# Threshold resummation in momentum space from effective theory

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TB, M.Neubert, hep-ph/0605050

TB, B. Pecjak and M. Neubert, hep-ph/0607228 ⟵ appeared today

# Why resummation?

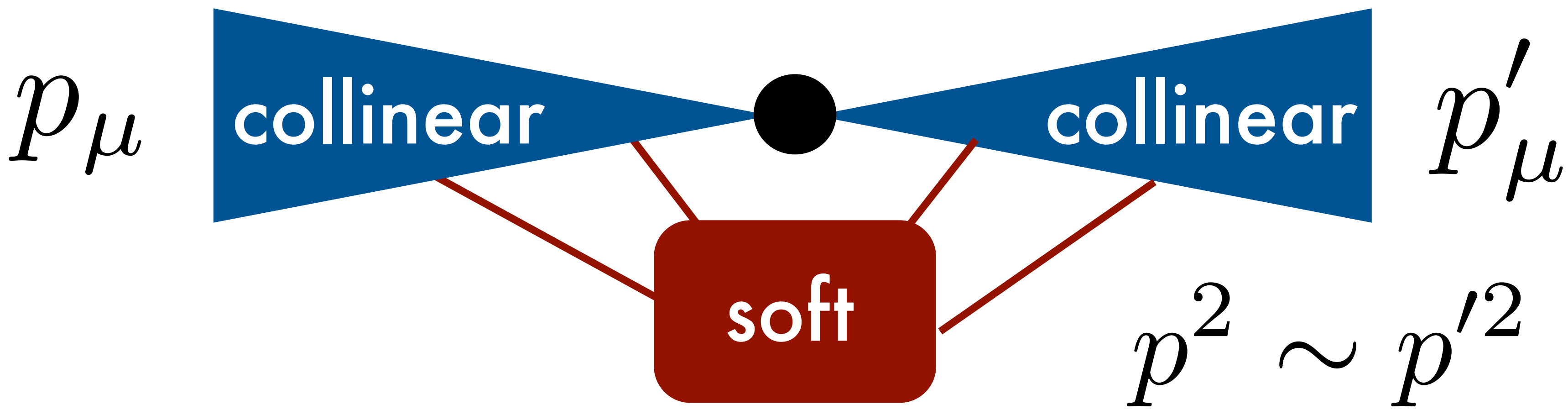
- Fixed order perturbation theory problematic for problems with widely separated scales $Q_1 >> Q_2$.
  - Large logarithms $\alpha_s^n \mathrm{Log}^n(Q_1/Q_2)$ and $\alpha_s^n \mathrm{Log}^{2n}(Q_1/Q_2)$. ← Sudakov logarithms
  - Scale in coupling? $\alpha_s(Q_1)$ or $\alpha_s(Q_2)$?
- Solution to both problems: integrate out physics at $Q_1$, solve RG, evolve to lower scale $Q_2$.

# Resummation for collider processes

- An old problem! In the past 20 year resummations were performed for many processes with scale hierarchies
  - DIS for $x \to 1$, Drell-Yan and Higgs production for $Q^2/s \to 1$, for $Q_T^2/Q^2 \to 0$.
  - $e^+e^-$ event shapes, hadronic event shapes, ...
  - ...
  - LL for arbitray observables with MC.
- Will talk about a new method to perform resummation of large perturbative log's in collider processes.
  - Based on RG in Soft-Collinear Effective Theory

