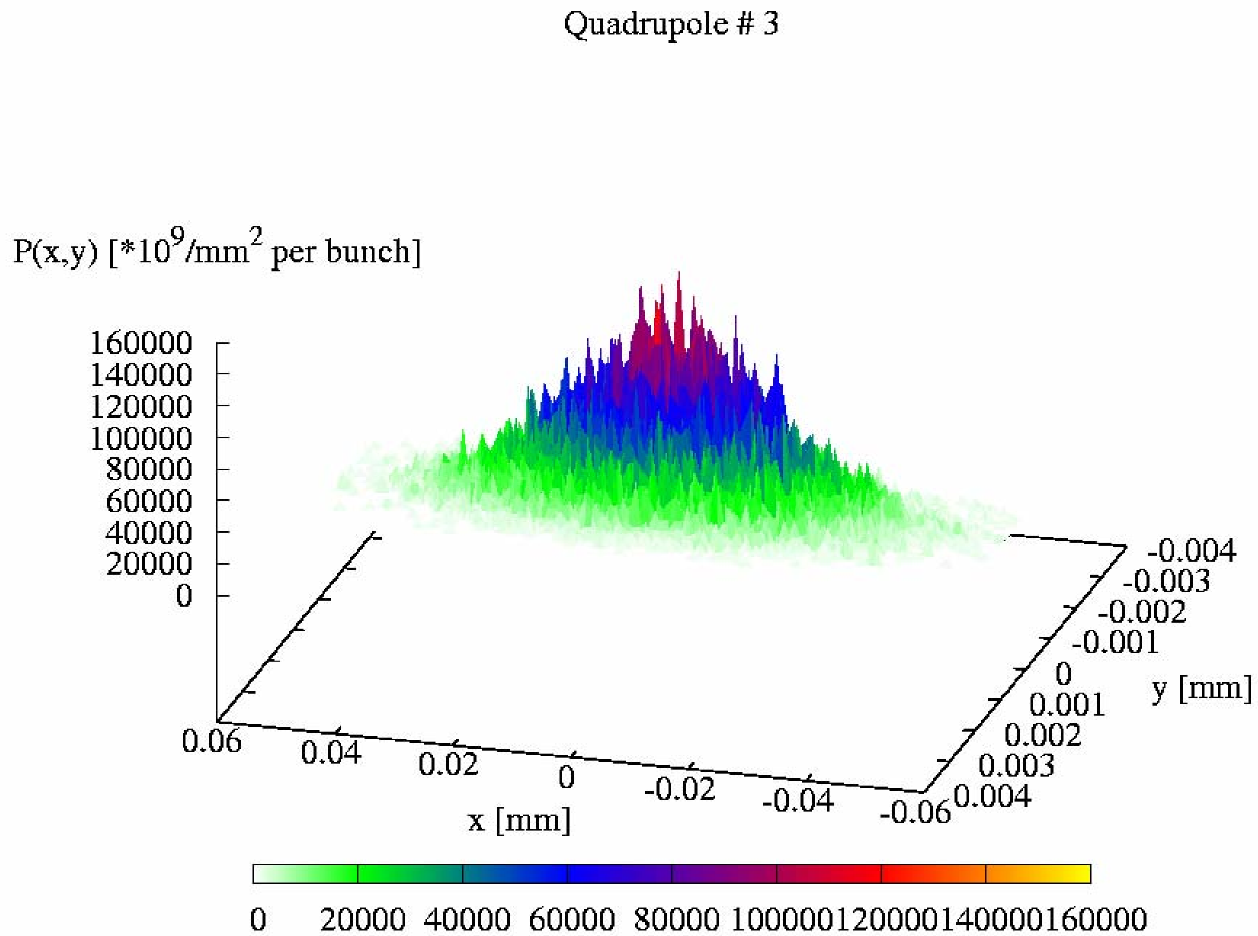
δ_0=1%


Quadrupole # 0
P(x,y) [*10^9/mm^2 per bunch]
1.2e+06
1e+06
800000
600000
400000
200000
0
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
x [mm]
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
y [mm]
0
200000
400000
600000
800000
1e+06
1.2e+06


Quadrupole # 1
P(x,y) [*10^9/mm^2 per bunch]
350000
300000
250000
200000
150000
100000
50000
0
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
x [mm]
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
y [mm]
0
50000
100000
150000
200000
250000
300000
350000

