

# Phenomenological Indication of the Terascale Implications for the ILC

*Sven Heinemeyer, IFCA (CSIC – UCSA)*

Vancouver, 07/2006

based on collaboration with  
*J. Ellis, K. Olive and G. Weiglein*

1. Motivation and models
2. Precision Observables in the MSSM
3. Fits and ILC reach
4. Conclusions

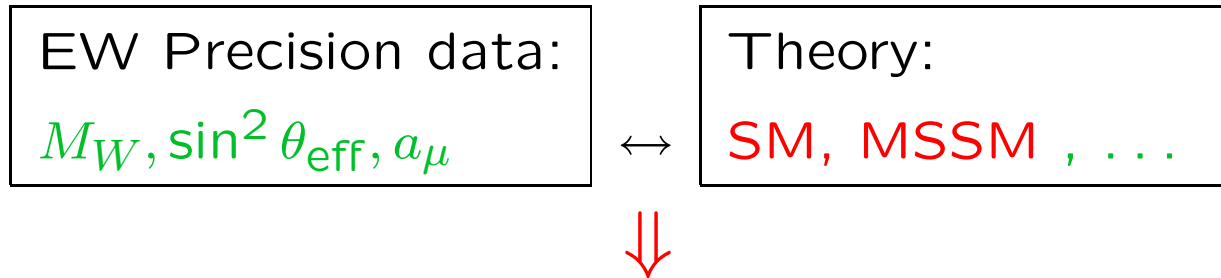
# 1. Motivation and models

What do we know about the SUSY mass scale?

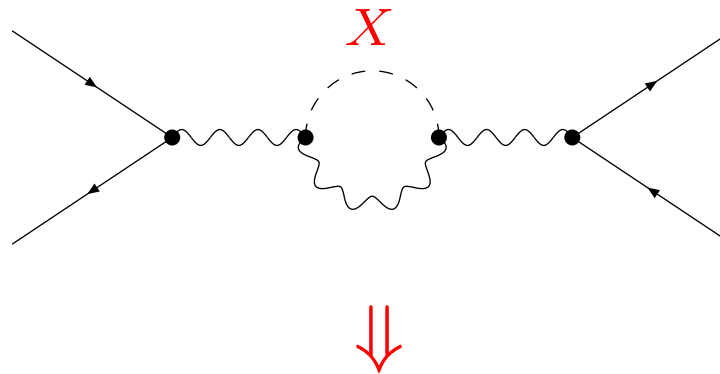
1. Coupling constant unification  $\Rightarrow M_{\text{SUSY}} \approx 1 \text{ TeV}$
2. LSP should be cold dark matter  $\Rightarrow M_{\text{SUSY}} \lesssim 1 \text{ TeV}$
3. Indirect hints from existing data?
  - Focus on **CMSSM**, **NUHM**, **VCMSSM** and **GDM**  
small number of free parameters
  - hard constraint: **LSP** gives right amount of **cold dark matter**  
CMSSM: only thin **strips** allowed in the  $m_{1/2}-m_0$  plane  
VCMSSM: even  $\tan\beta$  determined  
NUHM, GDM: also strong constraints
  - Use existing data of  $M_W$ ,  $\sin^2\theta_{\text{eff}}$ ,  $\text{BR}(b \rightarrow s\gamma)$ ,  $(g-2)_\mu$ ,  $M_h$   
 $\Rightarrow \chi^2$  fit with these observables  
 $\Rightarrow$  best fit values for masses, couplings, ...

## Precision Observables (POs):

Comparison of electro-weak precision observables with theory:



