Bootstrapping Multiparton One-Loop Amplitudes

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with

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VLCW06 – July 21st, 2006

References

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[1] Z. Bern, L. J. Dixon, D. A. Kosower,
Phys. Rev. D 71, 105013 (2005) [hep-th/0501240];
Phys. Rev. D 72, 125003 (2005) [hep-ph/0505055];
Phys. Rev. D 73, 065013 (2006) [hep-ph/0507005].

[2] CFB, Z. Bern, L. J. Dixon, D. Forde, D. A. Kosower,
hep-ph/0604195.

[3] CFB, Z. Bern, L. J. Dixon, D. Forde, D. A. Kosower,
hep-ph/0607014.

[4] CFB, V. Del Duca, L. J. Dixon,
to appear.

The (In)Famous Les Houches 2005 Wishlist

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process wanted at NLO ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

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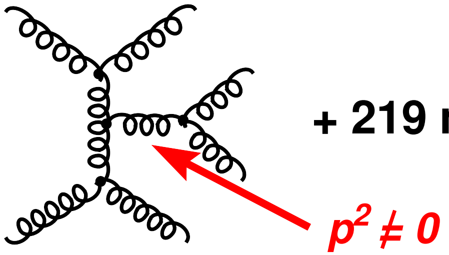
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

Large number of high-multiplicity processes that need to be computed!
The LHC turns on in 2007!

Feynman Graphs

- Feynman rules are too general, not optimized, do not take into account all symmetries of the theory
- Vertices and propagators involve gauge-dependent off-shell states
- Explosive growth of number of diagrams/terms

gluon legs	tree level	one loop
6	220	1,034
8	34,300	3,017,490
∞	worse than ∞	even worse than ∞



+ 219 more

Feynman Graphs

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Time to panic??

Feynman Graphs

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Time to panic?? – No!

➨ **(Semi)Numerical approaches and automatization**

MadEvent, ALPGEN, CompHEP, GRACE, HELAC/PHEGAS, . . .

Kramer, Soper, Nagy; Ellis, Giele, Glover, Zanderighi; Binoth, Ciccolini, Guillet, Heinrich, Kauer, Pilon, Schubert; Czakon; Anastasiou, Daleo; . . .

➨ **Recursion relations**

Color Ordering, Spinors and Twistors

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Summary and Outlook

- **Strip color information, only calculate diagrams with cyclic color ordering**
  $\Rightarrow$ **36 diagrams (not 220) for 6 gluons at tree level**

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- **Decompose one-loop QCD amplitudes**

$$A^{\text{QCD}} = A^{\mathcal{N}=4} - 4\, A^{\mathcal{N}=1} + A^{\text{scalar}} \qquad (1)$$

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$$A^{\text{QCD}} = A^{\mathcal{N}=4} - 4\,A^{\mathcal{N}=1} + A^{\text{scalar}} \quad (1)$$

- **Use the "right variables" to expose more symmetries - spinor helicity formalism**
  Transformation to Penrose's twistor space = "half Fourier transform" of spinors (only left-handed spinors transformed)
  $\Rightarrow$ **amazingly simple structure of scattering amplitudes** Witten; Nair; Roiban, Spradlin, Volovich

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  Transformation to Penrose’s twistor space = “half Fourier transform” of spinors (only left-handed spinors transformed)
  $\Rightarrow$ amazingly simple structure of scattering amplitudes
  Witten; Nair; Roiban, Spradlin, Volovich
- “Recycle” known amplitudes via recursion relations
  Berends, Giele; Mahlon; Cachazo, Svrcek, Witten; Britto, Cachazo, Feng, Witten

Recursion Relations at Tree Level

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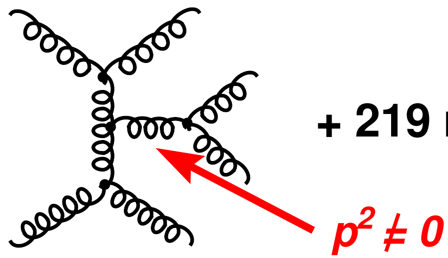