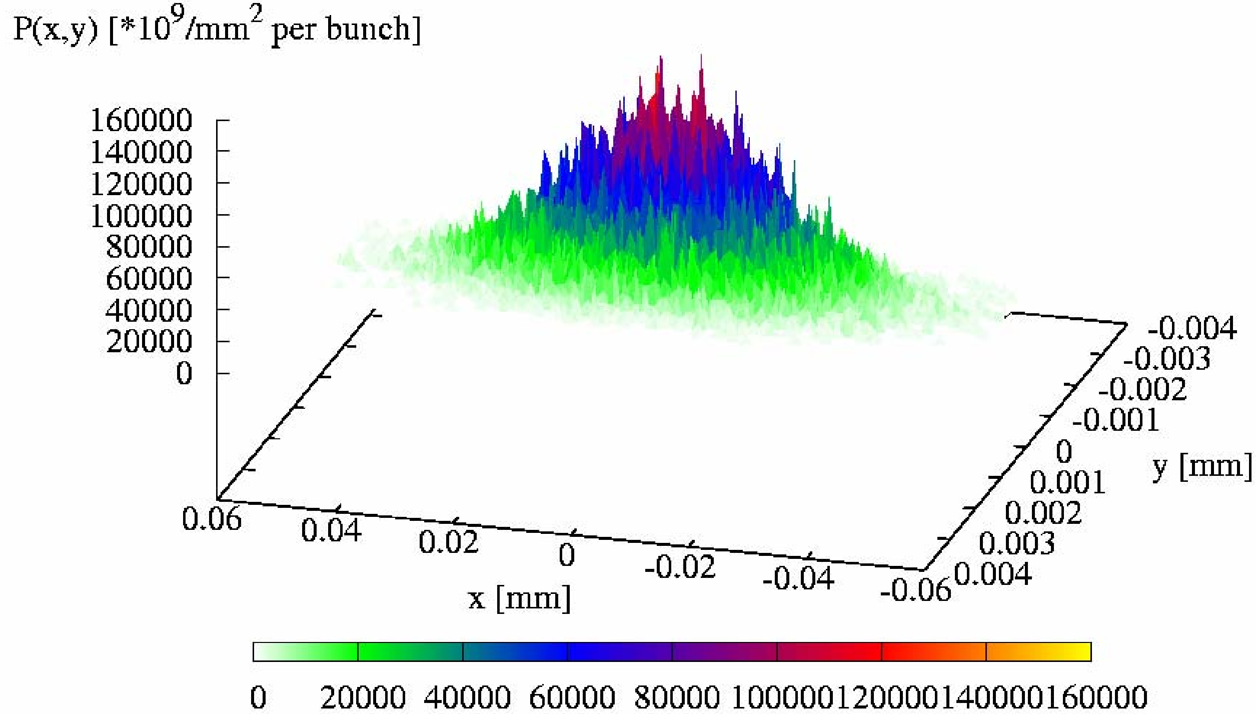
Quadrupole # 2



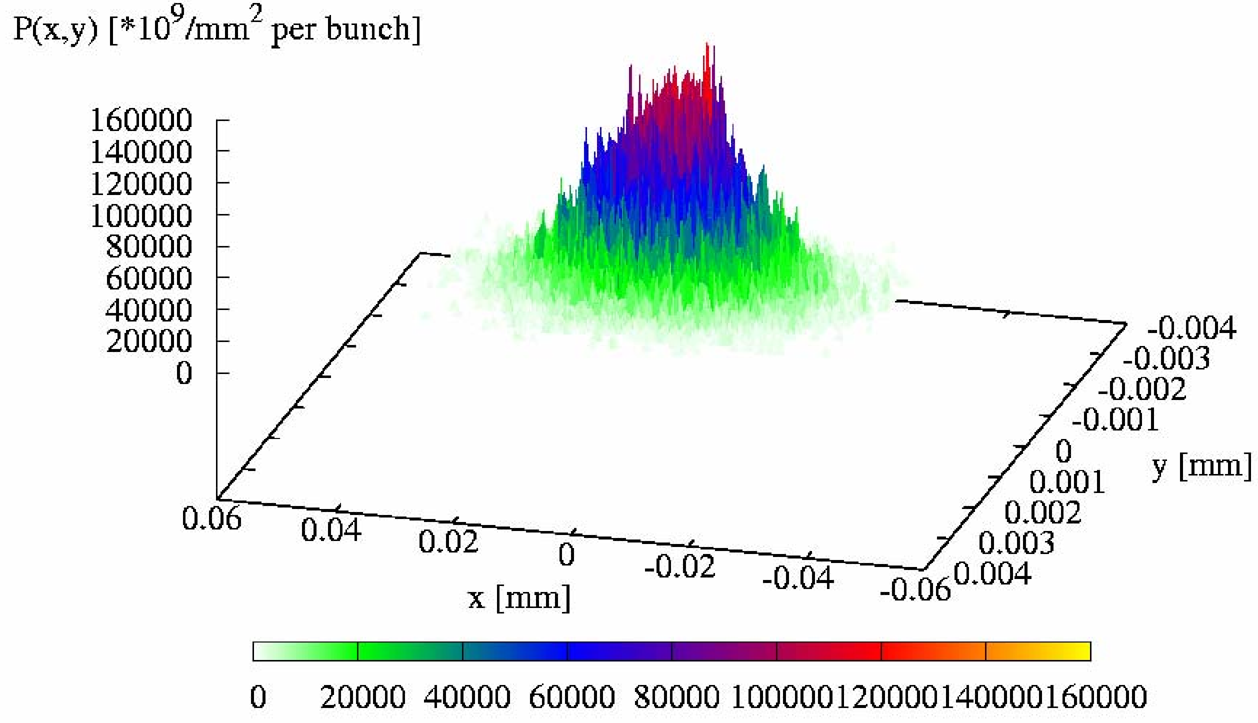


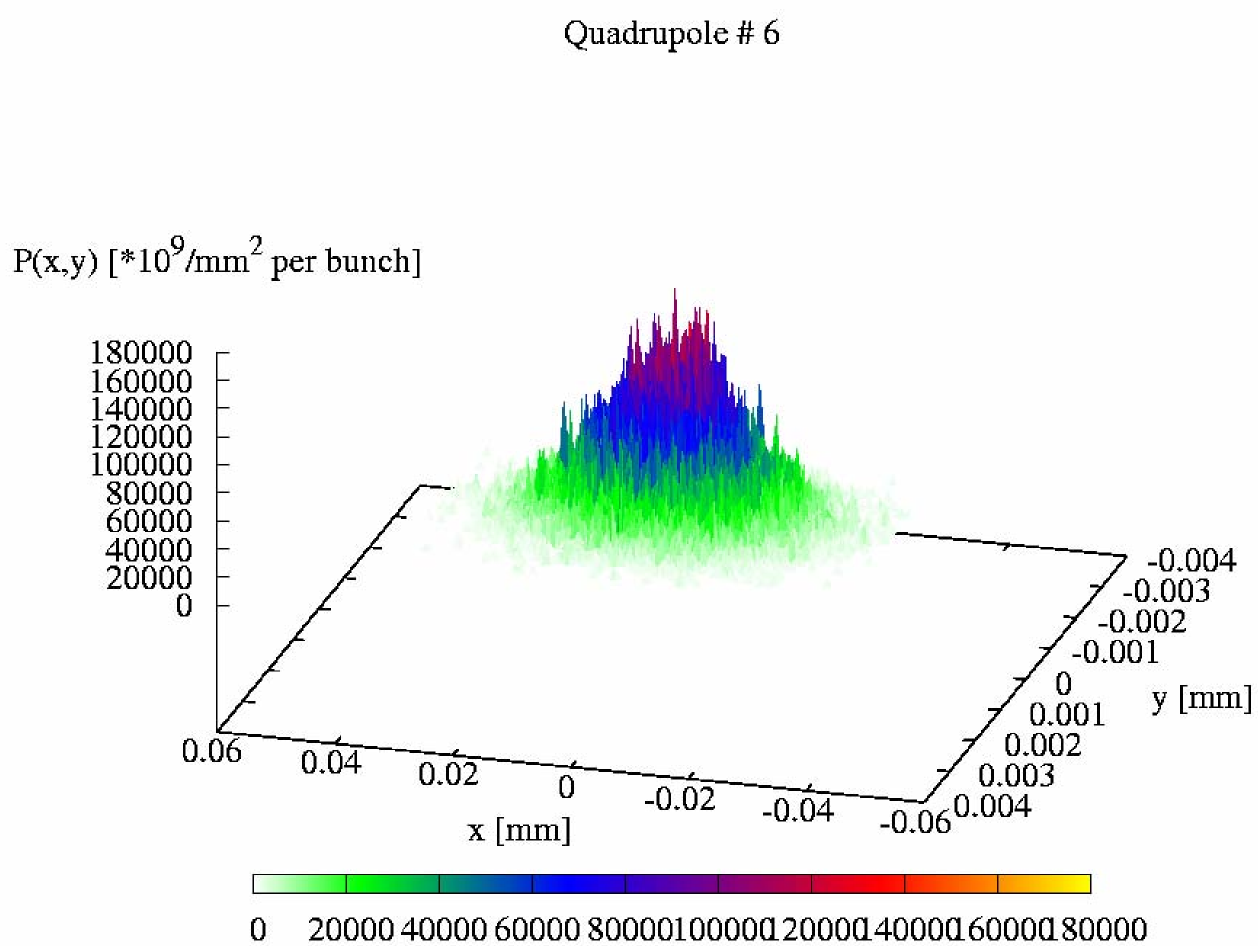
Quadrupole # 3
P(x,y) [*10^9/mm^2 per bunch]
160000
140000
120000
100000
80000
60000
40000
20000
0
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
x [mm]
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
y [mm]
0
20000
40000
60000
80000
100000
120000
140000
160000