Test of theory at quantum level: Sensitivity to loop corrections

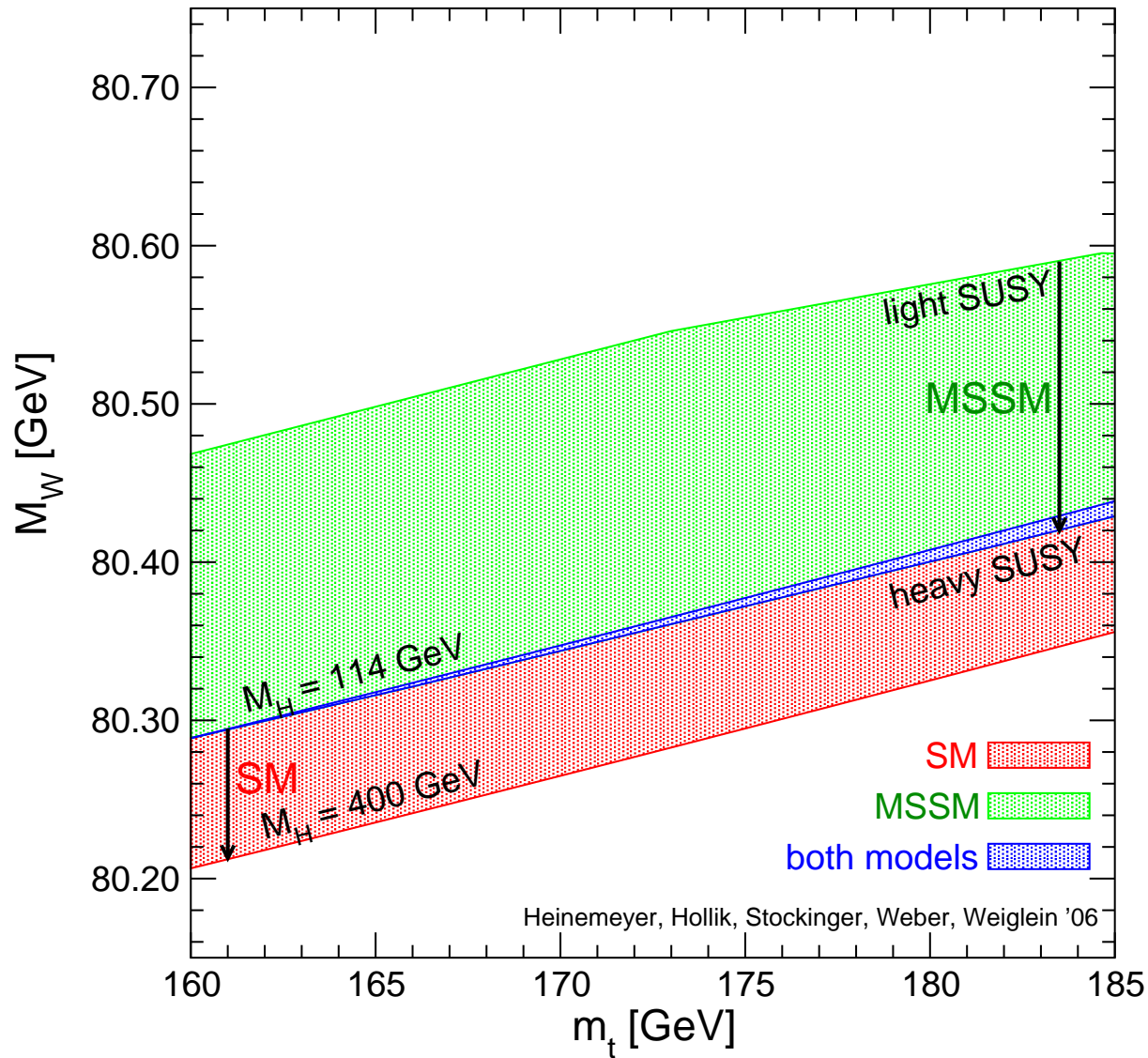


Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

Example: Prediction for  $M_W$  in the **SM** and the **MSSM** :

[S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '06]



