Higher Curvature Effects in the ADD and RS Models

hep-ph/0603242
+ work in progress

- ADD + Classic RS have Many common features:

  (i) localized SM fields (on a boundary)

  (ii) Bulk has constant curvature:

  ADD (Minkowski), RS (AdS_5)

  (iii) Gravity in Bulk described by Einstein-Hilbert action:

possible bulk cosmo constant

$$S = \frac{M^{D-2}}{2} \int d^D x \sqrt{g} \left\{ R + (\text{a constant}) \right\}$$

fundamental scale

Ricci scalar

- How is ADD/RS phenomenolgy altered if we give up (iii) + consider more general actions??

i.e., $\boxed{R \to F}$ ... a well-behaved function of invariants

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VLC '06

* Why do this ??

- in both models $\sqrt{s} \sim M_{(eff)}$ are probed + we know EH is an effective theory $< M$ ... so 'correction' terms should be present..

- strings predict such terms sub-leading in $1/M^2$ etc

- such terms have been considered for other reasons e.g, cosmology/dark energy issues

Here, to be tractable, we restrict ourselves to

$$F(R,P,Q) \; : \quad P \equiv R_{AB}R^{AB} \; ; \quad Q \equiv R_{ABCD}R^{ABCD}$$

Ricci tensor — Curvature tensor

- a fairly general case ..

.. which has been considered in cosmo studies ...

* What do we want to know ???

→ graviton KK properties : masses, wave functions, matter couplings, propagators, ...

(not, e.g, self-couplings of the gravitons)

↑↑ forces us to consider cubic etc terms

Then, to obtain these quantities in a constant curvature background# (which we have here)

* It is sufficient to expand F to second order in the invariants:

background value

$$\boxed{F = F_0 + \sum_i (X_i - X_{i0}) F_{X_i} + \frac{1}{2}\sum_{ij}(X_i - X_{i0})(X_j - X_{j0}) \cdot F_{X_i X_j} + \text{higher order dropped terms}}$$

$X_i = (R, P, Q)$

$\partial_{X_i} F|_{\text{background}}$

background value

$$\boxed{S_{eff} \rightarrow \frac{M^{D-2}}{2}\int d^D x \sqrt{g}\,\left\{\Lambda + a_1 R + a_2 R^2 + a_3 C + a_4 GB\right\}}$$

Weyl scalar $\equiv C_{ABCD} C^{ABCD}$

Gauss-Bonnet term

... where $\{\Lambda, a_i\}$ are functions of F_{X_i}, $F_{X_i X_j}$, F_0 + $R_0 (= \langle R \rangle_{\text{background}})$ $\begin{cases} = 0 & \text{in ADD} \\ = -20k^2 & \text{in interval RS} \end{cases}$

[Next]

$$\Rightarrow \boxed{GB \equiv R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \equiv R^2 - 4P + Q}$$

#{to be precise, in a maximally symmetric background}

Without making any further assumptions we obtain

$$
\begin{aligned}
\Lambda &= F_0 - R_0 F_R + R_0^2(F_{RR}/2 - \sigma F_P - \tau F_Q) + R_0^3(\sigma F_{PR} + \tau F_{QR}) \\
&+ R_0^4(2\sigma\tau F_{PQ} + \sigma^2 F_{PP} + \tau^2 F_{QQ})/2 \\
a_1 &= F_R - R_0 F_{RR} - R_0^2(\sigma F_{RP} + \tau F_{RQ}) \\
a_2 &= \beta F_P + \epsilon F_Q + F_{RR}/2 - R_0(\beta F_{RP} + \epsilon F_{RQ}) - R_0^2[(\tau\beta + \epsilon\sigma)F_{PQ} + \sigma\beta F_{PP} + \tau\epsilon F_{QQ}] \\
a_3 &= \alpha F_P + \delta F_Q - R_0(\alpha F_{RP} + \delta F_{RQ}) - R_0^2[(\tau\alpha + \delta\sigma)F_{PQ} + \sigma\alpha F_{PP} + \tau\delta F_{QQ}] \\
a_4 &= -\alpha F_P + \gamma F_Q + R_0(\alpha F_{RP} - \gamma F_{RQ}) + R_0^2[(\tau\alpha - \gamma\sigma)F_{PQ} + \sigma\alpha F_{PP} - \tau\gamma F_{QQ}], \qquad (11
\end{aligned}
$$

where we have defined $\sigma = (n+4)^{-1}$, $\tau = 2(n+4)^{-1}(n+3)^{-1}$, $\delta = 4\alpha = (n+2)/(n+1)$ $4\beta = (n+4)/(n+3)$, $\gamma = -(n+1)^{-1}$ and $\epsilon = (n+3)^{-1}$. For the case of $n = 0$ this reproduces th

¶Note that the quadratic terms in the Taylor expansion naturally involve factors of P^2 and Q^2 which are actuall fourth order in the (dynamical) curvature; we drop these terms for consistency in the discussion which follows.

Field Content (in D-dimensions!)

- massless tensor field (usual gravitons etc)
- massive tensor ghost (Yikes!)
- massive scalar (tachyonic?)

.. many ways to see this ...