# Soft-collinear effective theory

Bauer, Pirjol, Stewart '00



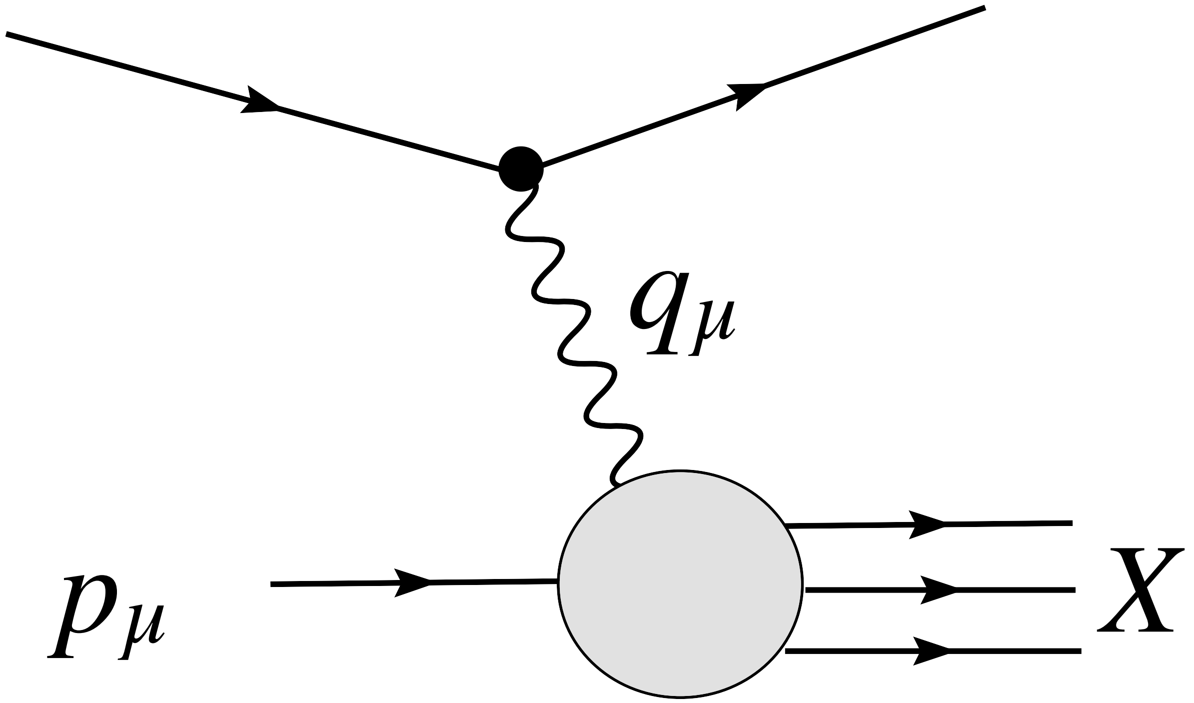

$$p^2 \sim p'^2 \ll 2p \cdot p'$$

- Eff. theory to analyze processes involving large momentum transfers and small invariant masses
- Originally developed to analyze *B*-meson decays to light hadrons
  - $B \to \pi\pi$, $B \to X_u\, l\nu$, ...

# Work in progress

- So far, we have analyzed only simplest process, DIS for $x \to 1$ (as well as inclusive $B$-decays)
  - High precision: Next-to-next-to-next-to-leading logarithmic accuracy ($N^3LL$)
  - Detailed comparison with standard approach
  - Drell-Yan process and Higgs production for $Q^2/s \to 1$ underway. (See also Idilbi, Ji and Yuan, hep-ph/0605068)
- Bauer and Schwartz: interesting proposal to improve MCs with eff. theory
  - Not yet implemented, tested only at LL accuracy.

# Kinematics for DIS




$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

- Are interested in the limit x→1, more precisely $Q^2 \gg Q^2(1-x) \gg \Lambda^2_{QCD}$

$$\approx M_X^2$$

# Factorization for DIS as x→1

Sterman '87

$$F_2^{\rm ns}(x,Q^2) = H(Q^2,\mu)\, Q^2 \int_x^1 \frac{dz}{z}\, J\Big(Q^2\,\frac{1-z}{z},\mu\Big)\, \frac{x}{z}\, \phi_q^{\rm ns}\Big(\frac{x}{z},\mu\Big)$$

| hard | x | jet | $\otimes$ | PDF |
|---|---|---|---|---|
| $Q^2$ | >> | $Q^2(1\text{-}x)$ | >> | $\Lambda^2$ |

- Rederivation in SCET had troubled history
  - Claims of nonfactorization, different form of factorization, non-perturbative factorization...
  - hep-ph/0607228 resolves these differences.
    - Proper identification of PDF as $x \to 1$ crucial.
- Resummation by solving RG equations for three parts.

# Traditional method: moment space

Sterman '87, Catani and Trentadue '89

$$F_{2,N}^{\rm ns}(Q^2) = \int_0^1 dx\, x^{N-1} F_2^{\rm ns}(x, Q^2)$$
$$= C_N(Q^2, \mu_f) \sum_q e_q^2\, \phi_{q,N}^{\rm ns}(\mu_f)$$

- Convolution in momentum space $\rightarrow$ product in moment space
- $x \rightarrow 1$ corresponds to $N \rightarrow \infty$. Perturbation theory contains $\alpha_s^n \,\mathrm{Log}^n(N)$ and $\alpha_s^n \,\mathrm{Log}^{2n}(N)$
- Split:

$$C_N(Q^2, \mu_f) = g_0(Q^2, \mu_f) \exp\left[G_N(Q^2, \mu_f)\right]$$

# Resummation in moment space

$$C_N(Q^2,\mu_f) = g_0(Q^2,\mu_f)\,\exp\left[G_N(Q^2,\mu_f)\right]$$

$$G_N(Q^2,\mu_f) = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}$$
$$\times\left[\int_{\mu_f^2}^{(1-z)Q^2}\frac{dk^2}{k^2}\,A_q(\alpha_s(k)) + B_q\big(\alpha_s(Q\sqrt{1-z})\big)\right]$$