Quadrupole # 4

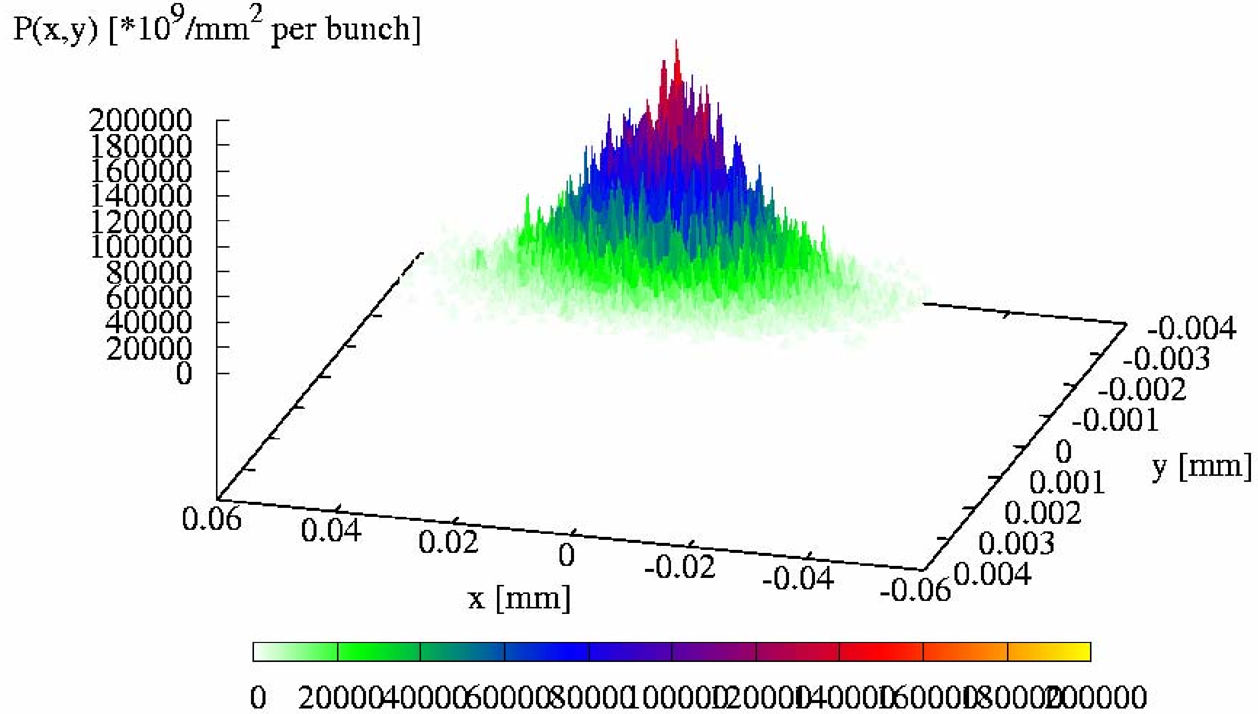


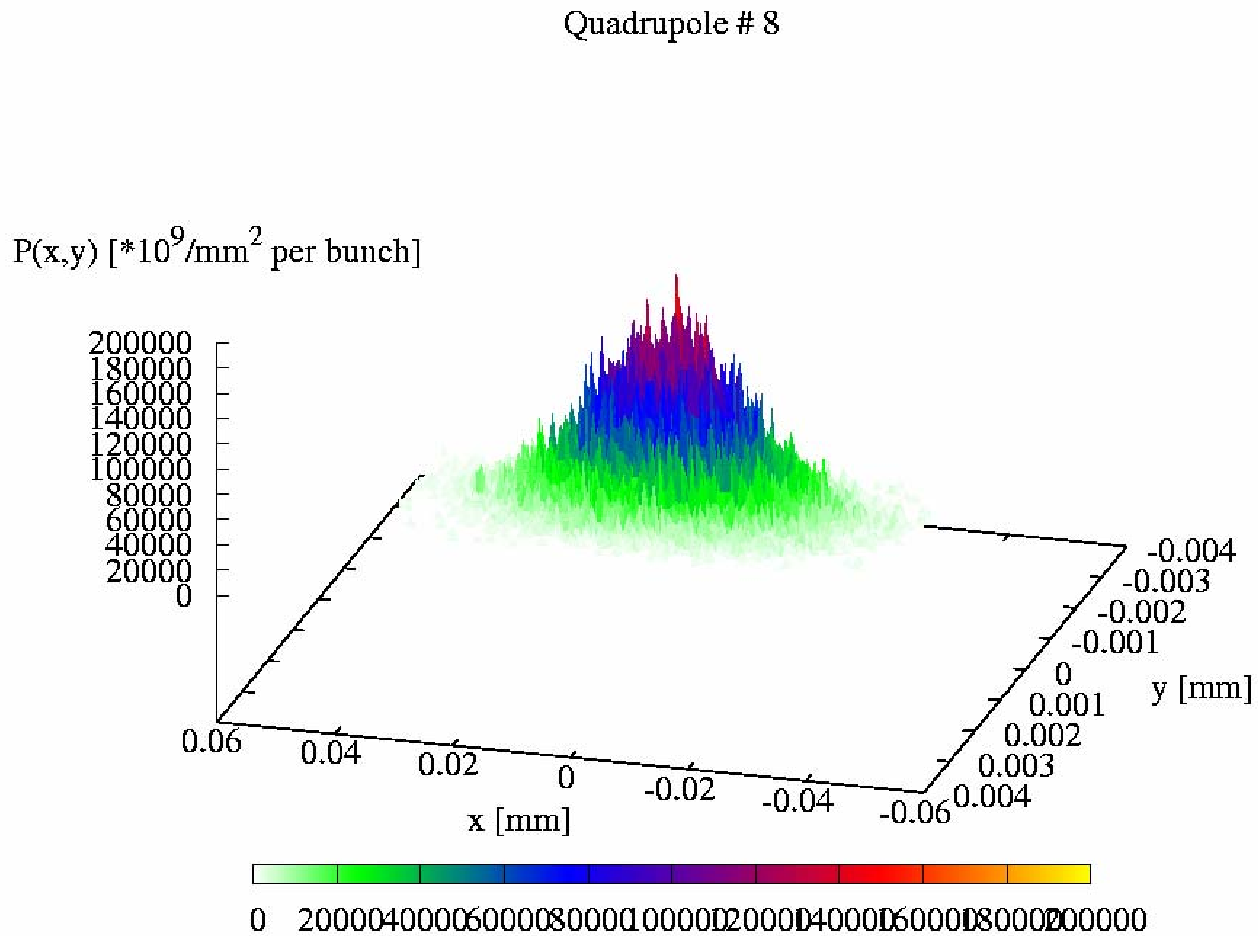
Quadrupole # 5

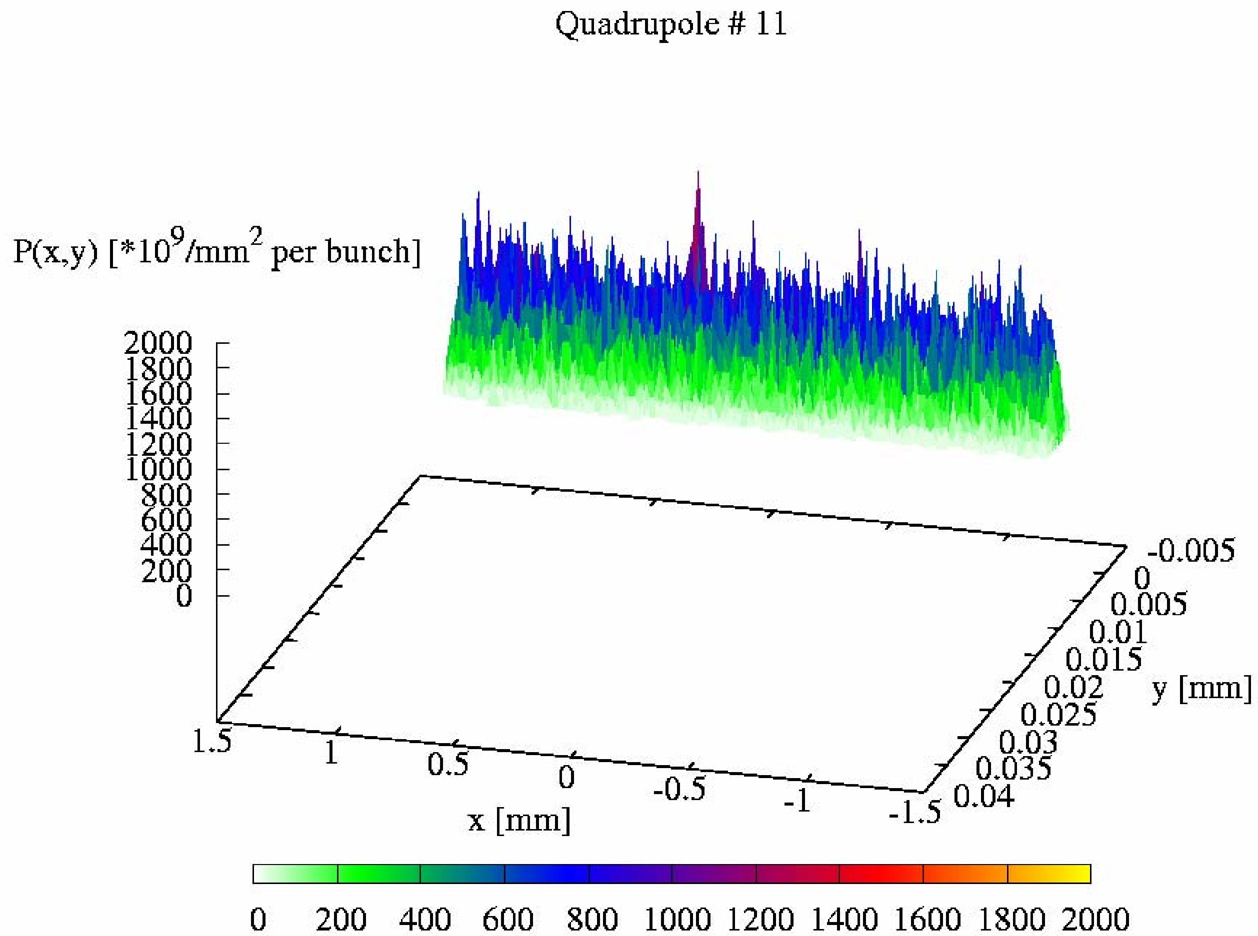



Quadrupole # 6
P(x,y) [*10^9/mm^2 per bunch]
180000
160000
140000
120000
100000
80000
60000
40000
20000
0
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
x [mm]
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
y [mm]
0 20000 40000 60000 80000 100000 120000 140000 160000 180000