$\Longrightarrow$ Consider gravitons being exchanged between localized SM sources (4D): $\boxed{T_{\mu\nu}}$. Then, eg, in

ADD (before KK-sums) (n extra dims)

$$\mathcal{A} = \text{"} \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+2)}{k^2 - m_n^2}^{**}$$

usual KK masses

this is the usual "graviton" exchange structure (eg, GRW) [gravitons + graviscalers]

ghost!! wrong sign!

$$- \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+3)}{k^2 - (m_n^2 + m_T^2)}$$

$\rightarrow$ massive in bulk tensor field

$$+ \frac{T^2/(n+2)(n+3)}{k^2 - (m_n^2 + m_s^2)} \text{"}$$

bulk mass for scalar field

- Remove tachyons $\therefore$ $m_s^2 > 0$ (demanded)
- Remove ghosts $\therefore$ $m_T^2 \to \infty$ $\to$ $F(R, P-4Q)$ only

$\hookrightarrow$ $\boxed{a_3 = 0}$

** $\boxed{T = \eta_{\mu\nu} T^{\mu\nu}}$

$\boxed{m_T^2 \sim (F_P + 4F_Q)^{-1}}$

.. a similar requirement in RS : $\boxed{F(R, P-4Q)}$

ADD : $\Lambda = 0 \rightarrow R_0, F_0 = 0$ (flat space)

$$F \rightarrow \underbrace{F_R}_{>0 \text{ (no KK ghosts)}} R + \left\{ -F_Q + \tfrac{1}{2} F_{RR} \right\} R^2 + F_Q \cdot G$$

$$\boxed{m_s^2 = \frac{(n+2) F_R}{4(n+3) \underbrace{(F_{RR}/2 - F_Q)}_{\geq 0 \text{ (no tachyons)}}}}$$

{ Note in "GB gravity" $F_{RR}/2 - F_Q = 0$ so $m_s^2 \rightarrow \infty$ (removed))

• m_s is naturally $O(M)$ so a new KK spectrum of scalars begins at $\sim$ TeV

(Demir + Tanyildizi '05)

?? effect ??

Small as " $T^2 / T_{\mu\nu} T^{\mu\nu}$ " $\sim \left(m^2_{external}/s \right)^2 \ll 1$ at LHC / ILC ...

The big effect is ... $\rightarrow$ $\boxed{\bar{M}_{pl}^2 = V_n M^{n+2} \underline{F_R}}$ ($F_R > 0$ recall)

from the zero mode graviton wavefunction normalization ...

* If $V_n = (2\pi R_c)^n$, R_c shifts for fixed M (+ $\bar{M}_{pl}$) as input.

- For fixed M, KK masses are shifted ...

$$\bar{M}_{pl}^2 = (2\pi R_c)^n M^{n+2} F_R$$

fixed — fixed

$$\longrightarrow \quad R_c \to R_c F_R^{-1/n}$$ so

$$\boxed{m_{KK} \to m_{KK} F_R^{+1/n}}$$ $\leftrightarrow$ KK Spectrum changes!

- In units of M graviton emission cross-sections are modified:

$$\boxed{d\sigma_{ADD} \to \underline{F_R^{-1}}\, d\sigma_{ADD}(M^2, s, t, u)}$$

- Similarly, graviton exchange amplitudes (neglecting the new scalars!) will

$$\boxed{\mathcal{A}_{KK} \Rightarrow F_R^{-1} \mathcal{A}_{KK}}$$

- we expect F_R to be $O(1)$ in most models...

(Note $F_R = 1$ in flat, R polynomial models)

$\Rightarrow$ $O(1)$ modifications of standard ADD..
(for M held fixed)

* RS on an interval

Λ_b = bulk cosmo const.

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

↑ warp factor

$\boxed{k \sim M}$

$\boxed{R_0 = \langle R\rangle_{AdS_5} = -20k^2}$

$R \to F(R, P-4Q)$

Trace of Einstein's Equation → $\boxed{224k^4 F_Q + 8k^2 F_R + F_0 = 2\Lambda_b/M^3}$ constraint

→ $k(M,\Lambda_b)$ is a derived parameter + sets the KK mass scale..

ex. $\boxed{F = R + \beta R^2/M^2}$

$$\to k^2 = \frac{3M^2}{40\beta}\left\{1 \pm \left(1 + \frac{40\Lambda_b}{9M^5}\beta\right)^{1/2}\right\}$$ two roots!

$\to -\Lambda_b/6M^3$ as $\beta \to 0$ (negative root)

(usual RS relationship)

$$S_{eff} = \int d^5x\sqrt{-g}\left\{-\Lambda_b + a_1\frac{M^3}{2}R + \frac{\alpha M}{2}G + \frac{\beta M}{2}R^2\right\}$$

dimensionless coefficients

e.g,

$$\frac{\beta}{M^2} = \underbrace{-F_Q + \frac{1}{2}F_{RR}}_{\text{ADD result}} - 20k^2 F_{RQ} - 280k^4 F_{QQ}$$ etc

are calculable!!

Scalar gets a bulk mass:

for any given model this is known!

$$\boxed{m_s^2 = \frac{3a_1}{16\beta} M^2}$$

$$= \frac{3}{8} \frac{F_R + 20k^2 F_{RR} + 280k^4 F_{RQ}}{F_{RR} - 2F_Q - 40k^2 F_{RQ} - 560k^4 F_{QQ}}$$

KK Scalar spectrum: $(2-\nu) J_\nu(x_{sn}) + x_{sn} J_{\nu-1}(x_{sn}) = 0$

$\nu^2 = 4 + m_s^2/k^2$ is large

$\rightarrow \boxed{m_{sn} = \underbrace{x_{sn}} k e^{-\pi k r_c}}$ $\boxed{x_{s_0} = ?}$

- if $\beta/a_1 = 1$, $k/M = 0.05$ $\rightarrow \frac{m_s}{k} \simeq 8.7 \rightarrow x_{s_0} \simeq 11$

$(x_1^{grav} = 3.83)$ $\rightarrow \approx \boxed{3\times \text{heavier}}$ than 1st graviton KK [Fig]