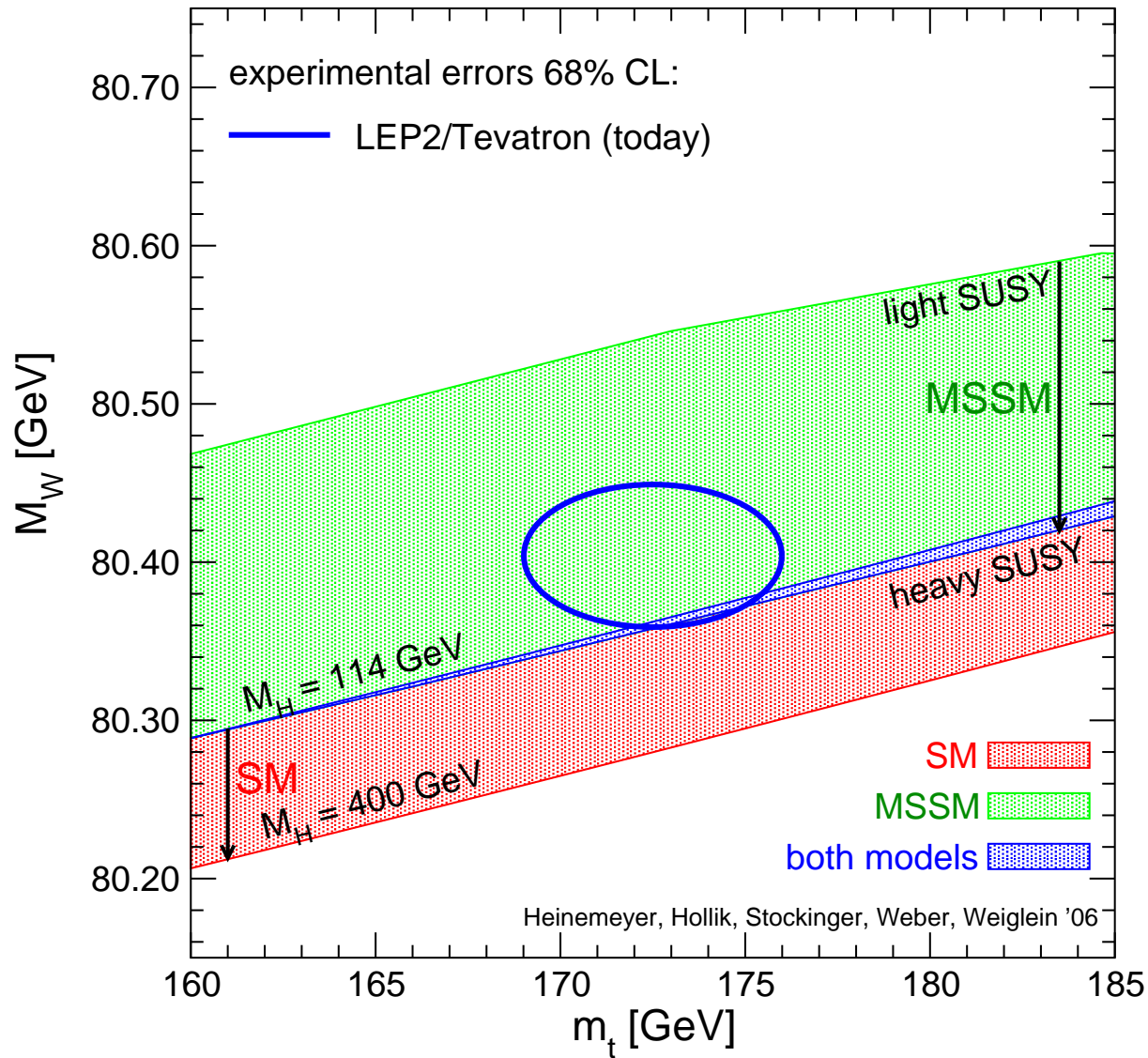
**MSSM band:**  
scan over  
SUSY masses