Complex continue spinors and momenta $\Rightarrow$ Amplitude function of complex parameter

$$A(z) = A(p_1, \ldots, p_j(z), p_{j+1}, \ldots, p_l(z), \ldots, p_n)$$

If $A(z \to \infty) \to 0$

Cauchy's theorem

$$\frac{1}{2\pi i}\oint_C \frac{dz}{z} A(z) = 0 \qquad (2)$$

$A(0)$ is the physical amplitude

Recursion Relations at Tree Level

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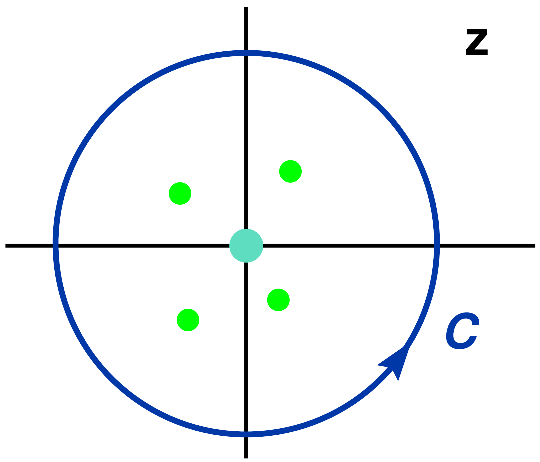
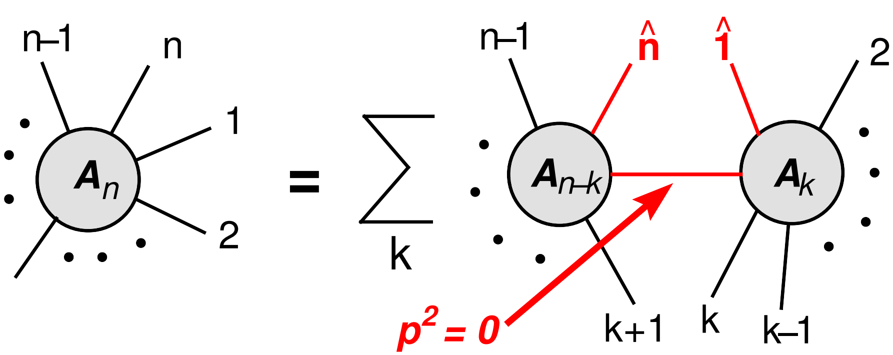
$$A(0) = -\sum_{\text{poles } \alpha} \underset{z=z_\alpha}{\text{Res}} \frac{A(z)}{z} \quad (3)$$

$$= \sum_{\text{configs}} A_L \frac{1}{P^2_{l\ldots m}} A_R$$



Poles in z correspond to physical factorizations

$$\frac{1}{\hat{P}^2_{l\ldots m}} = \frac{1}{P^2_{l\ldots m} - z \langle j^- | \not{P}_{l\ldots m} | k^- \rangle}$$



Britto, Cachazo, Feng, Witten

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On-Shell Recursions

Proof at tree level only relies on **Cauchy's theorem** and basic **factorization properties**.

See also: Draggiotis, Kleiss, Lazopoulos, Papadopoulos; Vaman, Yao

$\Rightarrow$ Many applications at tree level

- SUSY - processes with massless fermions Luo, Wen
- QCD - QCD is supersymmetric at tree level
- Massive scalars and fermions

  Badger, Glover, Khoze, Svrcek; Forde, Kosower; Schwinn, Weinzierl; Ferrario, Rodrigo, Talavera
- Higgs (top loop integrated out) Badger, Dixon, Glover, Khoze
- Gravity

  Bedford, Brandhuber, Spence, Travaglini; Cachazo, Svrcek; Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager

QCD at One Loop - A Disaster?

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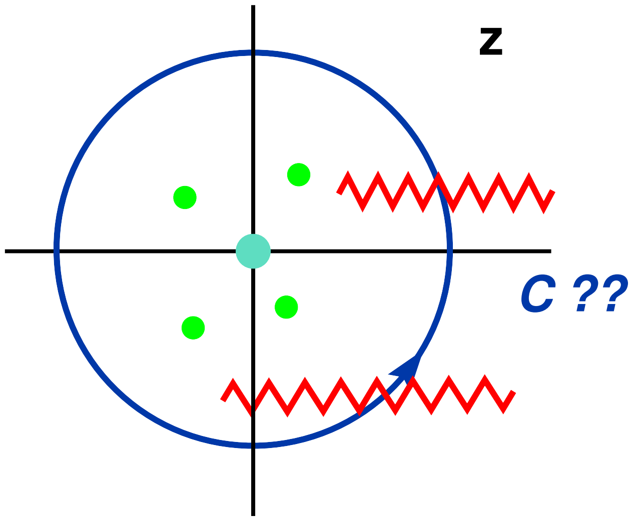

- **Branch cuts**

QCD at One Loop - A Disaster?