... as in ADD these scalars are more weakly coupled than gravitons by a factor

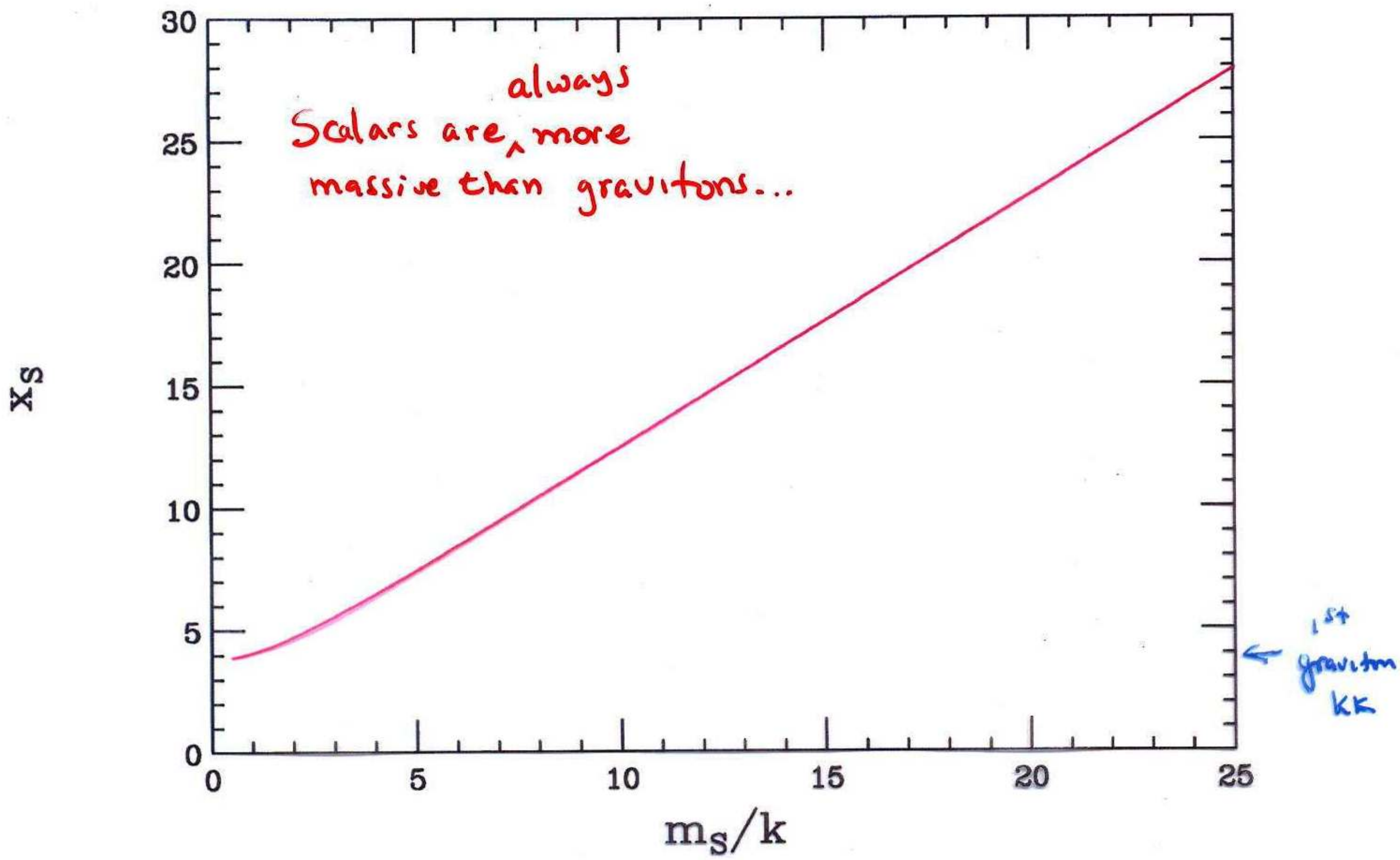
$\sim (m_{exr}^2/12s)$ in amplitude [Fig]

$\boxed{\text{Graviton sector}} \Rightarrow \boxed{\bar{M}_{Pl}^2 = \frac{M^3}{k} \cdot \mathcal{H}}$ (zero mode)