**overlap:**  
SM is MSSM-like  
MSSM is SM-like

**SM band:**  
variation of  $M_H^{\text{SM}}$

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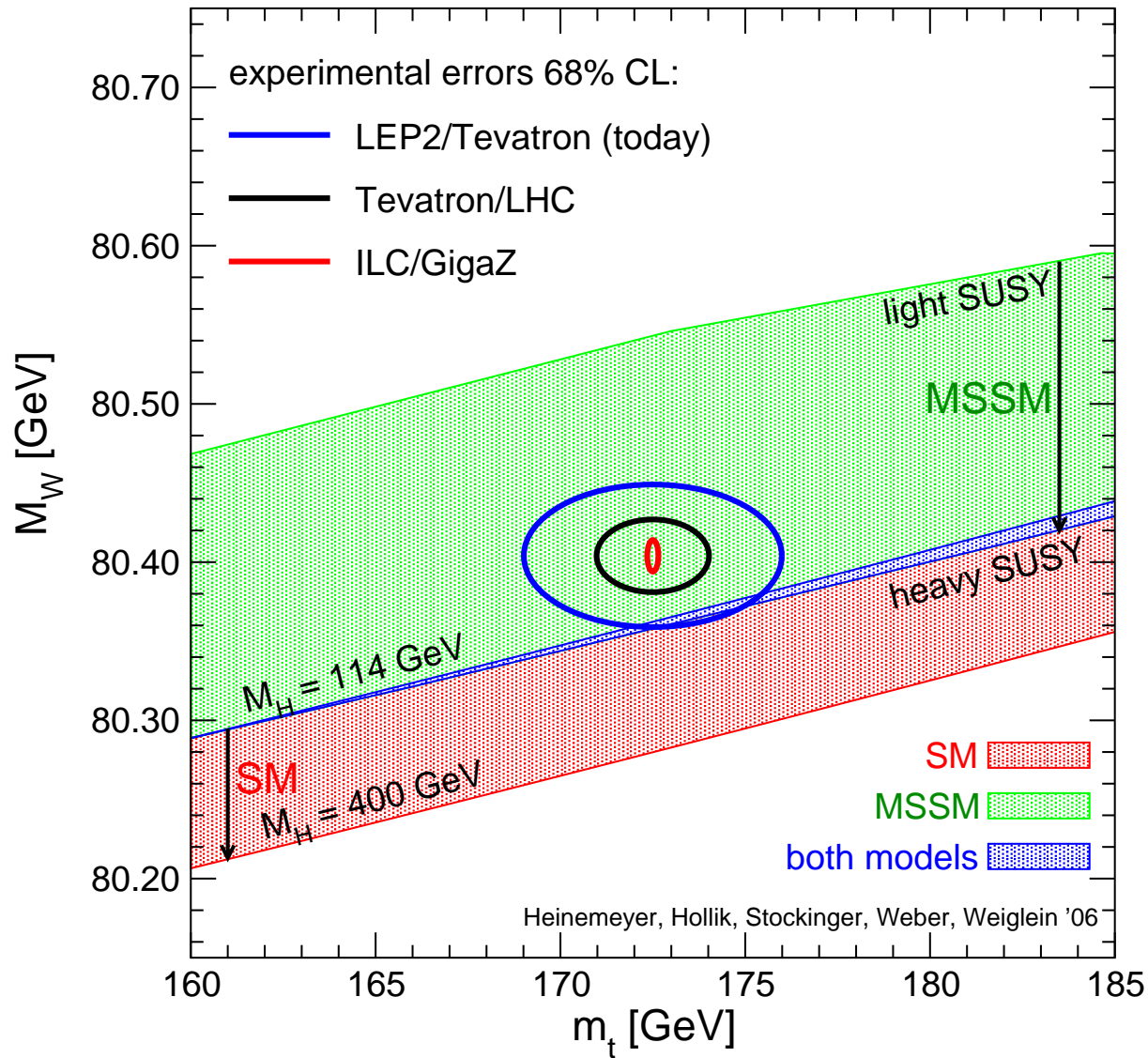
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## The models (I):

### CMSSM (or mSUGRA):

⇒ Scenario characterized by

$$m_0, m_{1/2}, A_0, \tan \beta, \text{sign } \mu$$

$m_0$  : universal scalar mass parameter

$m_{1/2}$  : universal gaugino mass parameter

$A_0$  : universal trilinear coupling

$\tan \beta$  : ratio of Higgs vacuum expectation values

$\text{sign}(\mu)$  : sign of supersymmetric Higgs parameter

} at the GUT scale

⇒ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is the lightest neutralino

## The models (I):

CMSSM (or mSUGRA)