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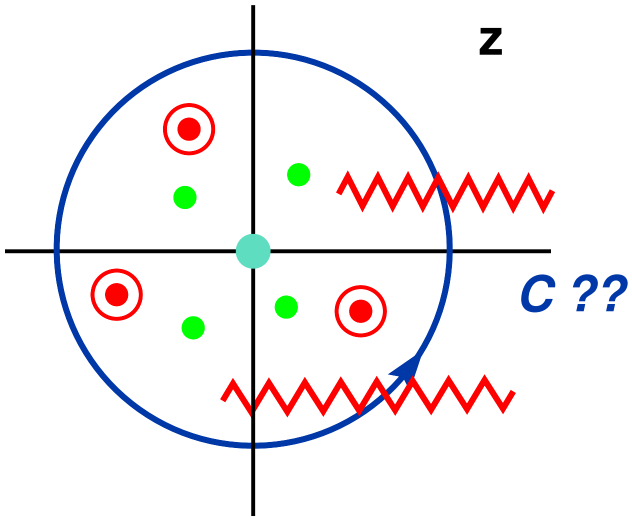

- Branch cuts
- Double poles, 'unreal poles' and nonstandard factorizations

QCD at One Loop - A Disaster?

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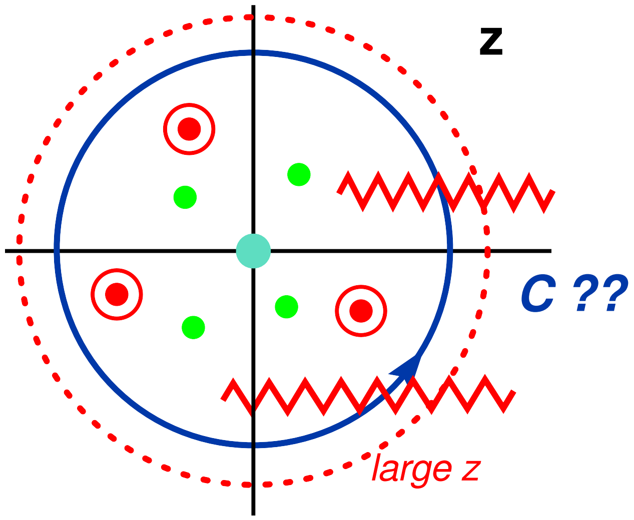

- **Branch cuts**
- **Double poles, ‘unreal poles’ and nonstandard factorizations**
- $A(z \to \infty) \neq 0$

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Bootstrap model

From Wikipedia, the free encyclopedia

In physics, the term **bootstrap model** is used for the class of theories that assume that very general consistency criteria are sufficient to determine the whole theory completely. In such theories, typically examples of quantum field theory, it is impossible to divide the objects and concepts to elementary and composite ones. See Geoffrey Chew. This strategy turned out to be successful only in the case of two-dimensional conformal field theory where many insights can indeed b derived by this method.

Here: very general consistency criteria

- **Cuts (unitarity)**
- **Poles (factorization)**

$$A(z) = C(z) + R(z) \tag{4}$$

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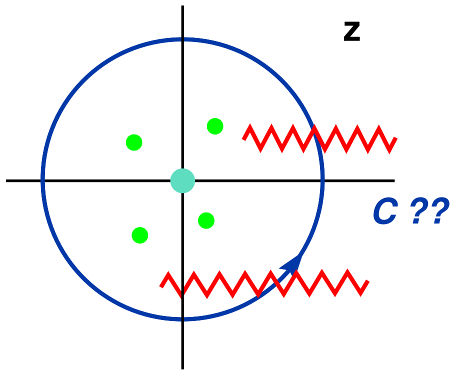
Cut Parts