$$\boxed{\mathcal{H} = F_R + 36k^2 F_Q + 1000k^4 F_{RQ} + 10080k^6 F_{QQ}}$$

e.g., For $R + \beta R^2/M^2$ $\mathcal{H} = 1 - 40\beta k^2/M^2$

Root For Lightest Scalar State



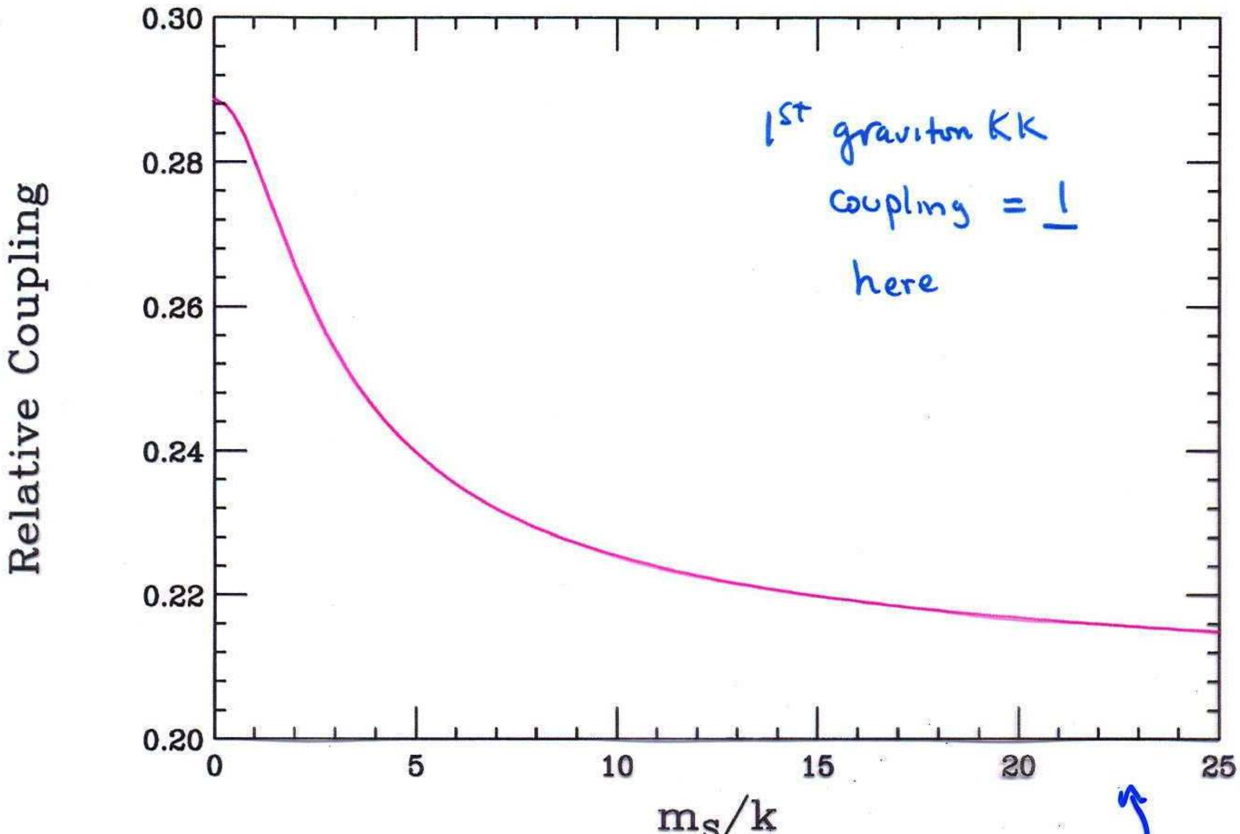
Bulk mass

$\boxed{\text{Note:}}$

- With $\bar{M}_{pl}^2 = \frac{M^3}{k} H(k)$ and $k = k(\Lambda, F)$ the graviton KK masses expressed as $m_n = x_n k e^{-k\pi r_c}$ are left invariant ... but k changes as a function of input parameters $\therefore$ shifting the KK masses

- with $\bar{M}_{pl}$ + M fixed, for any F, we can calculate both Λ + k $\rightarrow$ shifts in parameters from usual RS

Scalars couple more weakly than gravitons



Bulk mass

numerical factor ⊕ wave function ratios

$$\left(\mathcal{L}^{(n)}_{scalar} \equiv \frac{1}{\Lambda_\pi} (RC)\, T\, S_{(n)} \right)$$

These states will be difficult to observe at colliders!

Example of graviton KK mass shift:

$$\Rightarrow \int d^5x \sqrt{g} \left(\frac{M^3}{2} R - \Lambda_0 \right) \rightarrow \int d^5x \sqrt{g} \left[\frac{M^3}{2} \left(R + \frac{\beta}{M^2} R^2 \right) - \Lambda \right]$$

In standard RS: $k^2 \equiv k_0^2 = -\Lambda_0 / 6M^3$ as above..

$\Rightarrow$ $c \equiv k_0/\bar{M}_{pl}$ is a conventional model parameter

$\simeq 0.01 - 0.10$

Then

$$R = \frac{m_{KK}}{m_{KK}^{(0)}} = \left(80 \beta c^{4/3}\right)^{-1} \left[-1 + \left(1 + 160 \beta c^{4/3}\right)^{1/2} \right]$$

usual KK mass

$\Rightarrow$ [plot]

$\Rightarrow$ Significant shifts in mass!