⇒ Scenario character

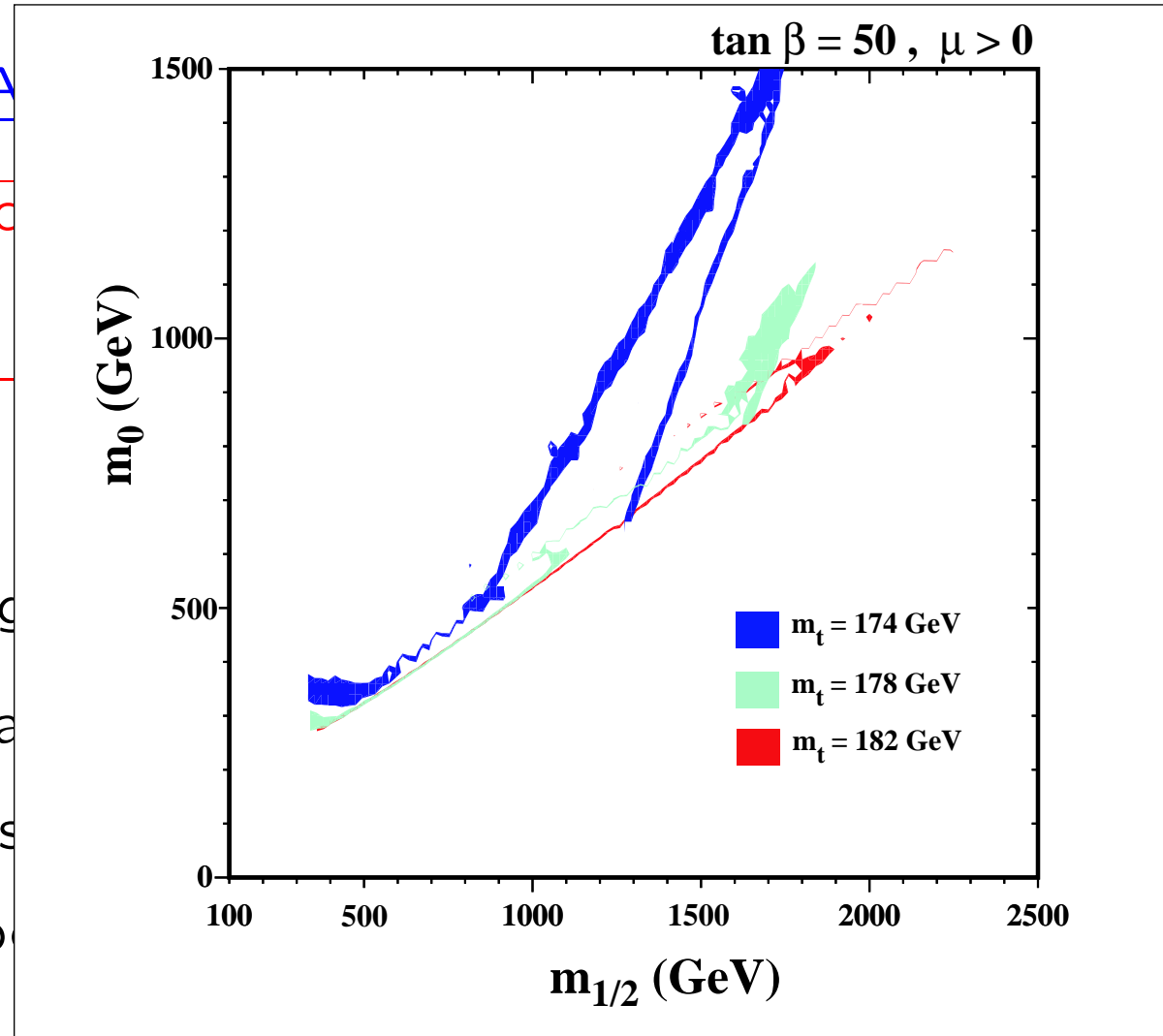
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⇒ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is the lightest neutralino



## The models (II):

NUHM: (Non-universal Higgs mass model)

⇒ besides the CMSSM parameters

$M_A$  and  $\mu$

Assumption:

no unification of scalar fermion and scalar Higgs parameters  
at the GUT scale

⇒ effectively  $M_A$  and  $\mu$  free parameters at the EW scale

⇒ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is the lightest neutralino

## The models (III):

### VCMSM: (Very Constrained MSSM)

⇒ In addition to CMSSM: assume relation between  $A_0$  and  $m_0$ :  
$$A_0/m_0 = 0, 3/4, 3 - \sqrt{3}, 2$$

Additional constraint also fixes  $\tan \beta$

Free parameters:  $m_{1/2}$ ,  $A_0/m_0$

$m_0$  and  $\tan \beta$  fixed via CDM constraint

Lightest SUSY particle (LSP) is the lightest neutralino

### GDM (mSUGRA): (Gravitino DM in mSUGRA)

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mSUGRA:  $m_{\text{gravitino}} = m_0 \Rightarrow$  gravitino can be the LSP