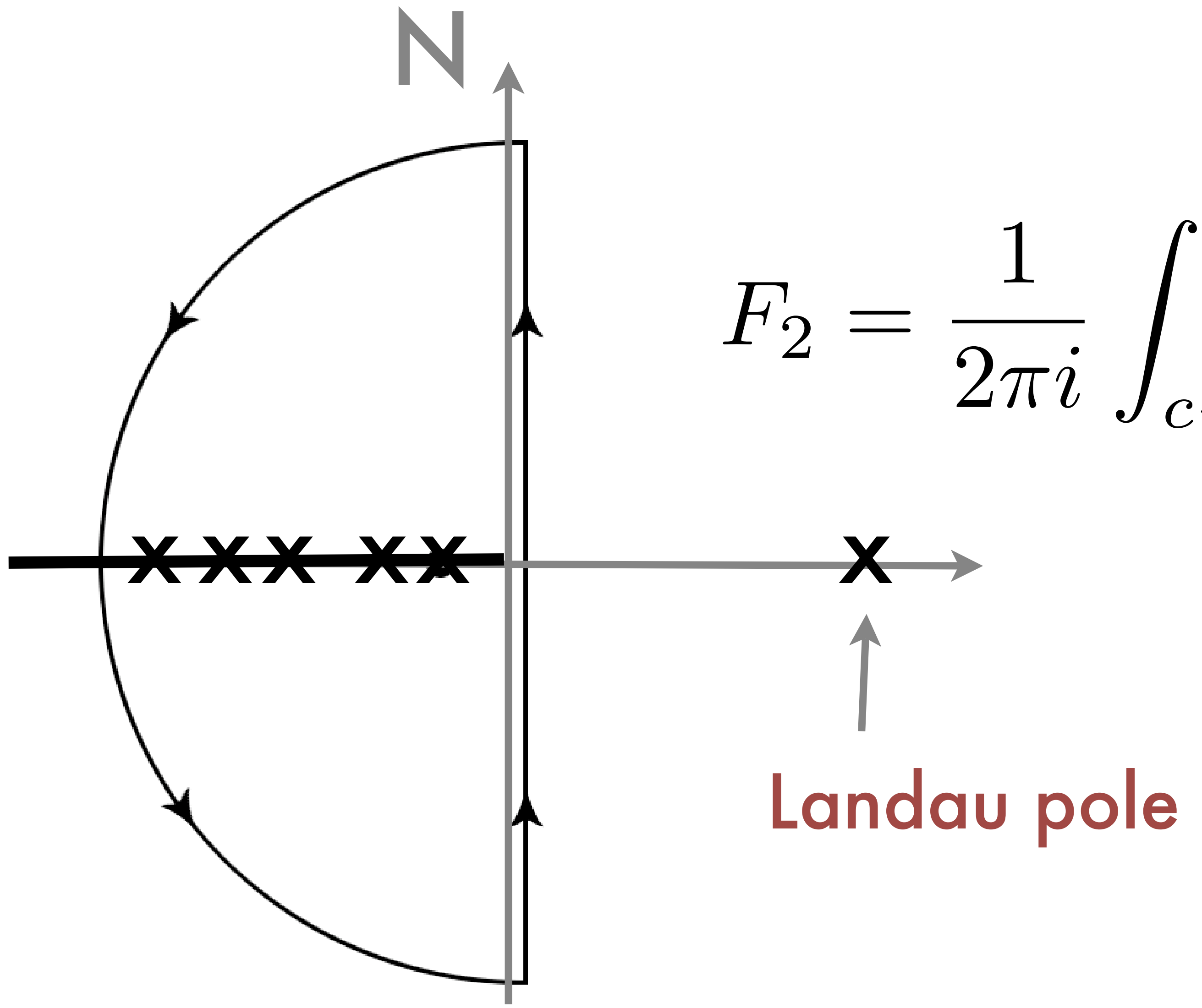
Landau pole

Cusp anomalous dim.

Anom. dim. of ??

- $A_q$, $B_q$ determined by matching to fixed order result. NNNLL: Moch, Vermaseren, Vogt '05

# Mellin Inversion




$$F_2 = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N}\phi_N\, C_N$$

- Can only be done numerically
- Problem with Fortran PDF's.

# Resummation in momentum space

- Match QCD onto Soft-Collinear Effective theory.
- Use RG evolution to resum logarithms.

$Q^2$ $\qquad\qquad$ $Q^2(1\text{-}x)$ $\qquad\qquad$ $\Lambda$

**match** $\rightarrow$ **run** $\rightarrow$ **match** $\rightarrow$ **run**

$$H(\mu_h) \times U_1(\mu_h, \mu_i) \times J(\mu_i) \otimes U_2(\mu_i, \mu_f) \otimes \phi(\mu_f)$$

from on-shell quark FF

from quark prop. in light-cone gauge

# First matching step

light-like Wilson lines

- Match QCD current onto EFT current

$$\big(\psi\gamma^\mu\psi\big)(x) \to \big(\bar{\xi}_{\bar{c}}W_{\bar{c}}\big)(x_-)\,\gamma^\mu\big(W^\dagger_{hc}\xi_{hc}\big)(x)$$

proton — jet

- Wilson coefficient $C_V$ and anomalous dimension $\gamma_V$ from on-shell matching
  - on-shell FF is known to 2 loops ($\to C_V$), divergencies to 3 loops ($\to \gamma_V$)
- Match QCD current onto EFT current

$$\frac{d}{d\ln\mu}C_V(Q^2,\mu) = \left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2}+\gamma^V(\alpha_s)\right]C_V(Q^2,\mu)$$

# Running to intermediate scale:

## Solution to the RG:

$$C_V(Q^2,\mu) = \exp\left[2S(\mu_h,\mu) - a_{\gamma^V}(\mu_h,\mu)\right]\left(\frac{Q^2}{\mu_h^2}\right)^{-a_\Gamma(\mu_h,\mu)} C_V(Q^2,\mu_h)$$