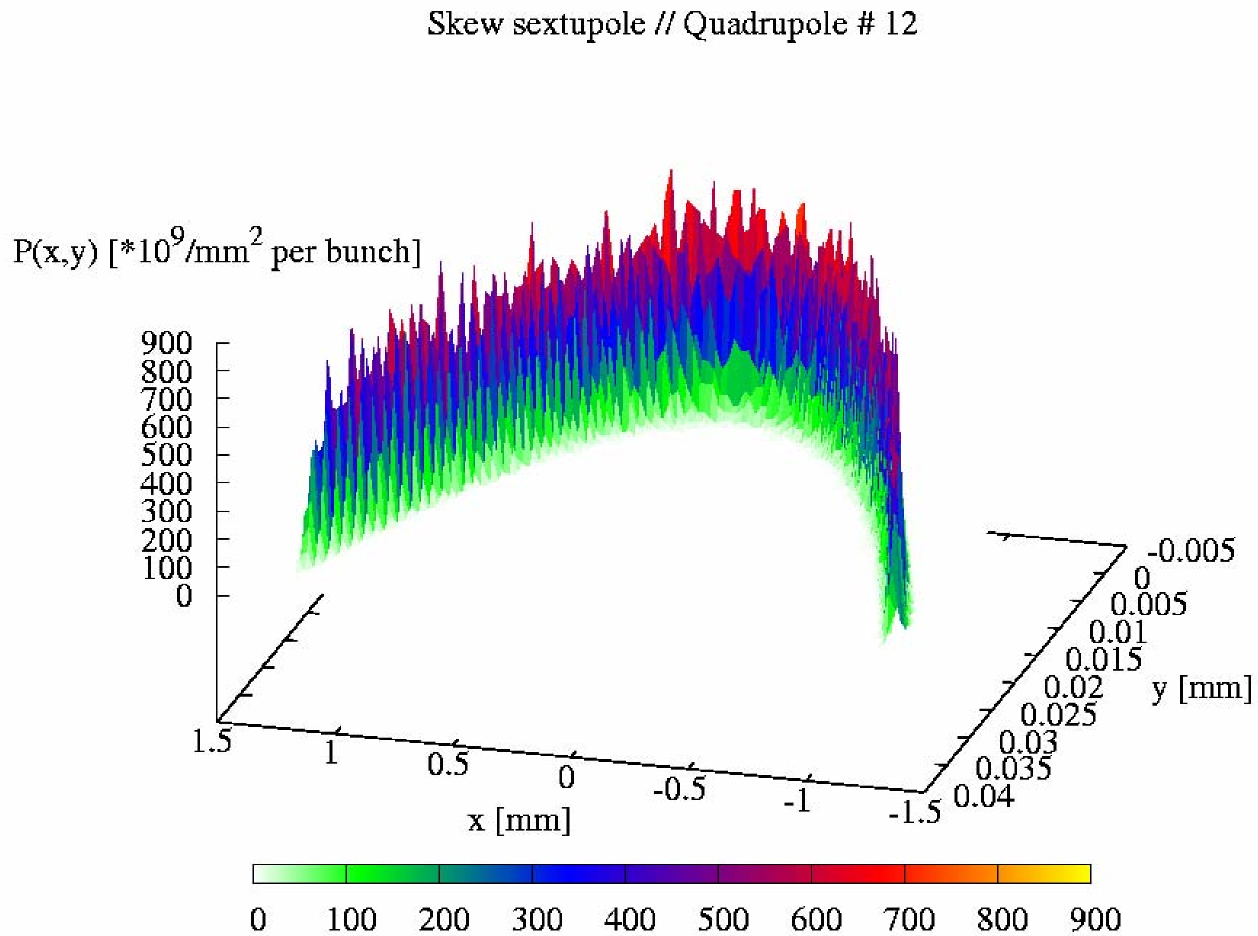
Quadrupole # 7

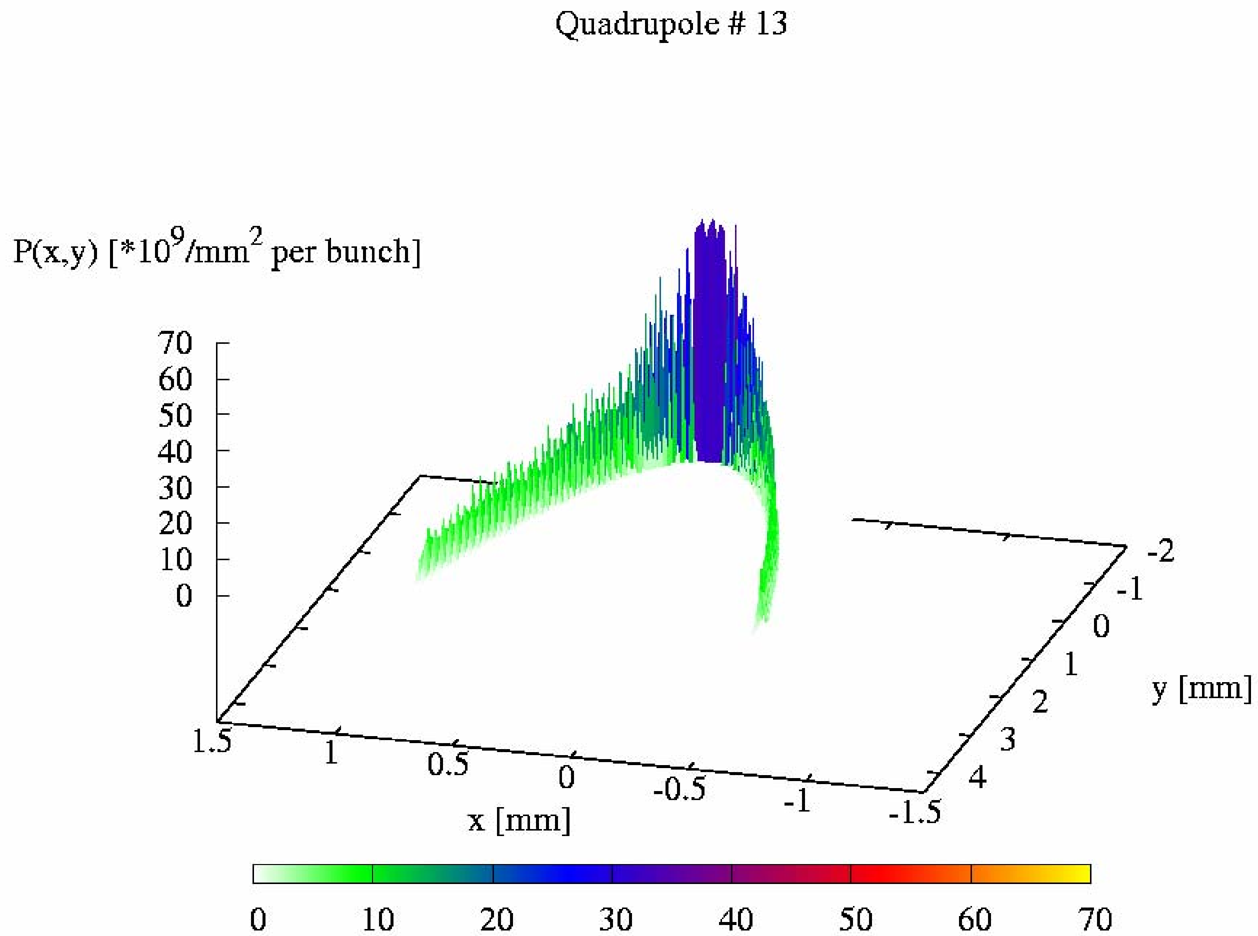



Quadrupole # 8
P(x,y) [*10^9/mm^2 per bunch]
200000
180000
160000
140000
120000
100000
80000
60000
40000
20000
0
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
x [mm]
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
y [mm]
0 20000 40000 60000 80000 100000 120000 140000 160000 180000 200000

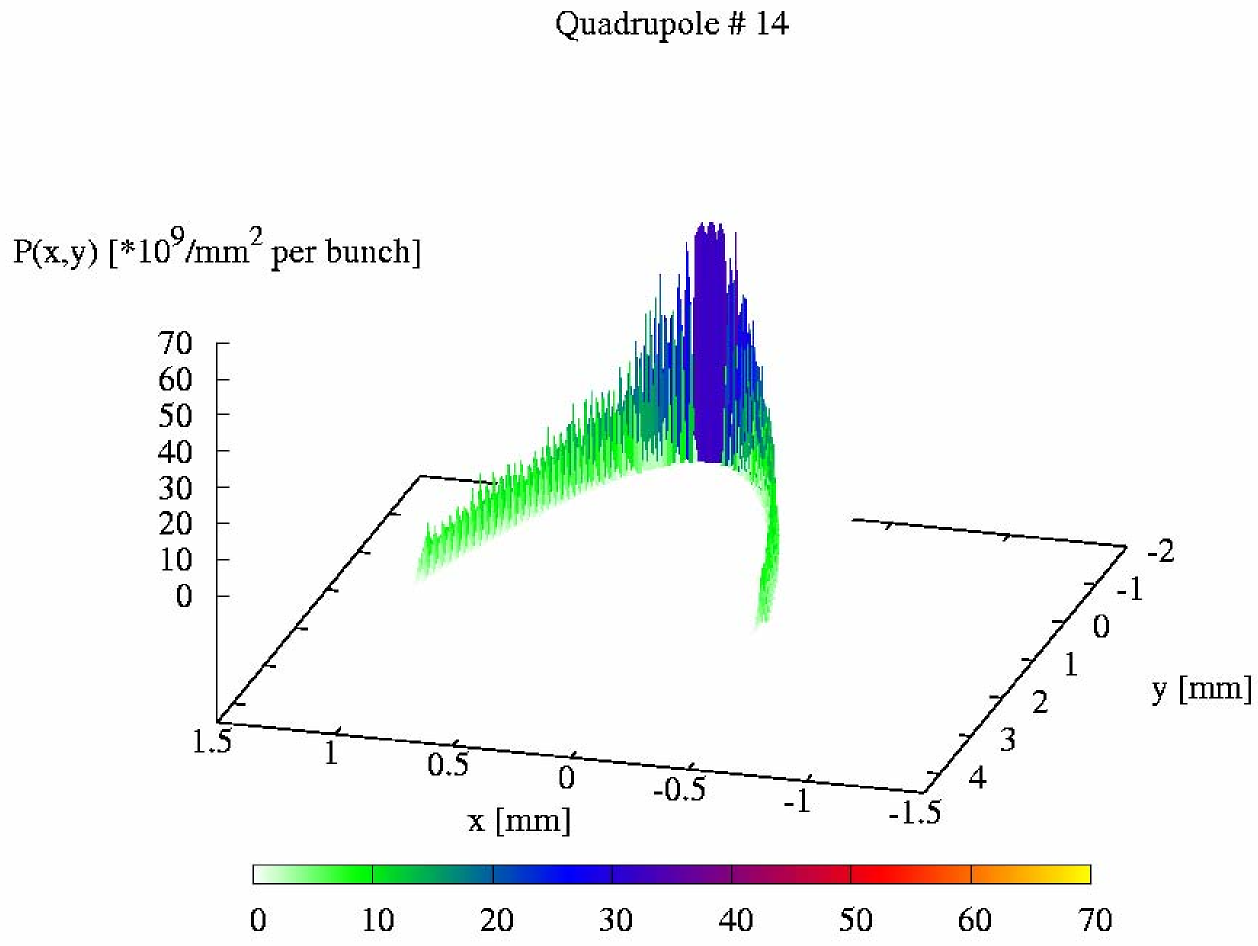


Quadrupole # 11
P(x,y) [*10^9/mm^2 per bunch]
2000
1800
1600
1400
1200
1000
800
600
400
200
0
1.5
1
0.5
0
-0.5
-1
-1.5
x [mm]
-0.005
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
y [mm]
0 200 400 600 800 1000 1200 1400 1600 1800 2000

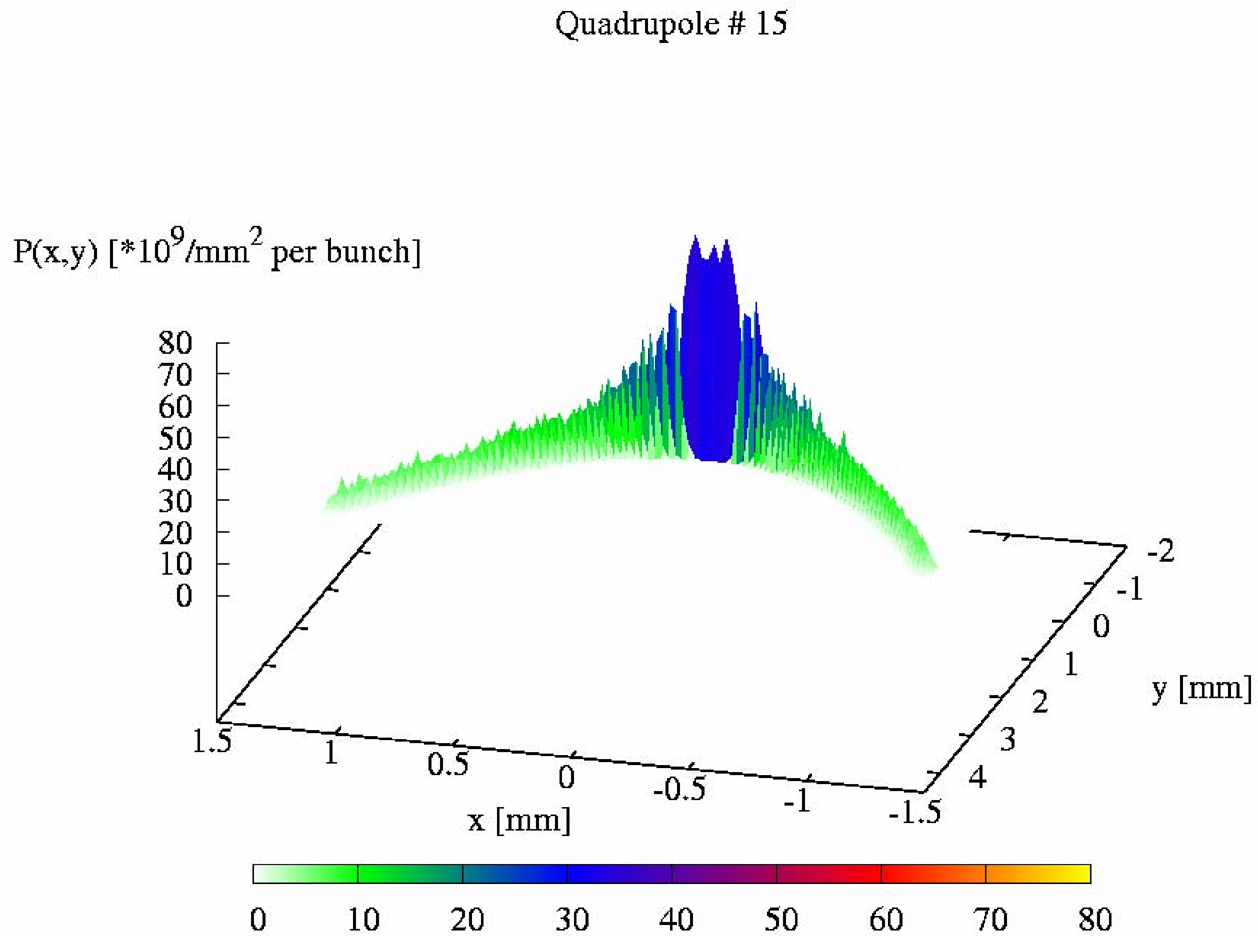

Skew sextupole // Quadrupole # 12
P(x,y) [*10^9/mm^2 per bunch]
900
800
700
600
500
400
300
200
100
0
1.5
1
0.5
0
-0.5
-1
-1.5
x [mm]
-0.005
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
y [mm]
0 100 200 300 400 500 600 700 800 900