Free parameters:  $m_{1/2}$ ,  $A_0/m_0$

Lightest SUSY particle (LSP) is the gravitino

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Additional constraint a  
Free parameters:  $m_{1/2}$   
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Lightest SUSY particle

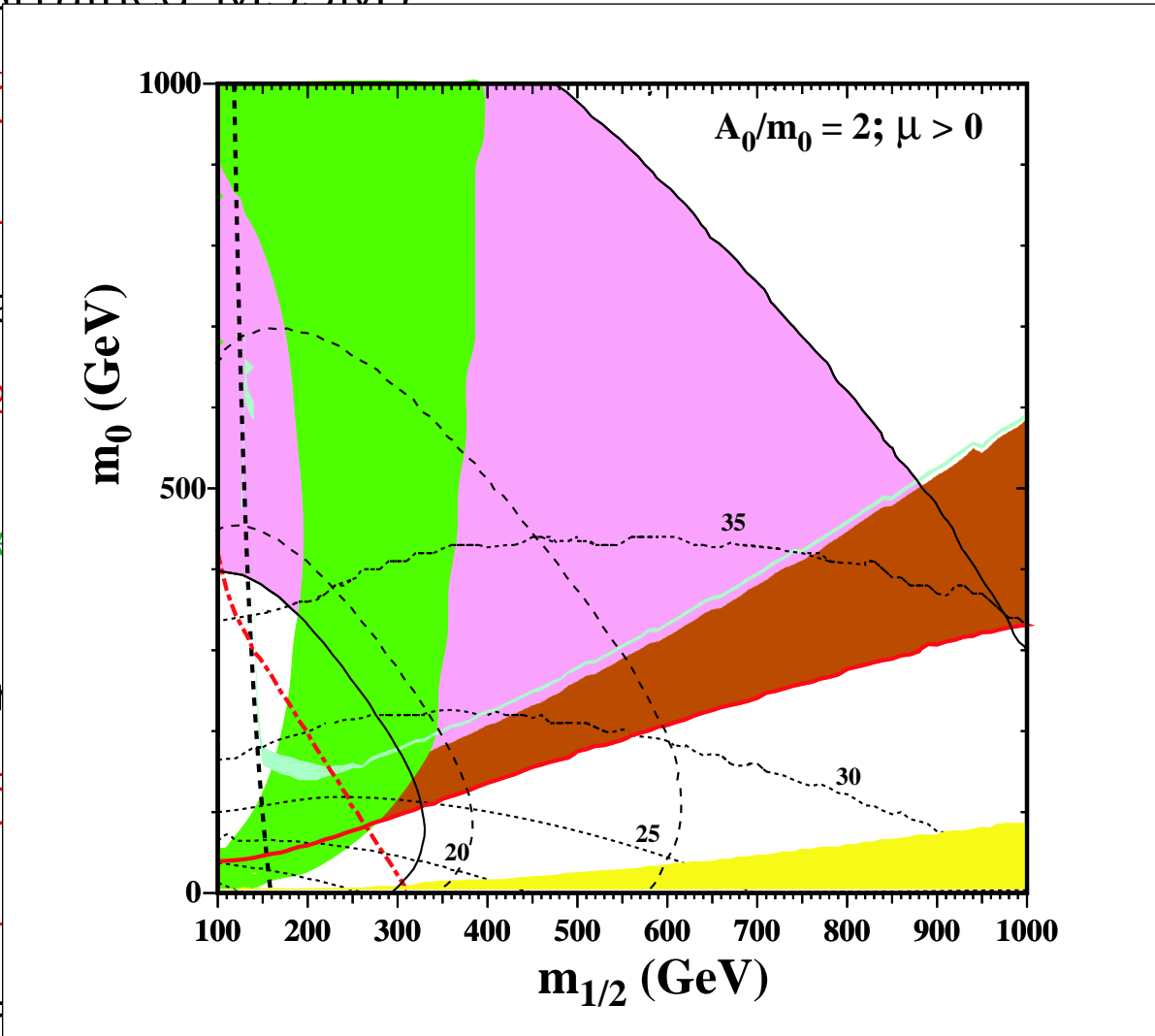
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Free parameters:  $m_{1/2}$ ,  $A_0/m_0$