Cut Parts

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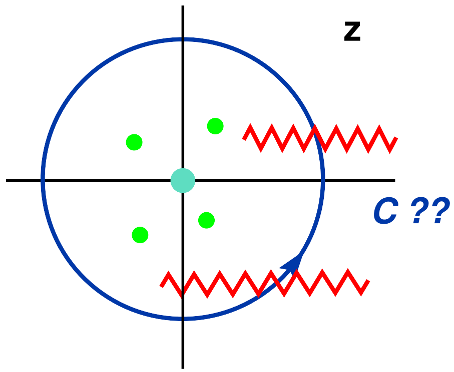

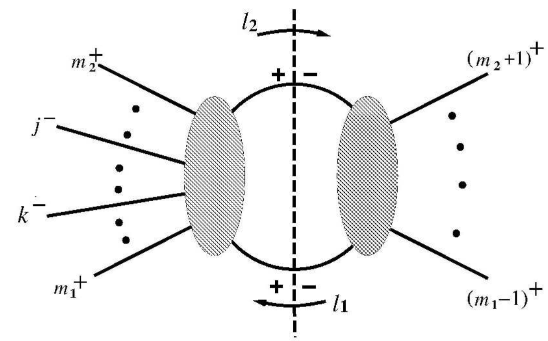

$C(0)$ **contains only Li, ln, π^2 – cut-constructible! via (generalized) unitarity**

$$\int d\text{LIPS}(-l_1, l_2) A^{\text{tree}}(-l_1, m_1, .., m_2, l_2) A^{\text{tree}}(-l_2, m_2+1, .., m_1-1, l_1)$$

Trees “recycled” into loops

Bern, Dixon, Dunbar, Kosower; Bedford, Brandhuber, McNamara, Spence, Travaglini; Quigley, Rozali; Britto, Buchbinder, Cachazo, Feng, Mastrolia; Bern, Bidder, Bjerrum-Bohr, Dixon, Dunbar, Ita, Perkins

On-Shell Recursion for Rational Parts

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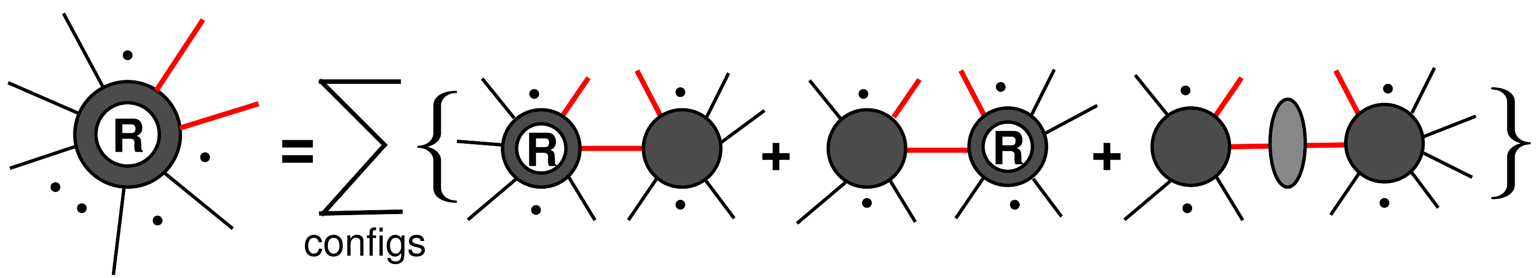
$$A(z) = C(z) + R(z) \qquad \Bigg| \qquad \frac{1}{2\pi i}\oint_C \frac{dz}{z}$$

$$A(0) = C(0) + \text{Inf}\,R - \sum_{\text{poles }\alpha} \operatorname*{Res}_{z=z_\alpha} \frac{R(z)}{z}$$

$$= C(0) + \text{Inf}\,R + \sum_{\text{configs}} A_L \frac{1}{P^2_{l\ldots m}} A_R$$



Loops "recycled" into loops

(ignoring slight subtleties with spurious singularities)

Bern, Dixon, Kosower

Non-Standard Factorizations

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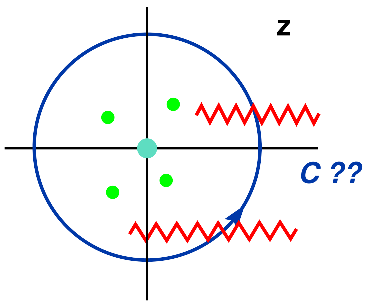

$$A(0) = C(0) \qquad - \sum_{\text{poles }\alpha} \operatorname*{Res}_{z=z_\alpha} \frac{R(z)}{z}$$