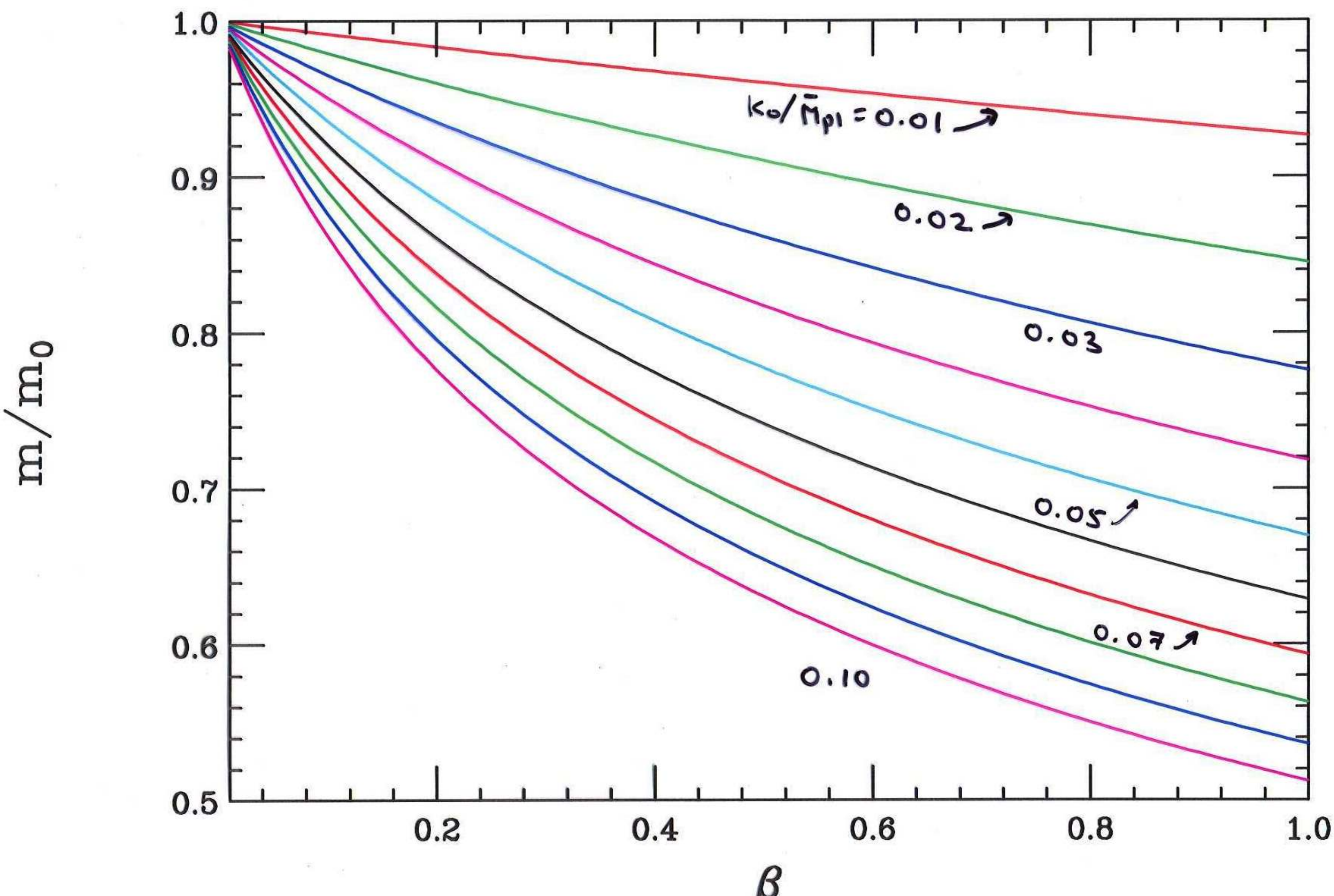
Similarly

$$\frac{\Lambda}{\Lambda_0} = -3 \left(80 \beta c^{4/3}\right)^{-1} \left[\left(1 - \frac{40}{3} \beta c^{4/3} R^2 \right)^2 - 1 \right]$$

... is also shifted to maintain consistancy

$\Rightarrow$ [plot]

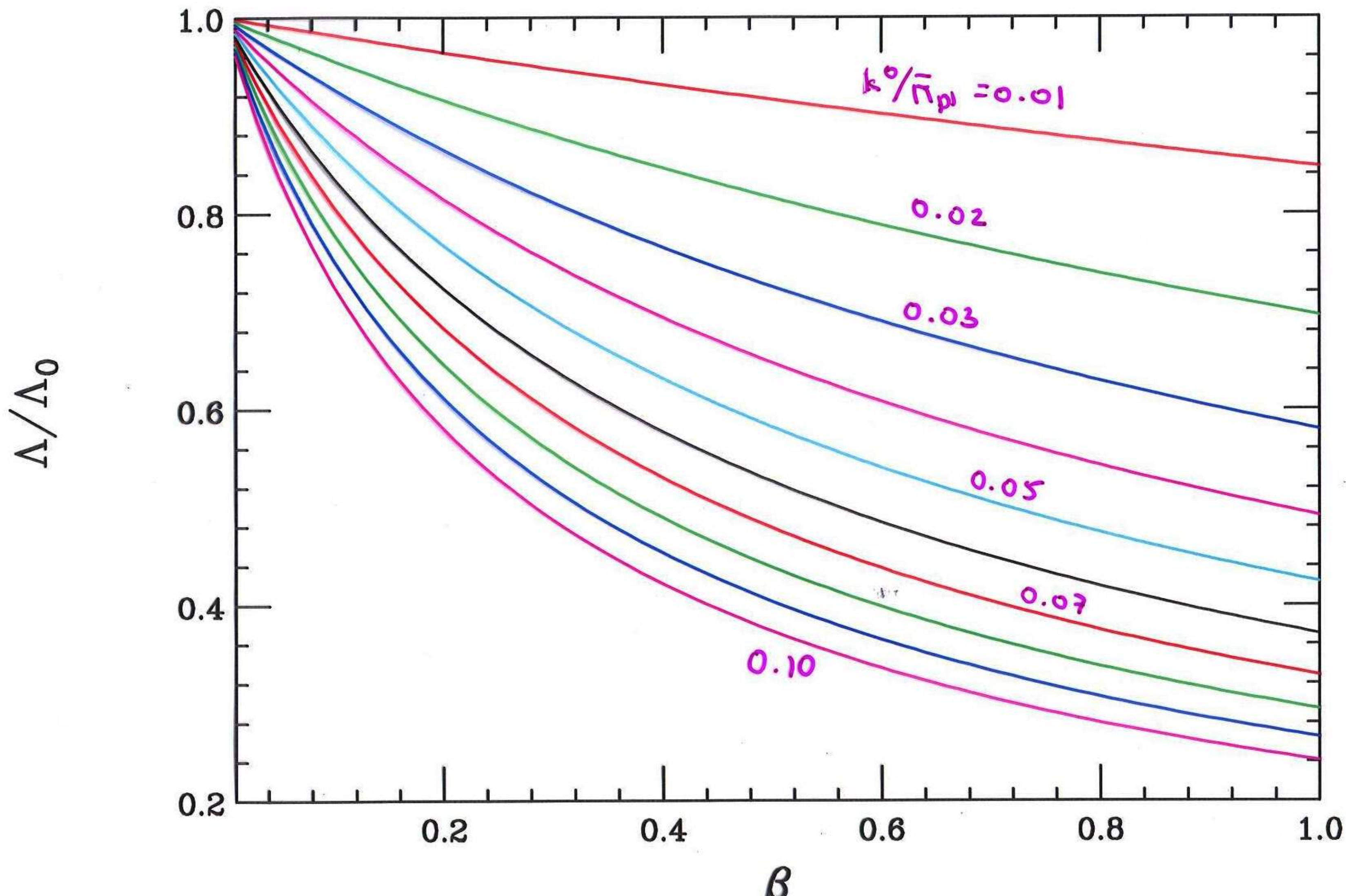
KK mass shifts



... quite sizeable!

10's of %!