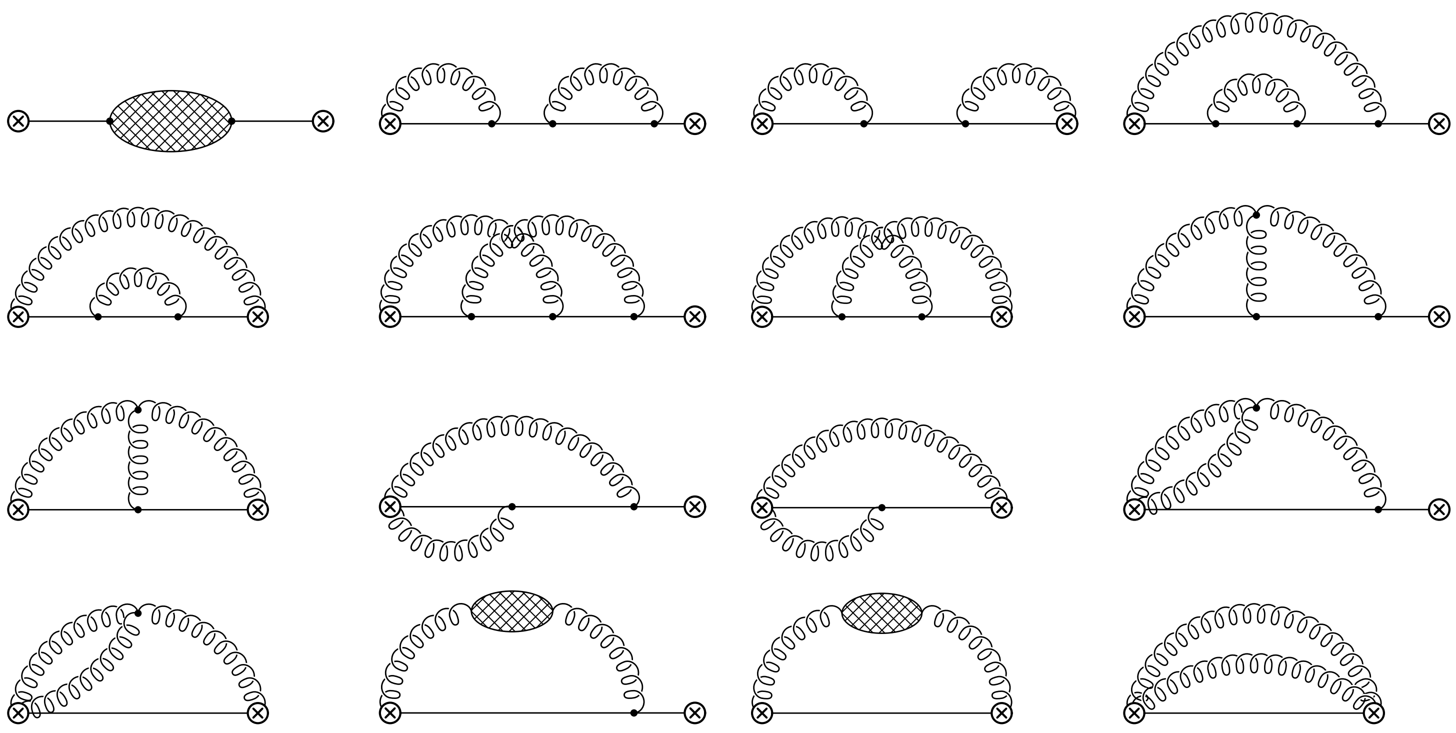
$$S(\nu,\mu) = -\int\limits_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}\int\limits_{\alpha_s(\nu)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')}\,, \qquad a_\Gamma(\nu,\mu) = -\int\limits_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$$

# Jet-function

$$J(p^2) = \frac{1}{\pi} \mathrm{Im}\, i \int d^x e^{-ipx} \langle 0|\, \mathrm{T} \left[ W^\dagger(0) \xi_{hc}(0)\, \bar{\xi}_{hc} W(x) |0 \right]\; |0\rangle$$

- Propagator in light-cone gauge.
- Have evaluated $J(p^2)$ to 2 loops.

TB, M.Neubert, hep-ph/0603140



# RG evolution of the jet-function

$$\frac{dJ(p^2,\mu)}{d\ln\mu} = -\left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{p^2}{\mu^2} + 2\gamma^J(\alpha_s)\right] J(p^2,\mu) - 2\Gamma_{\rm cusp}(\alpha_s)\int_0^{p^2} dp'^2\,\frac{J(p'^2,\mu)-J(p^2,\mu)}{p^2-p'^2}$$

$$J(p^2,\mu) = \exp\left[-4S(\mu_i,\mu) + 2a_{\gamma^J}(\mu_i,\mu)\right] \times \widetilde{j}(\partial_\eta,\mu_i)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\,\frac{1}{p^2}\left(\frac{p^2}{\mu_i^2}\right)^{\eta},$$

$$\eta = 2\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_{\rm cusp}[\alpha_s(\mu)] = 2a_\Gamma(\mu_i,\mu).$$

- Associated jet-function $\widetilde{j}$ is Laplace transform of $J(p^2,\mu_{\rm i})$.