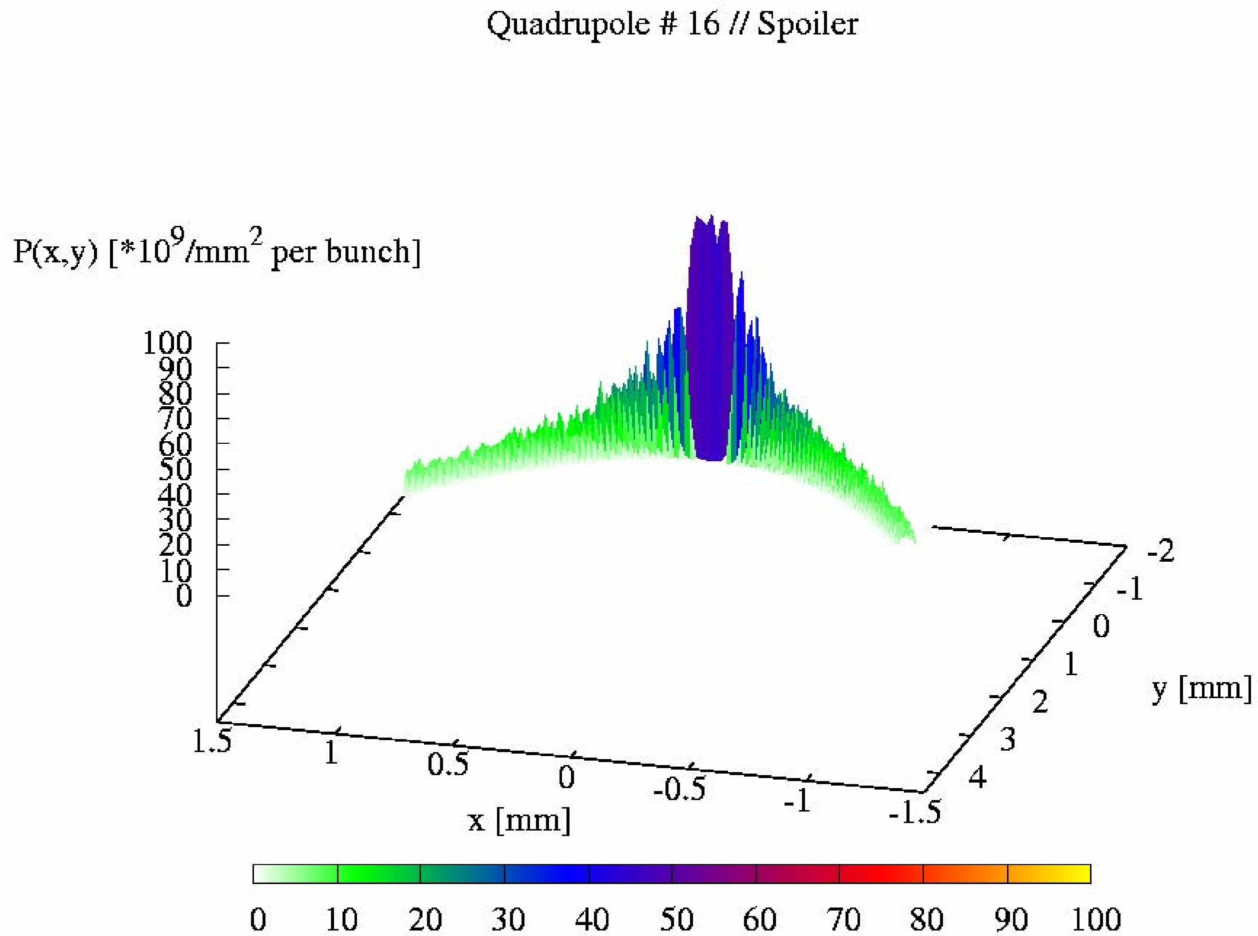


Quadrupole # 13
P(x,y) [*10^9/mm^2 per bunch]
70
60
50
40
30
20
10
0
1.5
1
0.5
0
-0.5
-1
-1.5
x [mm]
-2
-1
0
1
2
3
4
y [mm]
0
10
20
30
40
50
60
70



Quadrupole # 14
P(x,y) [*10^9/mm^2 per bunch]
70
60
50
40
30
20
10
0
1.5
1
0.5
0
-0.5
-1
-1.5
x [mm]
-2
-1
0
1
2
3
4
y [mm]
0
10
20
30
40
50
60
70

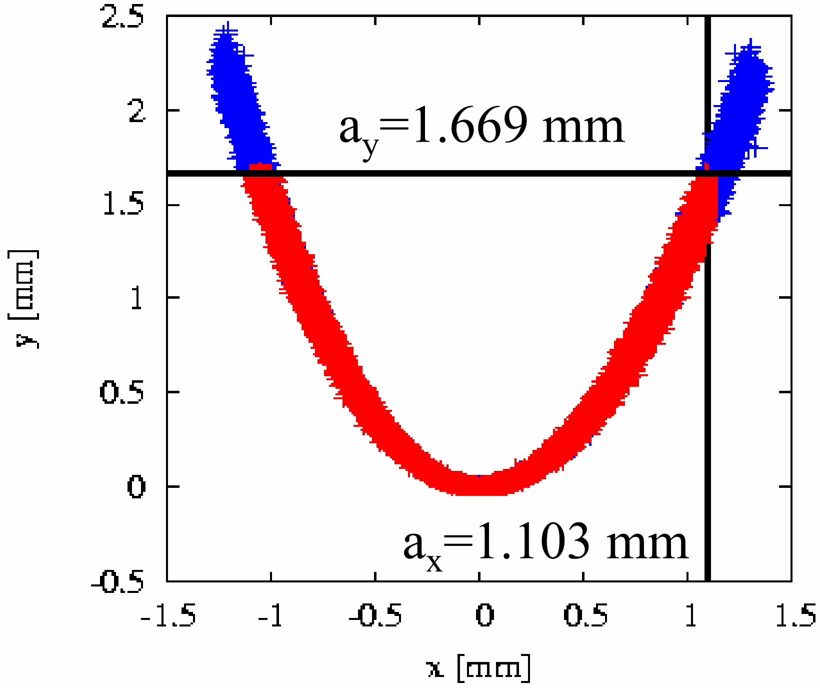

Quadrupole # 15
P(x,y) [*10^9/mm^2 per bunch]
80
70
60
50
40
30
20
10
0
1.5
1
0.5
0
-0.5
-1
-1.5
x [mm]
-2
-1
0
1
2
3
4
y [mm]
0
10
20
30
40
50
60
70
80

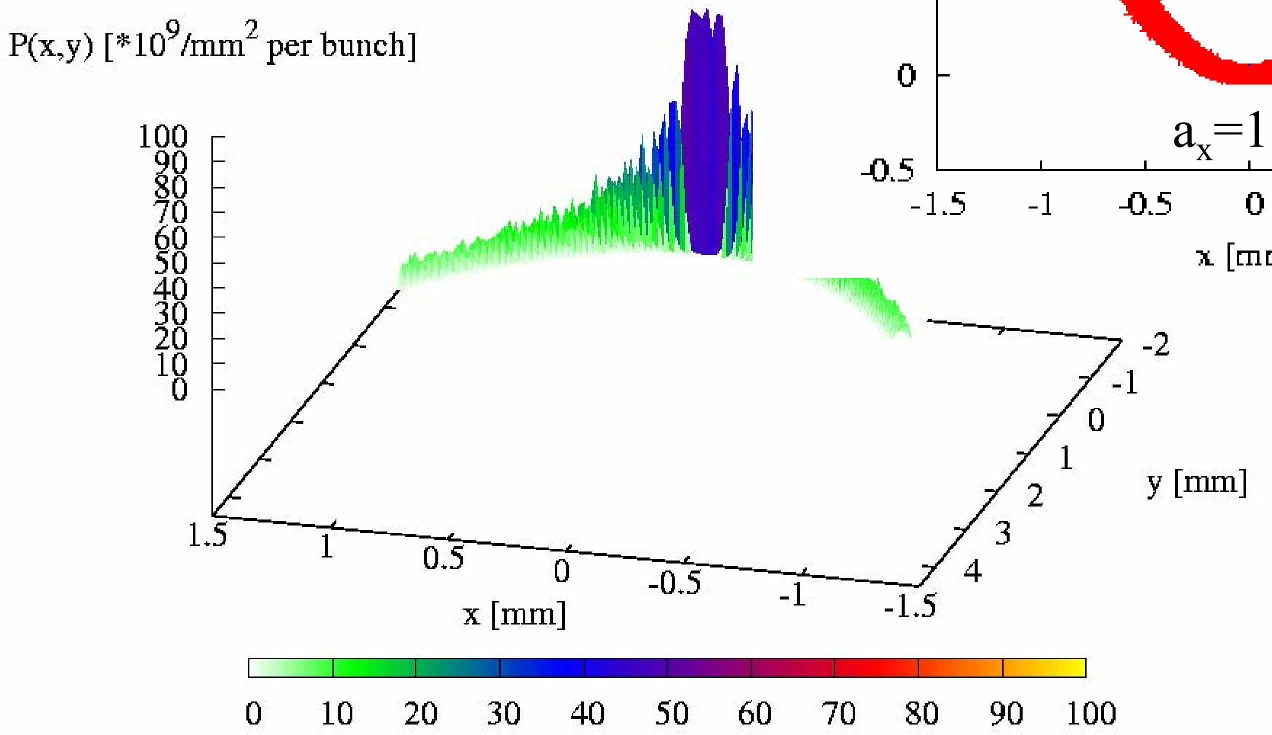

Quadrupole # 16 // Spoiler
P(x,y) [*10^9/mm^2 per bunch]
100
90
80
70
60
50
40
30
20
10
0
1.5
1
0.5
0
-0.5
-1
-1.5
x [mm]
-2
-1
0
1
2
3
4
y [mm]
0 10 20 30 40 50 60 70 80 90 100