Λ shifts



very sizeable

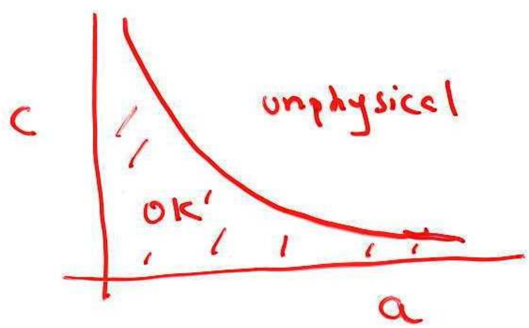
This approach will work for __any__ "crazy" model:

$$F = R \cdot \exp\left\{ a\left(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}\right)/M^4 \right\}$$

with $a \sim O(1)$ parameter... __then__

- $H(k) = e^{120 a k^4/\kappa^4} \left\{ 1 + 680 a \left(k/M\right)^4 + 198400 a^2 \left(k/\kappa\right)^8 \right\}$

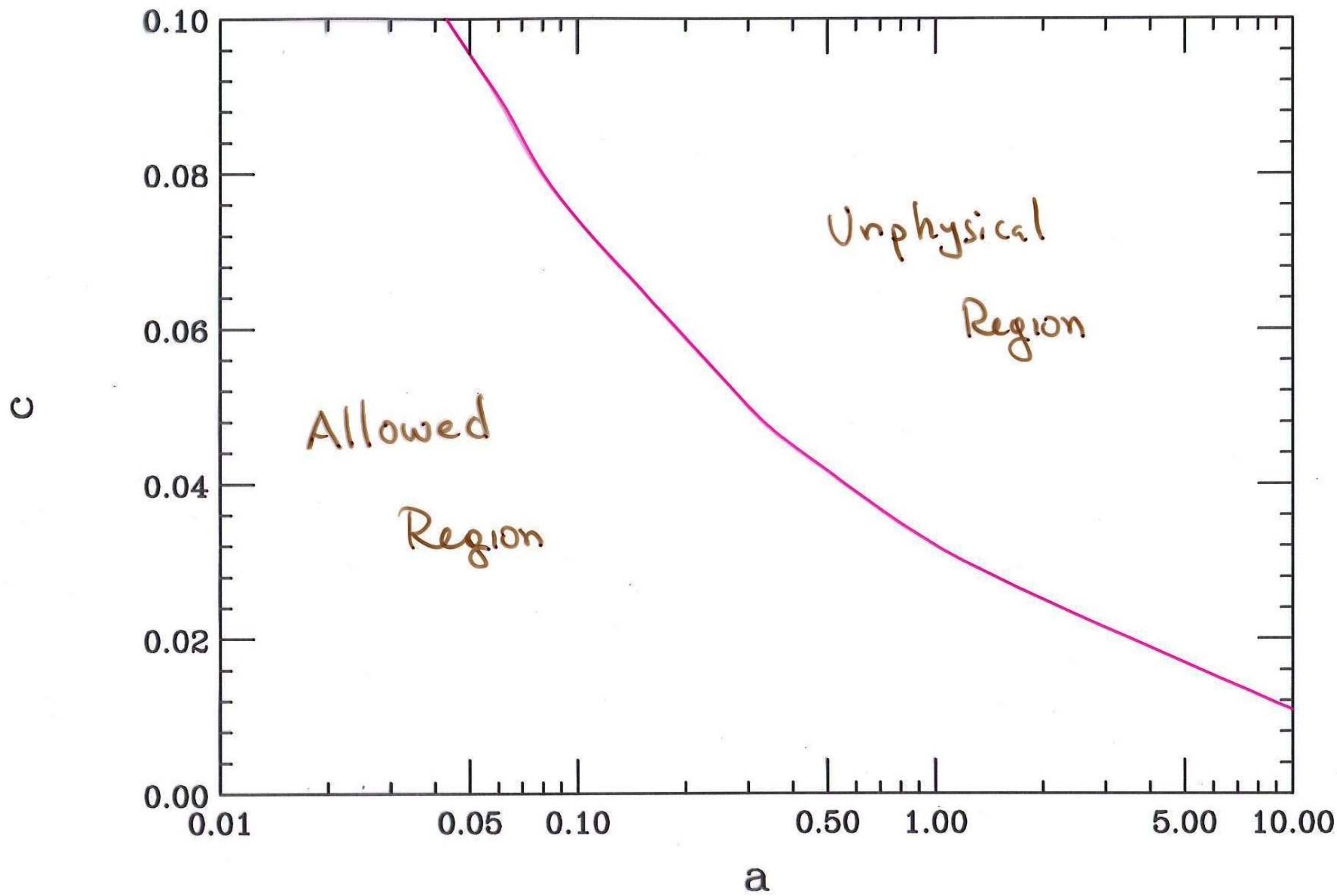
.. In this case the $\left\{ a, \; k_0/\bar{M}_{pl} = c \right\}$ parameter space is restricted strongly by the requirement that F_R be __Real__ + > 0 ...



...

.. in the physical region the KK masses are found to __Increase__ ...

[Fig]