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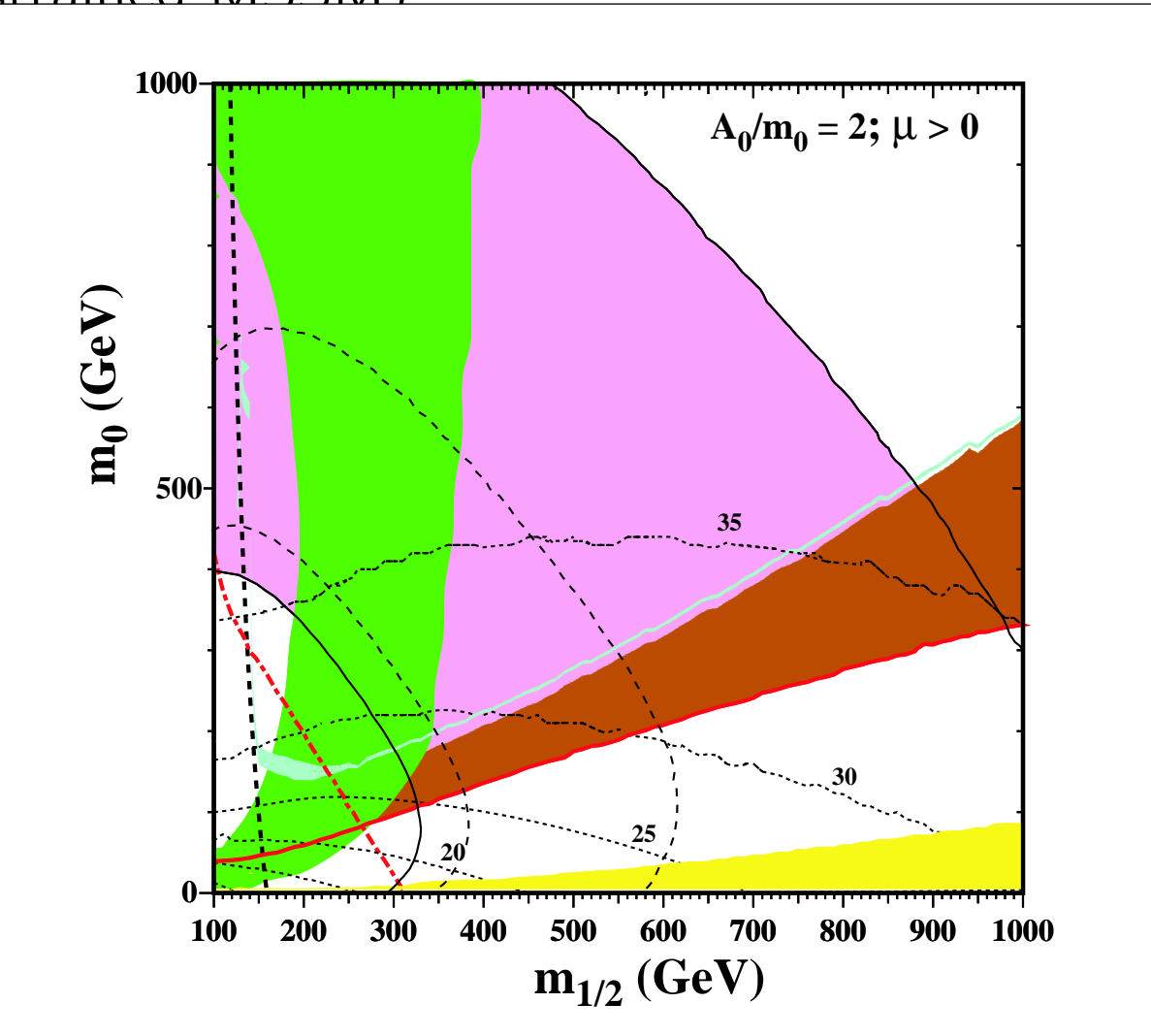
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Free parameters:  $m_{1/2}$ ,  $A_0/m_0$

Lightest SUSY particle (LSP) is the gravitino



## 2. Precision Observables in the MSSM

Precision observables:  $M_W$ ,  $\sin^2 \theta_{\text{eff}}$ ,  $m_h$ ,  $(g-2)_\mu$ ,  $b$  physics, ...