# Result

- Plug RG solutions into factorization theorem, assume $\phi_q(x,\mu_f) \sim (1-x)^{b(\mu_f)}$

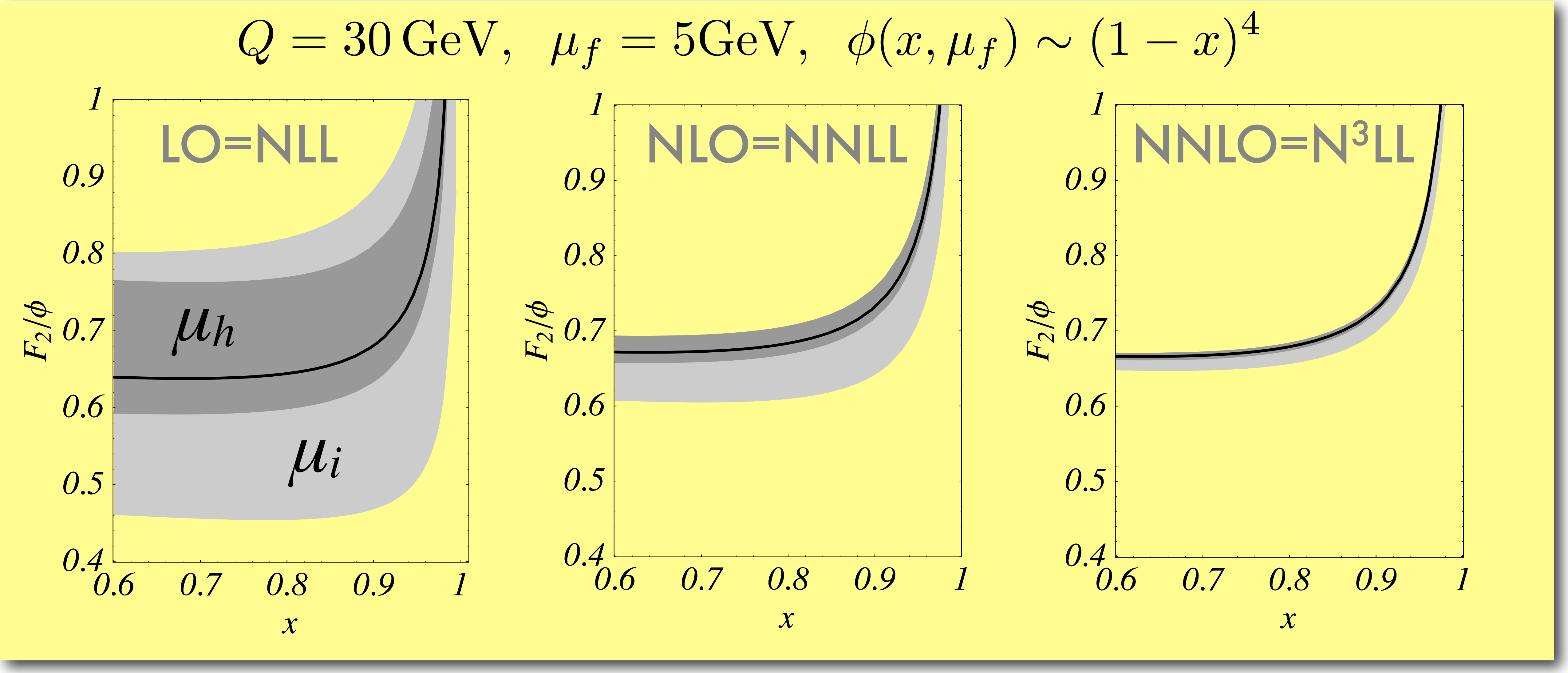
$$\frac{F_2^{\rm ns}(x,Q^2)}{\sum_q e_q^2\, x\, \phi_q^{\rm ns}(x,\mu_f)} = |C_V(Q^2,\mu_h)|^2\, U(Q,\mu_h,\mu_i,\mu_f)$$
$$\times (1-x)^\eta\, \tilde{j}\Big(\ln\frac{Q^2(1-x)}{\mu_i^2} + \partial_\eta, \mu_i\Big)$$
$$\times \frac{e^{-\gamma_E \eta}\,\Gamma(1+b(\mu_f))}{\Gamma(1+b(\mu_f)+\eta)}\,.$$

- Resummed result obtained after plugging in fixed order results for coefficient $C_V$, jet-function and anom. dimensions.