Unphysical Region
Allowed Region
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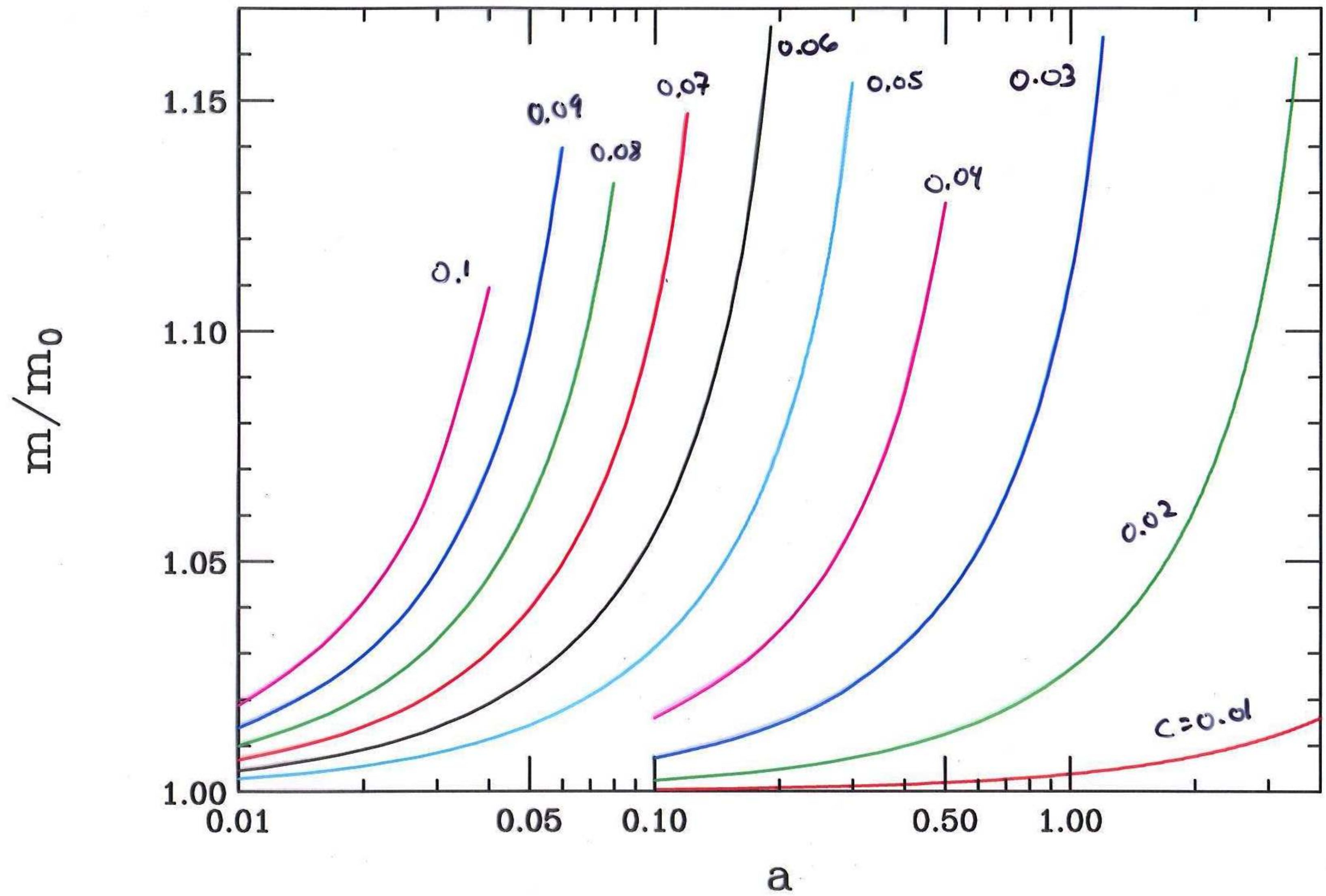
c
a

KK mass shift $F = R e^{a \cdot GB/M^4}$



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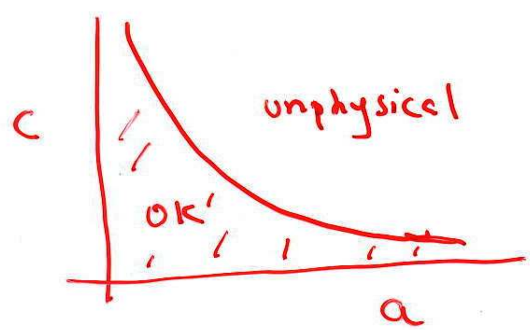