Collimation gap

Quadrupole # 16 //

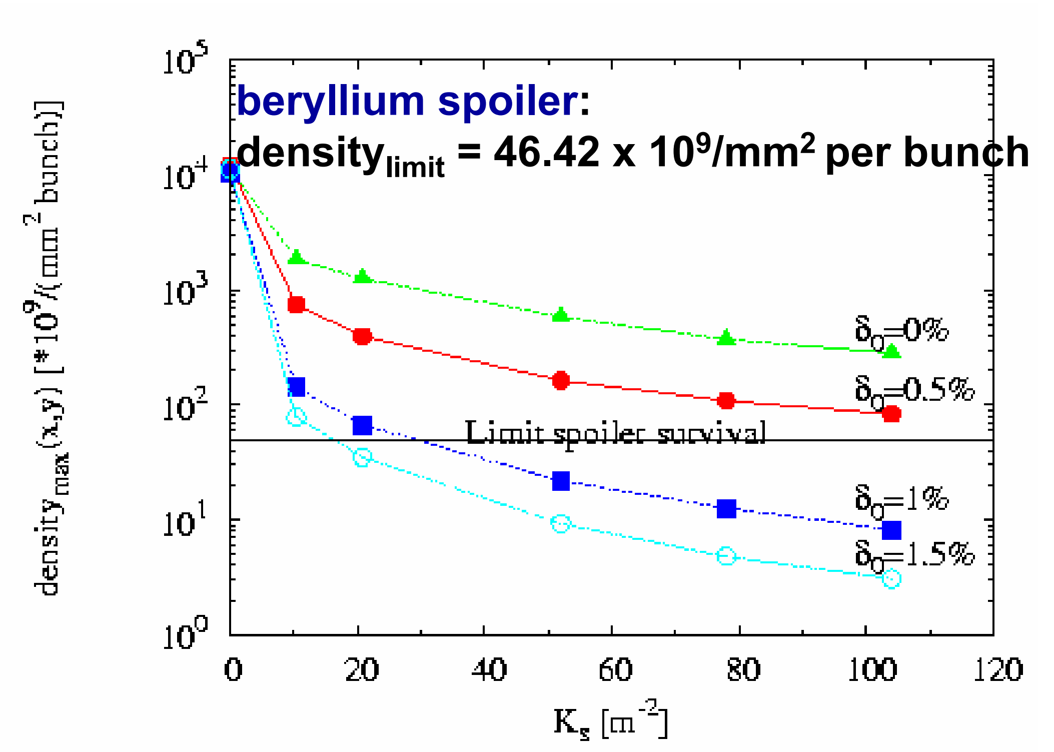
For a beam with 3% full width energy spread

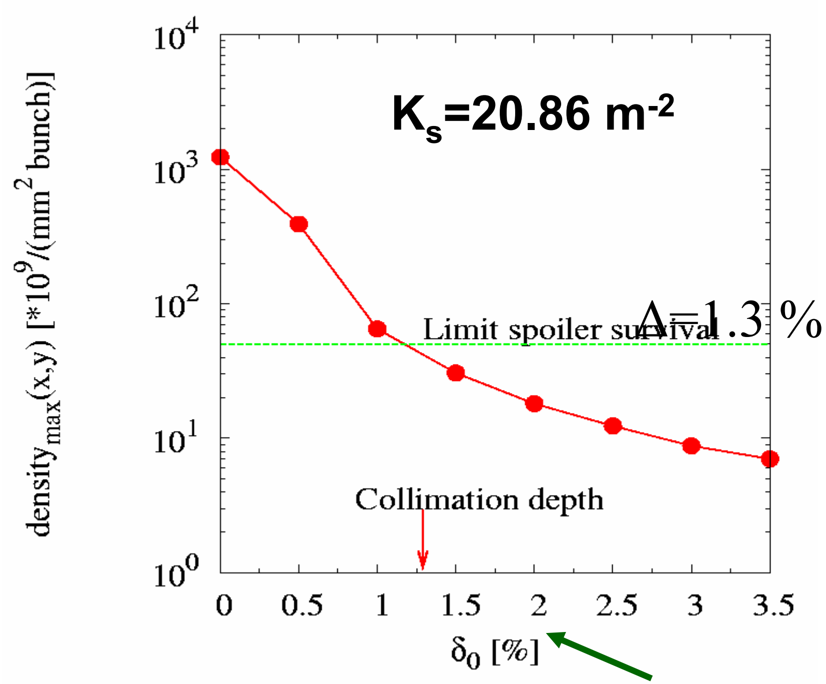




Beam peak density at the spoiler vs skew sextupole strength:

Beam is highly non-gaussian at the spoiler, and then it is the density which matters for the collimator survival, not the rms beam size. The spoiler survival is guarantee for off-momentum beams (>1%) using an integrated skew sextupole strength $K_2L \approx 20$ m-2

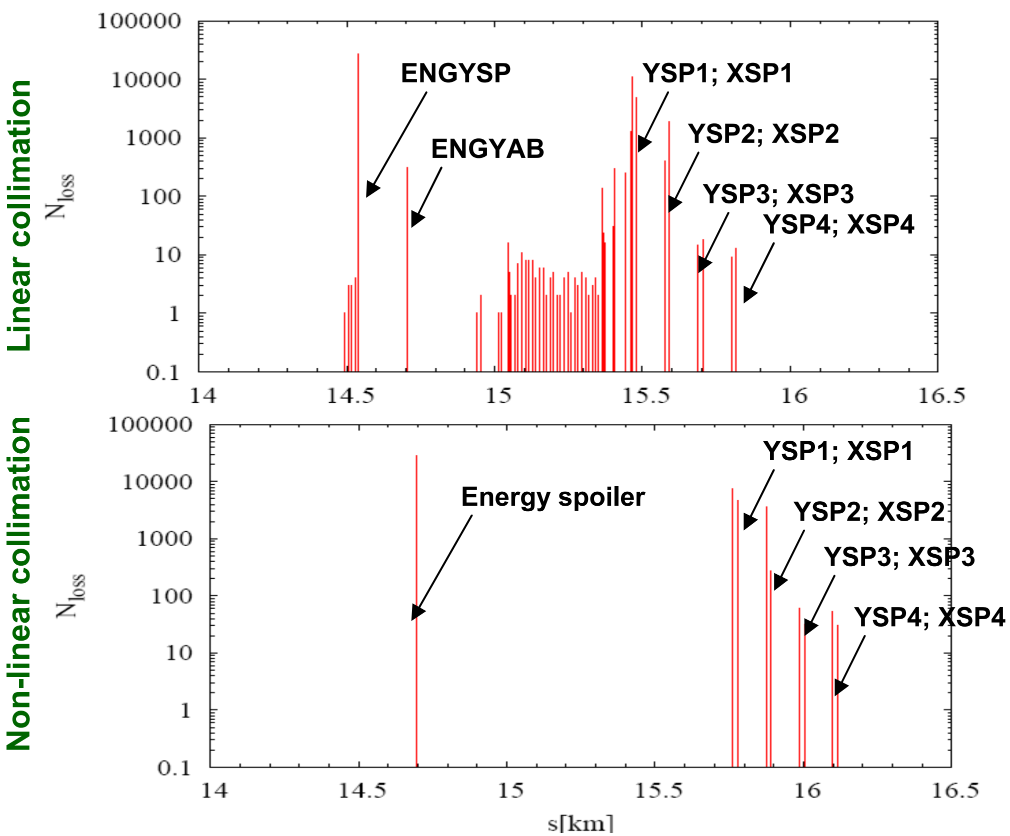


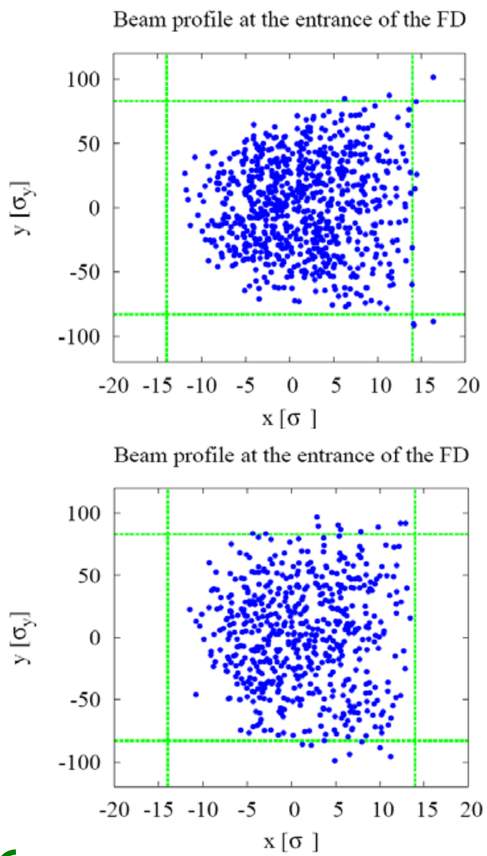


Average energy offset

Collimation efficiency and loss map

Tracking sample by using the code Placet with an input gaussian halo of $5x10^4$ macroparticles: $12.5\sigma_x$, $100\sigma_y$ (a 25% increase over collimation depth:$10\sigma_x$ and $80\sigma_y$) and 4% full width energy spread (energy collimation depth: ± 1.3%)





Aperture final doublet: $14\sigma_x$ and $83\sigma_y$

Cleaning efficiency: $\frac{\text{\# outside collimation depth at FD}}{\text{\# total initial halo}}$

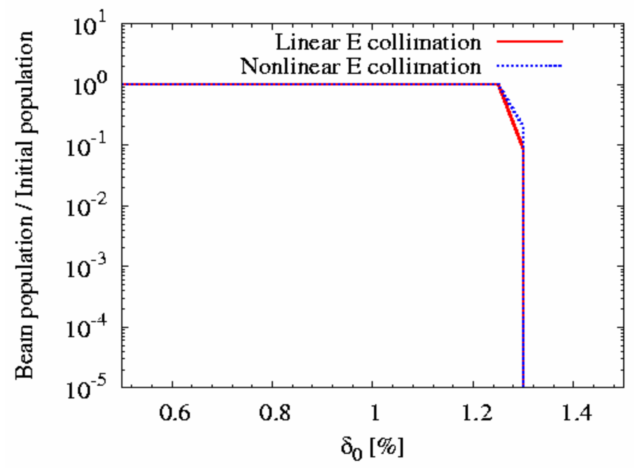
- **Linear collimation system ≈ $5.x10^{-4}$**
- **Nonlinear collimation system ≈ $5.5x10^{-4}$**

Collimation efficiency and machine protection

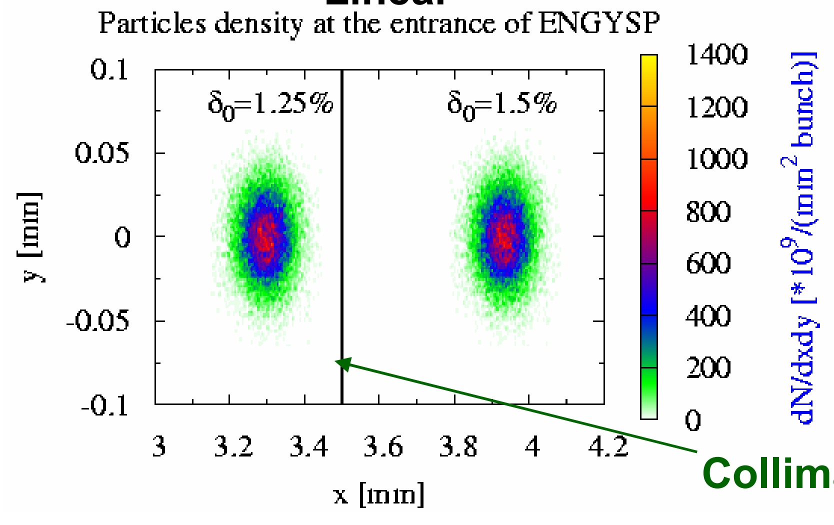
For failures scenarios mis-steered or errant beams will hit the energy spoiler

Tracking of gaussian beams of 10^5 macroparticles for different energy offsets and without energy spread. Beams with energy offset ≥ 1.3 % (energy collimation depth) are totally intercepted by the spoiler.