# Difference to traditional approach

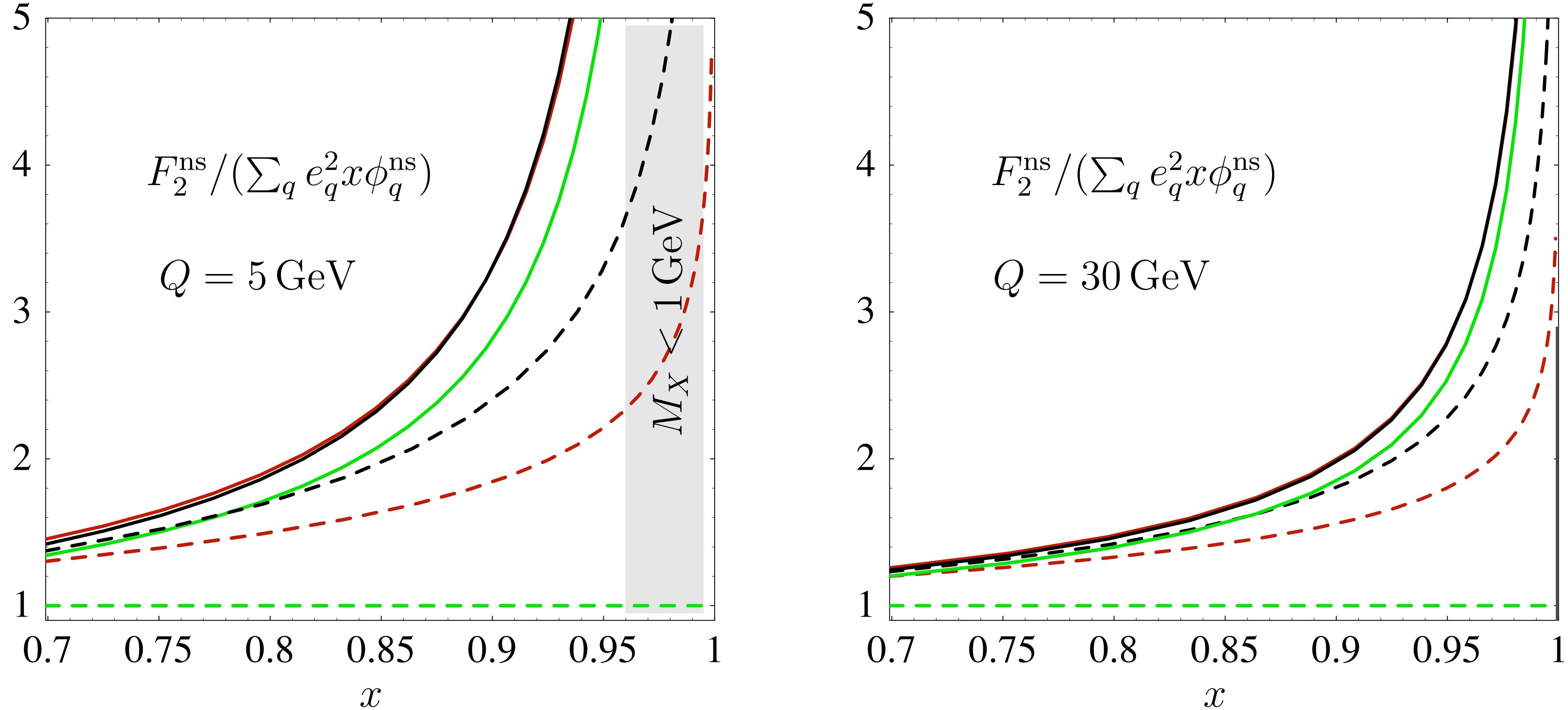
- Simple analytic result in momentum space
- No Landau pole ambiguities. No coupling below scales $\mu_h$, $\mu_i$ and $\mu_f$.
- Freedom to choose scales $\mu_h$, $\mu_i$ and $\mu_f$
  - Obtain fixed order for $\mu_h=\mu_i=\mu_f$. Trivial matching to fixed order result for generic $x$.
  - Set appropriate scales *after* integrating
    - Avoids large spurious power corrections discussed by Catani et al. hep-ph/9604351
  - Estimate uncertainties with scale variation