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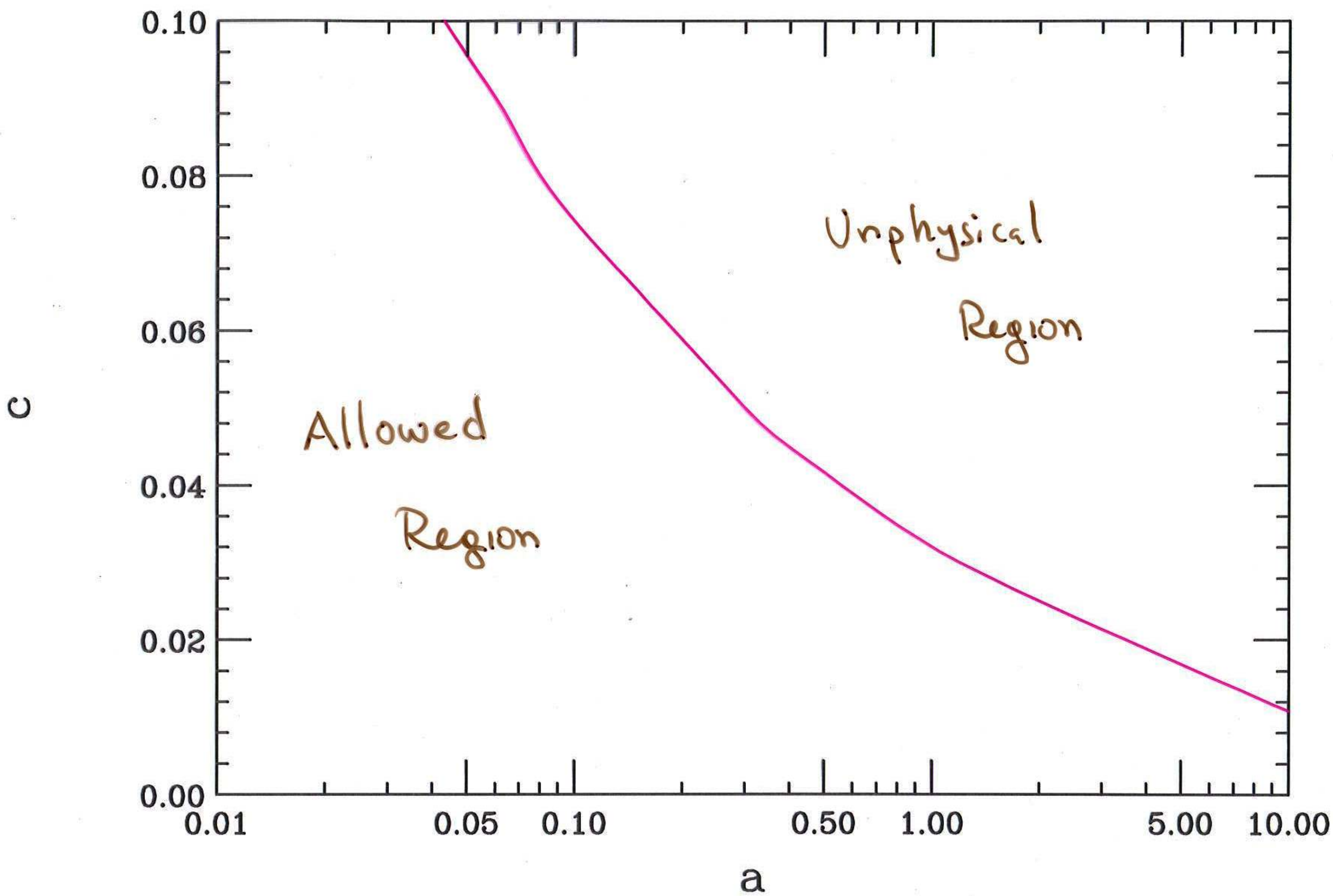
[Fig]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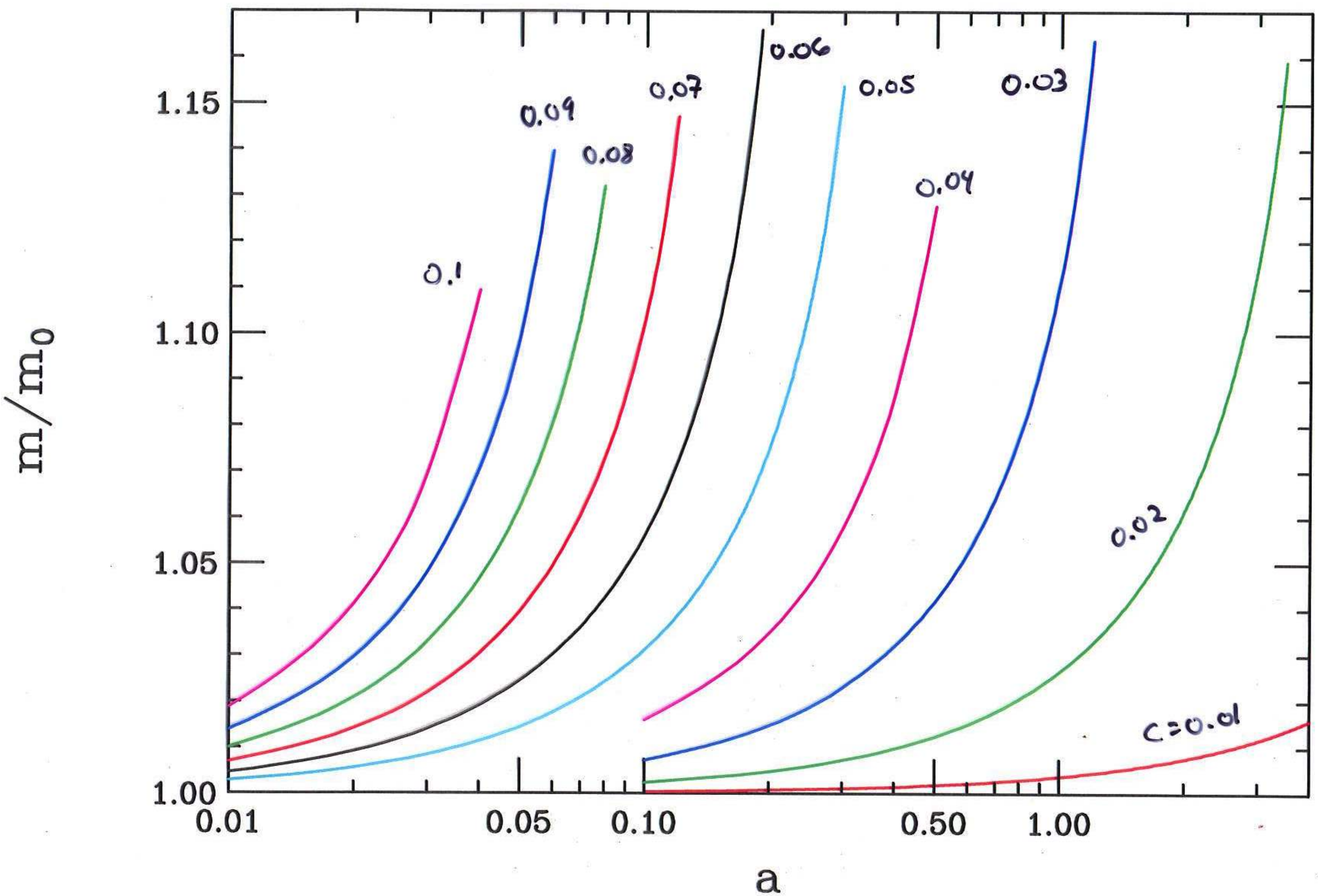
KK mass shift $\quad F = R e^{a \cdot GB/M^4}$



Summary / Conclusions

- It is possible to obtain ADD + RS-like sol's from more general gravity actions

- These lead to subtle alterations in the model predictions $\rightarrow$ scalar KK towers (not Higgs KK's)

$\Rightarrow$ Besides alterations in model relationships, these include rescaling of "classic" predictions involving graviton KK states $\rightarrow$ { mass + coupling shifts }

- Experimental observation of any of these effects provides info on a more fundametal theory than EH ...

$\Rightarrow$ Work in progress ...