Non-Standard Factorizations

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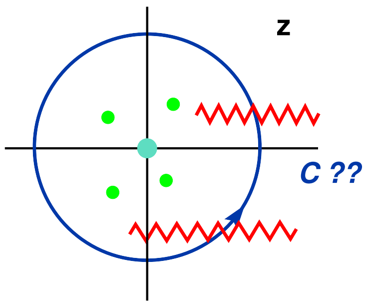

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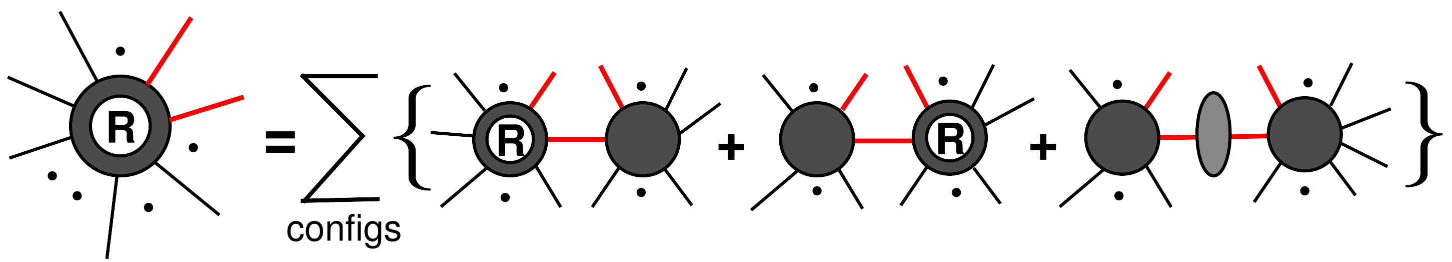

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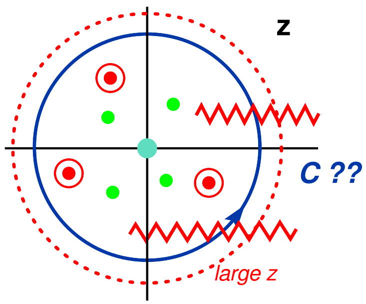
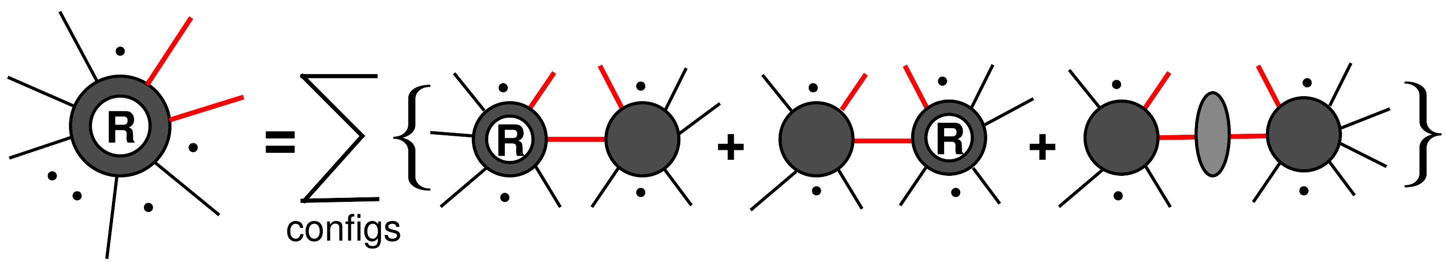
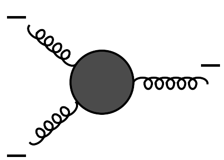
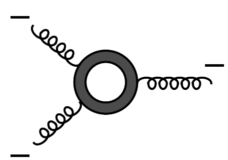


$$A(0) = C(0) + \text{Inf } R - \sum_{\text{poles } \alpha} \text{Res}_{z=z_\alpha} \frac{R(z)}{z} + ???$$

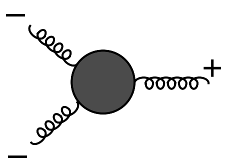


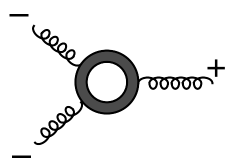


$= 0$



$\sim$ **double pole + ratl** $\times(--+)$**tree**





$\sim$ **rational** $\times(--+)$**tree**

Factorization properties unclear at one loop.