# Result for $F_2^{\mathrm{ns}}(\mathrm{x})/\phi_q(x)$



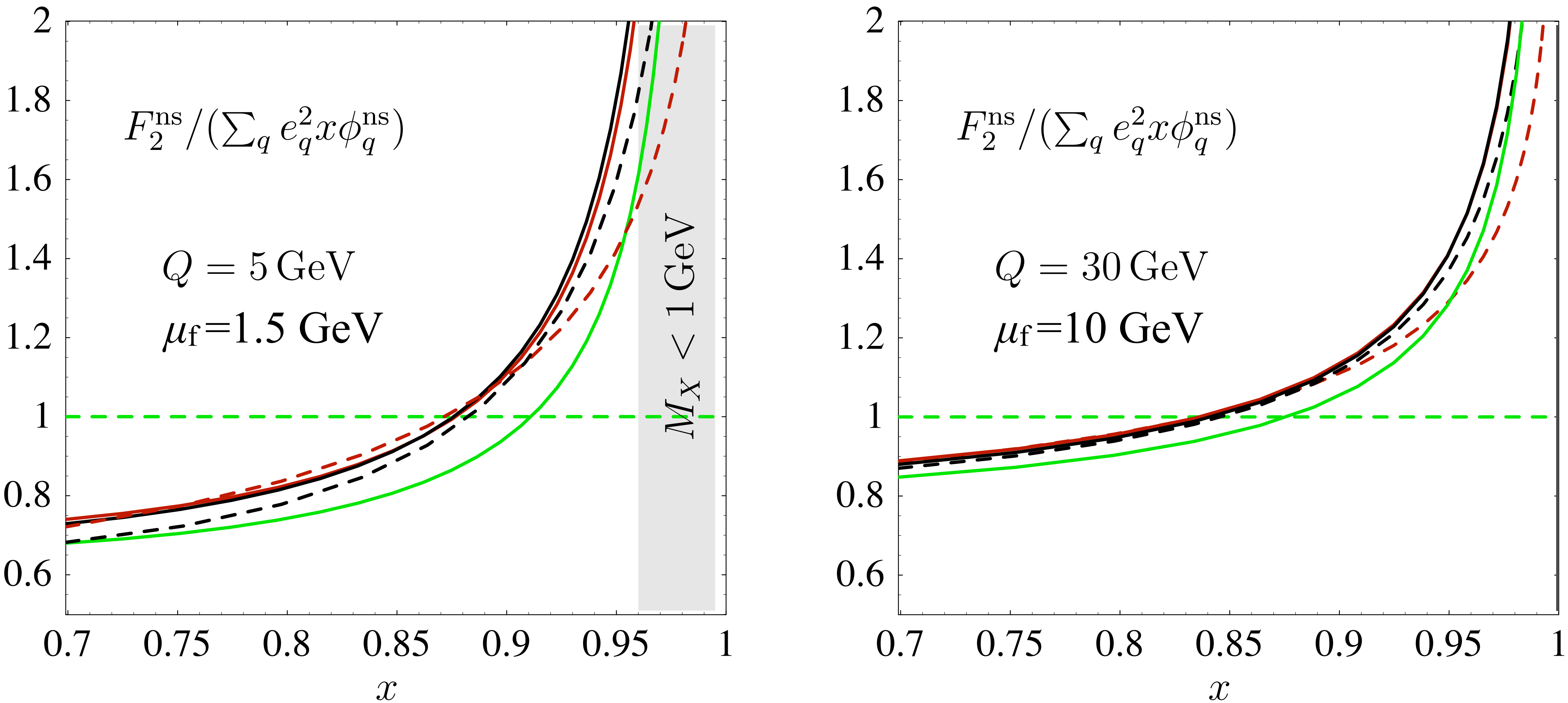

- Default scales: $\mu_h^2 = Q^2$ and $\mu_i^2 = Q^2(1\text{-x})$
  - Bands obtained by varying these scales a factor of two up and down.
  - Matching scales are fixed in traditional approach.

# Comparison with fixed order, $\mu_f = Q$



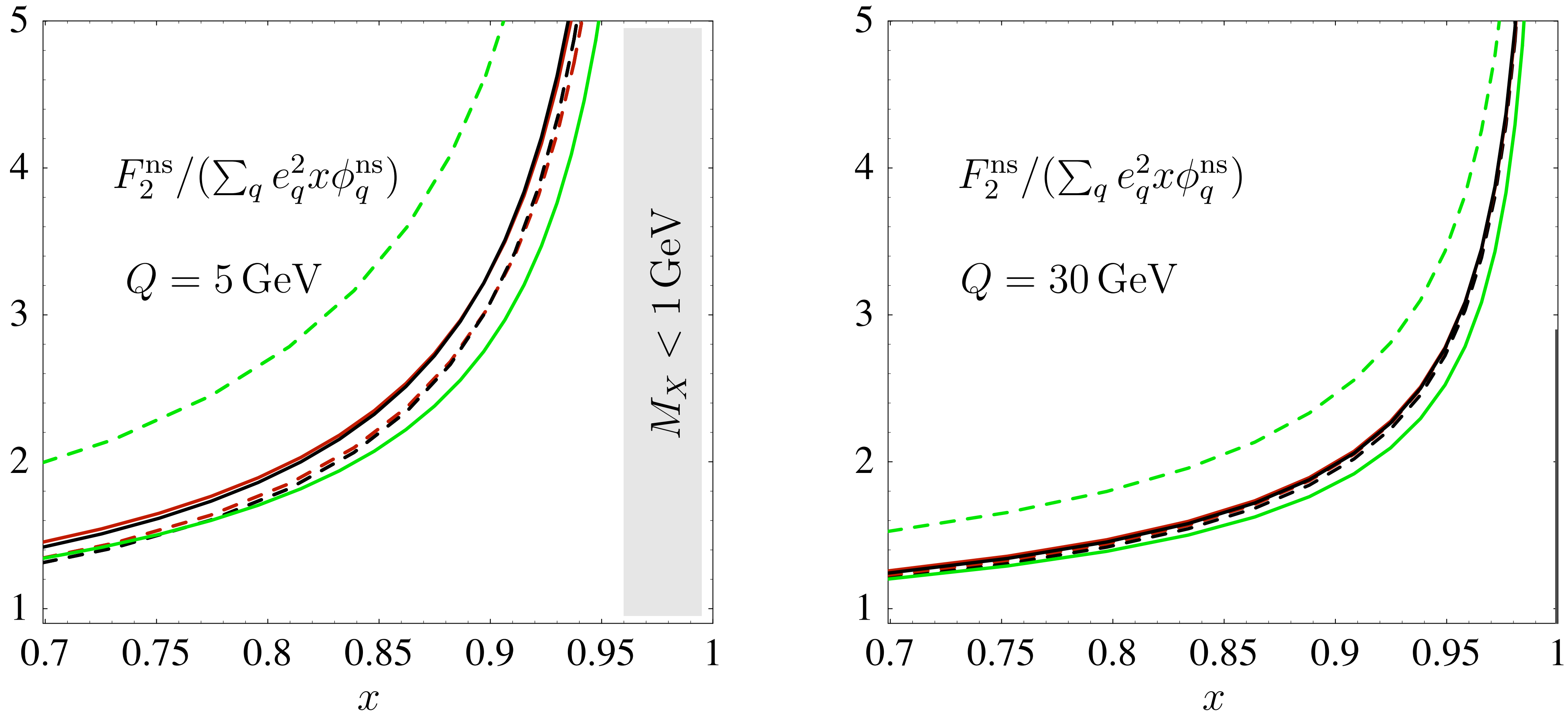

- LO (=NLL), NLO, NNLO
- Dashed: fixed order. Solid: resummed.
- Large K-factors.