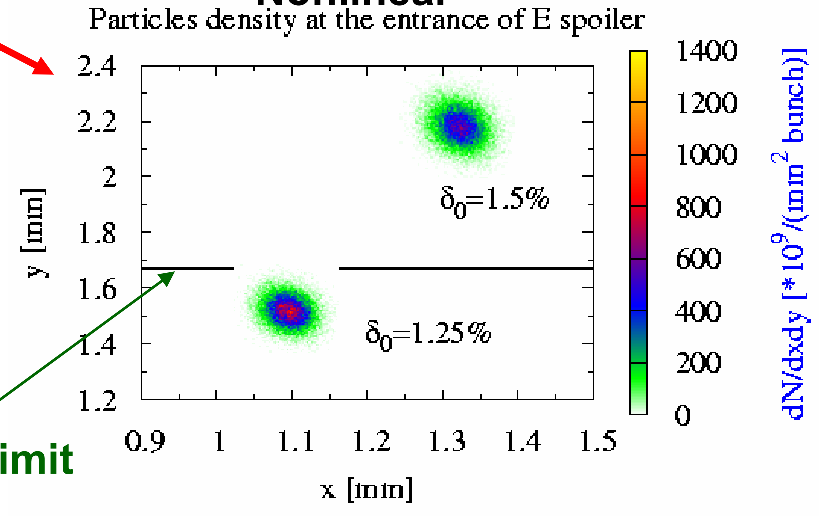
The nonlinear collimation system uses a vertical spoiler. Unlike the linear collimation system, the beam density is reduced by the nonlinear system as the beam energy offset increases. This helps to spoiler survival



Linear



Nonlinear



Collimation limit

Outlook

- A skew sextupole integrated strength of ≈ 20 m^{-2} has been chosen. For this value the spoiler survival is guaranteed for off-momentum beams (> 1 %).
- The chromatic behavior of the optics has been characterized. Good agreement between the simulated horizontal and vertical rms beam sizes at the spoiler as a function of the skew sextupole strength and the analytical expressions has been found.
- High chromatic aberrations of second, third and fourth order are found. The luminosity is degraded with the excitation of the skew sextupole.
- A local cancellation of higher order aberrations was made using two additional thin multipoles: a skew octupole and a normal sextupole. The luminosity was improved by more than a factor 2.
- Tracking studies with Placet show a comparable cleaning efficiency from linear and nonlinear collimation system.
- The nonlinear collimation system uses a vertical spoiler. Unlike the linear system, the beam density is reduced by the nonlinear system as the beam energy offset increases. This helps to spoiler survival.

Some remarks and Ongoing studies

- A shorter lattice(1.5 Km) is being studied
- We have only defined apertures for the collimators in the nonlinear collimation system. No aperture is defined for bendings and quadrupoles.
- The loss maps for the comparison between linear and nonlinear collimation systems have been created with a new version of Placet (in collaboration with H. Burkhardt and L. Neukermans).
- For a first test we have as input halo at the entrance of the BDS a gaussian distribution of particles with dimensions in the transversal planes 25% higher than the collimation depths.
- To avoid the dependence of the cleaning efficiency with the halo model it would be better to do a scan for different halo amplitudes as we have used for the cleaning efficiency study of the nonlinear collimation system of the LHC.
- More realistic simulations can be made using the incoming halo generated by beam-gas scattering in the Linac (from H. Burkhardt and L. Neukermans)

Some analytical calculations:

The Hamiltonian:

$$H_s = \frac{K_s}{3!}\left(y^3 - 3\left(x + D_{x,s}\delta\right)^2 y\right)$$

$$K_s = \frac{2B_T l_s}{(B\rho)a_s^2}$$

The deflection:

$$\Delta x' = -\frac{\partial H_s}{\partial x} = K_s\left(x + D_{x,s}\delta\right)y$$

$$\Delta y' = -\frac{\partial H_s}{\partial y} = -\frac{1}{2}K_s\left(y^2 - x^2 - D_{x,s}^2\delta^2 - 2D_{x,s}\delta x\right)$$

Some analytical calculations:

Position at the downstream spoiler:

$$x_{spo} = x_{0,spo} + R_{12}^{s,spo} \Delta x'$$

$$y_{spo} = y_{0,spo} + R_{34}^{s,spo} \Delta y'$$

Position at the downstream spoiler w/o skew sextupole:

$$x_{0,spo} = x_{\beta,spo} + D_{x,spo} \delta$$

$$y_{0,spo} = y_{\beta,spo}$$

Some analytical calculations:

The Hamiltonian:

$$H_s = \frac{K_s}{3!}\left(y^3 - 3\left(x + D_{x,s}\delta\right)^2 y\right)$$

$$K_s = \frac{2 B_T l_s}{(B\rho)a_s^2}$$

The deflection:

$$\Delta x' = -\frac{\partial H_s}{\partial x} = K_s\left(x + D_{x,s}\delta\right)y$$

$$\Delta y' = -\frac{\partial H_s}{\partial y} = -\frac{1}{2}K_s\left(y^2 - x^2 - D_{x,s}^2\delta^2 - 2D_{x,s}\delta x\right)$$

Some analytical calculations:

Position at the downstream spoiler:

$$x_{spo} = x_{0,spo} + R_{12}^{s,spo} \Delta x'$$

$$y_{spo} = y_{0,spo} + R_{34}^{s,spo} \Delta y'$$

Position at the downstream spoiler w/o skew sextupole:

$$x_{0,spo} = x_{\beta,spo} + D_{x,spo} \delta$$

$$y_{0,spo} = y_{\beta,spo}$$

Some analytical calculations:

Beam size at the spoiler:

$$\sigma_x = \sqrt{\langle x_{spo}^2 \rangle - \langle x_{spo} \rangle^2}$$

$$\sigma_y = \sqrt{\langle y_{spo}^2 \rangle - \langle y_{spo} \rangle^2}$$

-Gaussian distribution for transverse
-Uniform flat momentum distribution for longitudinal

δ_0 average momentum offset

$$P(\delta) = \begin{cases} 0 & \delta < -\dfrac{\delta_{flat}}{2} + \delta_0 \\ \dfrac{1}{\delta_{flat}} & -\dfrac{\delta_{flat}}{2} + \delta_0 < \delta < \dfrac{\delta_{flat}}{2} + \delta_0 \\ 0 & \delta > \dfrac{\delta_{flat}}{2} + \delta_0 \end{cases}$$

Some analytical calculations:

The Hamiltonian:

$$H_s = \frac{K_s}{3!}\left(y^3 - 3\left(x + D_{x,s}\delta\right)^2 y\right)$$

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Some analytical calculations:

The Hamiltonian:

$$H_s = \frac{K_s}{3!}\left(y^3 - 3\left(x + D_{x,s}\delta\right)^2 y\right)$$

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Some analytical calculations:

Position at the downstream spoiler:

$$x_{spo} = x_{0,spo} + R_{12}^{s,spo} \Delta x'$$

$$y_{spo} = y_{0,spo} + R_{34}^{s,spo} \Delta y'$$

Position at the downstream spoiler w/o skew sextupole:

$$x_{0,spo} = x_{\beta,spo} + D_{x,spo} \delta$$

$$y_{0,spo} = y_{\beta,spo}$$

Some analytical calculations:

Beam size at the spoiler:

$$\sigma_x = \sqrt{\langle x_{spo}^2 \rangle - \langle x_{spo} \rangle^2}$$

$$\sigma_y = \sqrt{\langle y_{spo}^2 \rangle - \langle y_{spo} \rangle^2}$$

-Gaussian distribution for transverse
-Uniform flat momentum distribution for longitudinal

δ_0 average momentum offset

$$P(\delta) = \begin{cases} 0 & \delta < -\frac{\delta_{flat}}{2} + \delta_0 \\ \frac{1}{\delta_{flat}} & -\frac{\delta_{flat}}{2} + \delta_0 < \delta < \frac{\delta_{flat}}{2} + \delta_0 \\ 0 & \delta > \frac{\delta_{flat}}{2} + \delta_0 \end{cases}$$

Some analytical calculations:

Beam size at the spoiler for uniform flat momentum distribution:

$\beta\varepsilon << D_x\delta$

$$\sigma_x \approx \sqrt{D_{x,spo}^2 \frac{\delta_{flat}^2}{12} + R_{12}^{s,spo\,2} K_s^2 D_{x,s}^2 \left(\frac{\delta_{flat}^2}{12} + \delta_0^2 \right) \beta_{y,s} \varepsilon_y}$$

$$\sigma_y \approx \sqrt{\frac{1}{4} R_{34}^{s,spo\,2} K_s^2 D_{x,s}^4 \left(\frac{\delta_{flat}^4}{180} + \frac{1}{3} \delta_{flat}^2 \delta_0^2 \right)}$$

Some analytical calculations:

For spoiler survival:

$$\sigma_{r,\min} = \sqrt{\sigma_x \sigma_y}$$

↓

$120\mu m$ minimum beam size

[S. Fartoukh et al.,"Heat Deposition by Transient Beam Passage in the Spoilers" CERN SL 2001 012 AP (2001)]

Some analytical calculations:

The sextupolar deflection also yields a weak betatron collimation for horizontal or vertical amplitudes at collimation depth (units of σ) of:

$$n_x = \frac{D_{x,s}\Delta}{\sqrt{\beta_{x,s}\varepsilon_x}}, n_y = \frac{D_{x,s}\Delta}{\sqrt{\beta_{y,s}\varepsilon_x}}$$