Large-z Contributions

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Summary and Outlook

Can pick continuations to avoid either non-standard factorizations or $z \to \infty$ contributions, but in general not both!

- **Continuation $[j, l\rangle$ avoids non-standard factorizations**

$$A(0) = C(0) + \operatorname*{Inf}_{[j,l\rangle} R + R^{[j,l\rangle}_{\text{recurs}} \tag{5}$$

Large-z Contributions

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- **Continuation $[j, l\rangle$ avoids non-standard factorizations**

$$A(0) = C(0) + \operatorname*{Inf}_{[j,l\rangle} R + R^{[j,l\rangle}_{\text{recurs}} \tag{5}$$

- **Continuation $[a, b\rangle$ has no large-parameter contributions**

$$A(0) = C(0) + R^{[a,b\rangle}_{\text{recurs}} + \text{non-standard channels}^{[a,b\rangle} \tag{6}$$

The Bootstrap Formalism

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Summary and Outlook

Solution $\Rightarrow$ use two continuations!

Extract large-parameter contributions of primary continuation from auxiliary relation (6)

$$(7)\ A(0) = C(0) + R^{[a,b\rangle}_{\text{recurs}} + \text{non-standard}^{[a,b\rangle} \quad \Big| \quad \operatorname*{Inf}_{[j,l\rangle}$$

$$\operatorname*{Inf}_{[j,l\rangle} R = \operatorname*{Inf}_{[j,l\rangle} R^{[a,b\rangle}_{\text{recurs}}$$

$$(8) \qquad \text{if} \quad \operatorname*{Inf}_{[j,l\rangle} \left[\text{non-standard channels}^{[a,b\rangle}\right] = 0$$

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Extract large-parameter contributions of primary continuation from auxiliary relation (6)

(7) $A(0) = C(0) + R^{[a,b\rangle}_{\text{recurs}} + \text{non-standard}^{[a,b\rangle} \Big|\ \underset{[j,l\rangle}{\text{Inf}}$

$$\underset{[j,l\rangle}{\text{Inf}}\, R = \underset{[j,l\rangle}{\text{Inf}}\, R^{[a,b\rangle}_{\text{recurs}}$$

(8) if $\underset{[j,l\rangle}{\text{Inf}} \left[\text{non-standard channels}^{[a,b\rangle}\right] = 0$

The complete bootstrap

$$A(0) = C(0) + R^{[j,l\rangle}_{\text{recurs}} + \underset{[j,l\rangle}{\text{Inf}}\, R^{[a,b\rangle}_{\text{recurs}}$$

Passes all nontrivial checks!

CFB, Bern, Dixon, Forde, Kosower

Results

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- ☑ All-multiplicity formulae for $(+ + \ldots +)$, $(- + \ldots +)$ one-loop gluon amplitudes (also with a fermion pair)

  Bern, Dixon, Kosower

- ☑ All-multiplicity formulae for $(+ \ldots + - + \ldots + - + \ldots +)$ one-loop gluon amplitudes

  Forde, Kosower; CFB, Bern, Dixon, Forde, Kosower

- ☑ All-multiplicity formulae for $(- - - + \ldots +)$ one-loop gluon amplitudes

  CFB, Bern, Dixon, Forde, Kosower

- ☑ Some all-multiplicity results for parts of Higgs plus gluons (and fermion pair) at NNLO (effective theory - top loop integrated out)

  CFB, Del Duca, Dixon

All of the above $\ll \infty$ pages

- ☑ Working algorithm for all other configurations of one-loop gluon amplitudes!

  CFB, Bern, Dixon, Forde, Kosower

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- ❑ Understand complex factorization at one loop and beyond + connection to Lagrangian?
- ❑ Higher loops?
- ❑ Massive partons (external fermions, scalars, . . .)
- ❑ Automatization
- ❑ Attack the wishlists...

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- Results
- To-Do List
- Summary

"One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane."

in J. Schwinger, "Particles, Sources, and Fields", Vol. I.