# Comparison with fixed order, low $\mu_f$



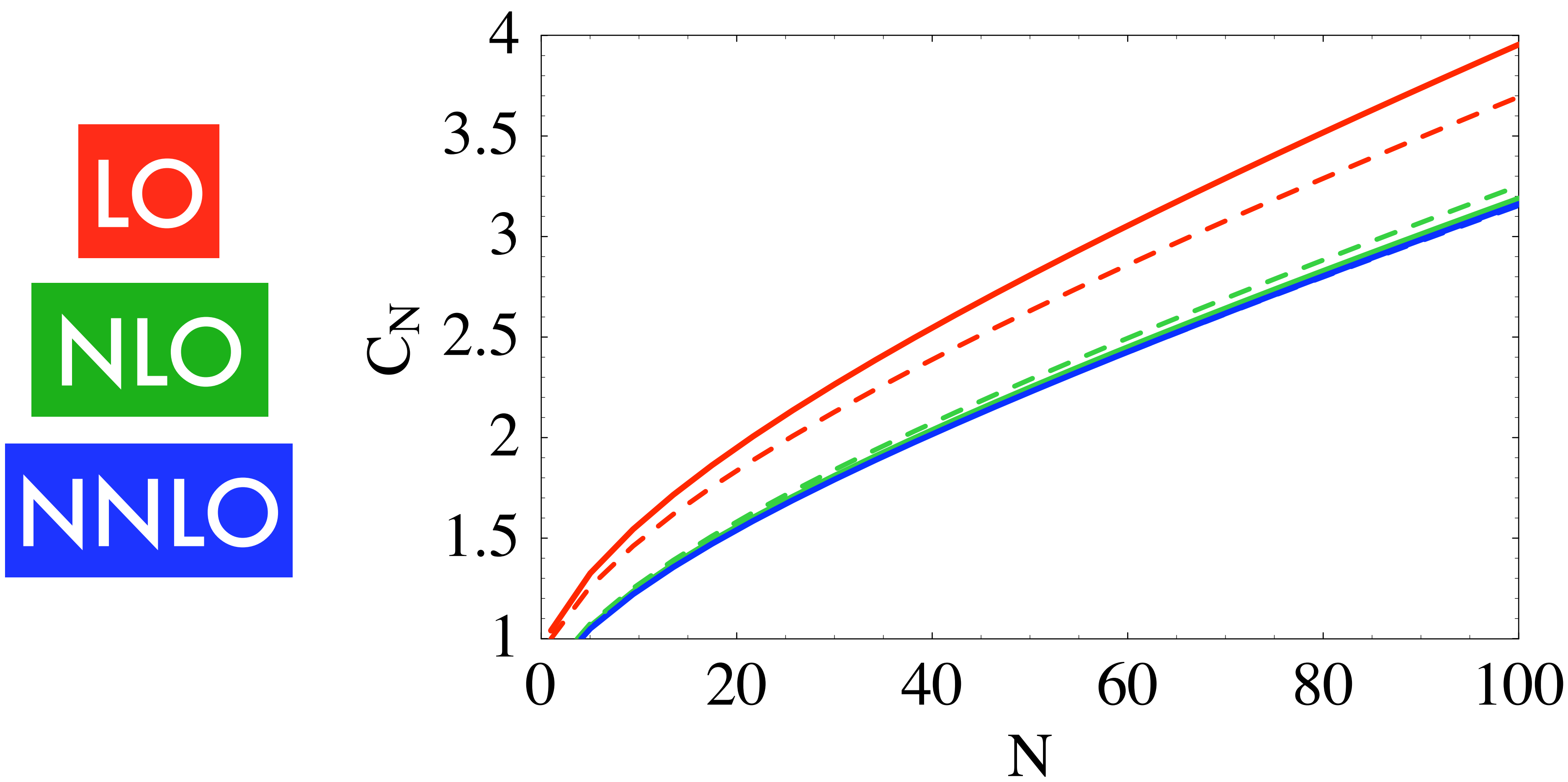

- LO (=NLL), NLO, NNLO
- Dashed: fixed order. Solid: resummed.
- Fixed order with $\mu = \mu_f$ fairly close to resummed result!

# Comparison with moment space result




- Dashed: Mellin inverted moment space results. Solid: momentum space results.
- Only small numerical differences (different scale choice, 1/N corrections in moment space).
- Faster convergence of momentum space results.

# Moments $C_N = F_{2,N}/\phi_N$




- $Q = 30$ GeV, $\mu_h = Q$, $\mu_i^2 = Q^2/N$, $\mu_f = 5$ GeV.
- Solid: EFT, default scale. Dashed: Moch, Vermaseren, Vogt, hep-ph/0506288.
- Note: NNLO indistinguishable.

# Connection with standard approach

- Can compare EFT expression for moments with standard results. The two agree provided that

$$\left(1+\frac{\pi^2}{12}\nabla^2+\ldots\right)B_q(\alpha_s)=\gamma^J(\alpha_s)+\nabla\,\ln\widetilde{j}(0,\mu)$$
$$-\left(\frac{\pi^2}{12}\nabla-\frac{\zeta_3}{3}\nabla^2+\ldots\right)\Gamma_{\rm cusp}(\alpha_s)\,,\qquad \nabla = d/d\ln\mu^2.$$

  - fulfilled with two-result from explicit calculation of $J(p^2)$.

# Summary

- Traditionally, resummation for hard processes is performed in moment space.
  - Landau poles (in Sudakov exponent and Mellin inversion)
  - Mellin inversion only numerically
- Solving RG equations in SCET, we have obtained resummed expressions directly in momentum space.
  - Clear scale separation. No Landau pole ambiguities.
  - Analytic expressions for resummed rates.
  - Simple connection with fixed order expressions.
- Same technology should be applicable to many other processes.
  - Threshold resummation for DY and Higgs production under way.