Additionally we can collimate (in the other betatron phase) using the linear optics:

$$n_x^{(2)} = \frac{a_x}{\sqrt{\beta_{x,spo}\varepsilon_x}} \approx \frac{D_{x,spo}\Delta}{\sqrt{\beta_{x,spo}\varepsilon_x}}$$

$$n_y^{(2)} = \frac{a_y}{\sqrt{\beta_{y,spo}\varepsilon_y}} \approx \frac{1}{2}\frac{\left|K_s R_{34}^{s,spo}\right| D_{x,s}^2\Delta^2}{\sqrt{\beta_{y,spo}\varepsilon_y}}$$

$\pm\Delta$

Energy collimation depth (units of δ)

Some analytical calculations:

The achievable value of D_{xs} is limited by the emittance growth $\Delta(\gamma\varepsilon_x)$ due to SR in the dipole magnets:

$$\Delta(\gamma\varepsilon_x) \approx \left(4\times10^{-8}\, m^2 GeV^{-6}\right) E^6 I_5 < f\varepsilon_x$$

7%

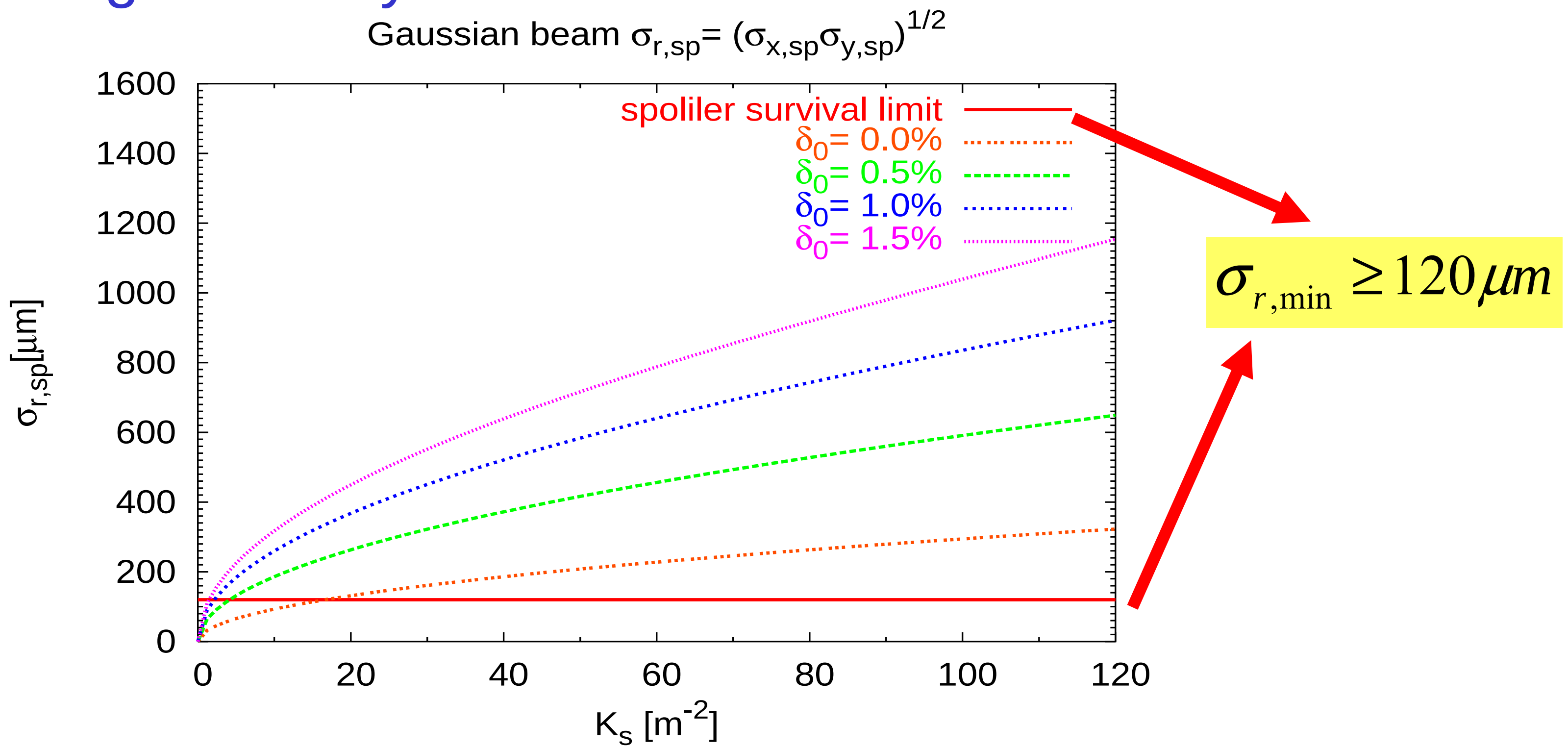
1.0×10^{-19} m

f: fraction of the initial emittance

I_5: radiation integral

r.m.s beam size at the spoiler vs skew sextupole strength:

Taking the analytical formulas:



Spoiler survival is guaranteed for off-momentum beams (>1%) for a integrated sextupole strength $K_2L \sim 20\ m^{-2}$

Some analytical calculations:

-I transformation between the pair of skew sextupoles:

$$\begin{pmatrix} x \\ p_x \\ y \\ p_y \\ ct \\ \delta \end{pmatrix}_{S2} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & R_{16} \\ 0 & -1 & 0 & 0 & 0 & R_{26} \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & +1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & +1 \end{pmatrix} \begin{pmatrix} x \\ p_x \\ y \\ p_y \\ ct \\ \delta \end{pmatrix}_{S1}$$

$$\begin{pmatrix} D_x \\ D_{p_x} \\ D_y \\ D_{p_y} \\ 0 \\ 1 \end{pmatrix}_{S2} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & R_{16} \\ 0 & -1 & 0 & 0 & 0 & R_{26} \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & +1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & +1 \end{pmatrix} \begin{pmatrix} D_x \\ D_{p_x} \\ D_y \\ D_{p_y} \\ 0 \\ 1 \end{pmatrix}_{S1}$$

Some analytical calculations:

-I transformation between the pair of skew sextupoles:

without bends between skews

$$x_{S2} = -x_{S1} + R_{16}^{S1,S2}\delta$$
$$y_{S2} = -y_{S1}$$
$$D_{x_{S2}} = -D_{x_{S1}} + R_{16}^{S1,S2}$$

$$R_{16}^{S1,S2} = 0$$

$$x_{S2} = -x_{S1}$$
$$y_{S2} = -y_{S1}$$
$$D_{x_{S2}} = -D_{x_{S1}}$$

-I transport + higher dispersion terms between the pair of skew sextupoles

$$x_{S2} = -x_{S1} + T_{166}^{S1,S2}\delta^2 + U_{1666}^{S1,S2}\delta^3$$

Some analytical calculations:

Beam size at the second skew sextupole:

$$\sigma_{x_{S2}} = \sqrt{\left\langle x_{S2}^2 \right\rangle - \left\langle x_{S2} \right\rangle^2}$$

uniform flat momentum distribution:

$$P(\delta) = \begin{cases} 0 & \delta < -\frac{\delta_{flat}}{2} + \delta_0 \\ \frac{1}{\delta_{flat}} & -\frac{\delta_{flat}}{2} + \delta_0 < \delta < \frac{\delta_{flat}}{2} + \delta_0 \\ 0 & \delta > \frac{\delta_{flat}}{2} + \delta_0 \end{cases}$$

δ_0 average momentum offset

Some analytical calculations:

$R_{16}^{S1,S2} = 0$ + $T_{166}^{S1,S2} \approx 0$, $U_{1666}^{S1,S2} \approx 0$ taking $\beta\varepsilon << D_{x_{S1}}\delta$

$$\sigma_{x_{S2}} \approx \left(D_{x_{S1}}^2 \frac{\delta_{flat}^2}{12} + (T_{166}^{S1,S2})^2 \left(\frac{\delta_{flat}^4}{180} + \frac{1}{3}\delta_{flat}^2 \delta_0^2 \right) + (U_{1666}^{S1,S2})^2 \left(\frac{\delta_{flat}^6}{448} + \frac{\delta_{flat}^4}{8}\delta_0^2 + \frac{3}{4}\delta_{flat}^2 \delta_0^4 \right) + 2T_{166}^{S1,S2} U_{1666}^{S1,S2} \left(\frac{\delta_{flat}^4 \delta_0}{24} + \frac{1}{2}\delta_{flat}^2 \delta_0^3 \right) - 2T_{166}^{S1,S2} D_{x_{S1}} \frac{\delta_{flat}^2 \delta_0}{6} - 2U_{1666}^{S1,S2} D_{x_{S1}} \left(\frac{\delta_{flat}^4}{80} + \frac{1}{4}\delta_{flat}^2 \delta_0^2 \right) \right)^{\frac{1}{2}}$$

Some analytical calculations:

$R_{26}^{S1,S2} = 0$ + $T_{366}^{S1,S2} \approx 0$, $U_{3666}^{S1,S2} \approx 0$ with $D_y = 0$

$$\sigma_{y_{S2}} \approx \left(\beta_y \varepsilon_y + (T_{366}^{S1,S2})^2 \left(\frac{\delta_{flat}^4}{180} + \frac{1}{3} \delta_{flat}^2 \delta_0^2 \right) + (U_{3666}^{S1,S2})^2 \left(\frac{\delta_{flat}^6}{448} + \frac{\delta_{flat}^4}{8} \delta_0^2 + \frac{3}{4} \delta_{flat}^2 \delta_0^4 \right) + 2 T_{366}^{S1,S2} U_{3666}^{S1,S2} \left(\frac{\delta_{flat}^4 \delta_0}{24} + \frac{1}{2} \delta_{flat}^2 \delta_0^3 \right) \right)^{\frac{1}{2}}$$

Computation of the beam sizes:

Given the transfer map between one location of the accelerator and the IP in the form:

$$x_{IP} = \sum X_{jklmn}\, x^j\, p_x^k\, y^l\, p_y^m\, \delta^n$$

and given the particle density at the initial location, the rms beam size at the IP is given by:

$$\sigma_{IP}^2 = \sum X_{jklmn} X_{j'k'l'm'n'} \int x^{j+j'}\, p_x^{k+k'}\, y^{l+l'}\, p_y^{m+m'}\, \delta^{n+n'}\, \rho dv$$

The integral is performed depending on ρ and the X_{jklmn} are obtained to arbitrary order from MADX-PTC (thanks to E. Forest, F. Schmidt, et al).