2 A) Theoretical prediction for  $M_W$  in terms

of  $M_Z$ ,  $\alpha$ ,  $G_\mu$ ,  $\Delta r$ :

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$

$\Updownarrow$

loop corrections

→ more details in talk by S.H. earlier today

2 B) Effective mixing angle:

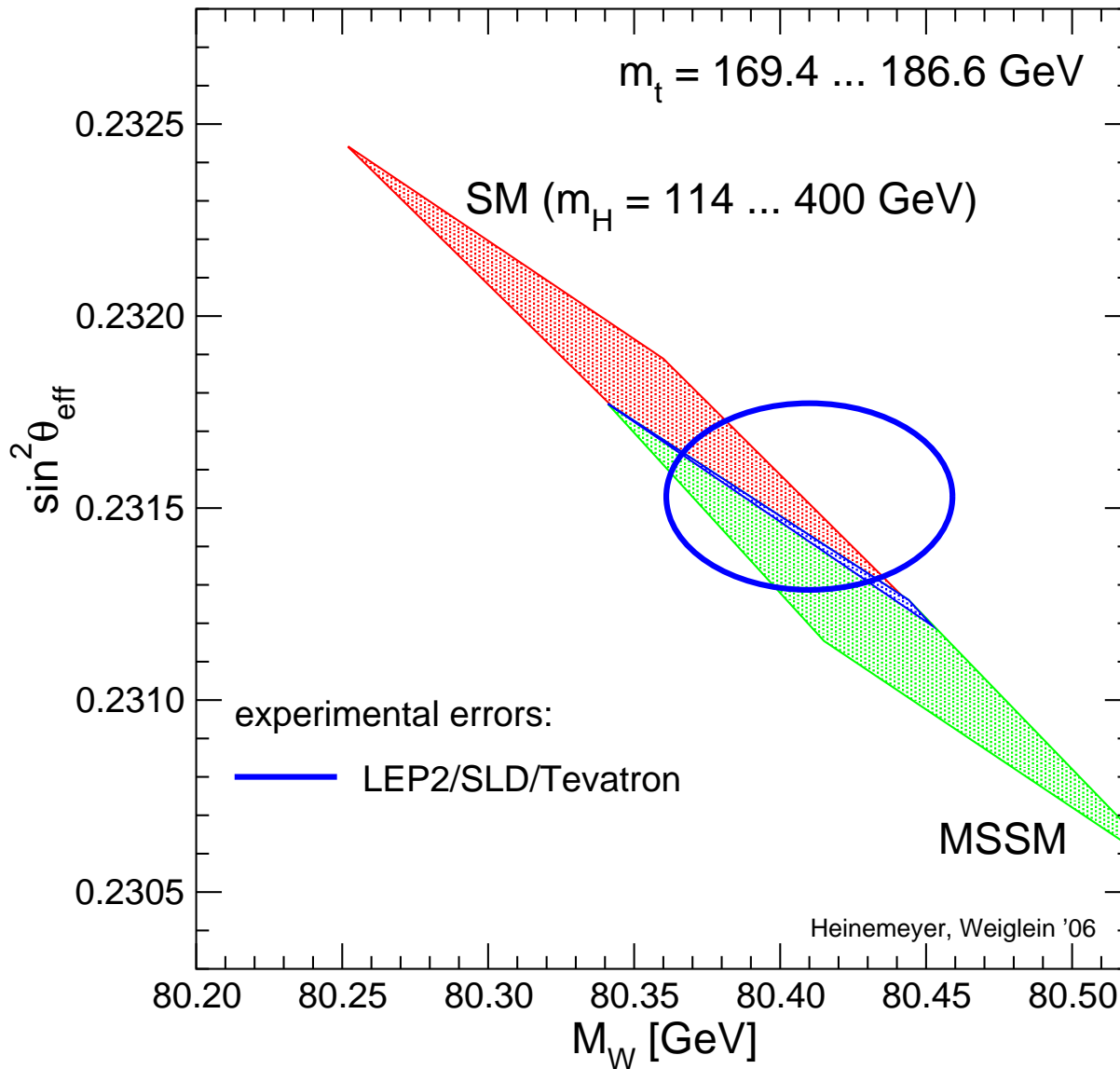
$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left( 1 - \text{Re} \frac{g_V^f}{g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

Example of application:

Prediction for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  in the SM and the MSSM :

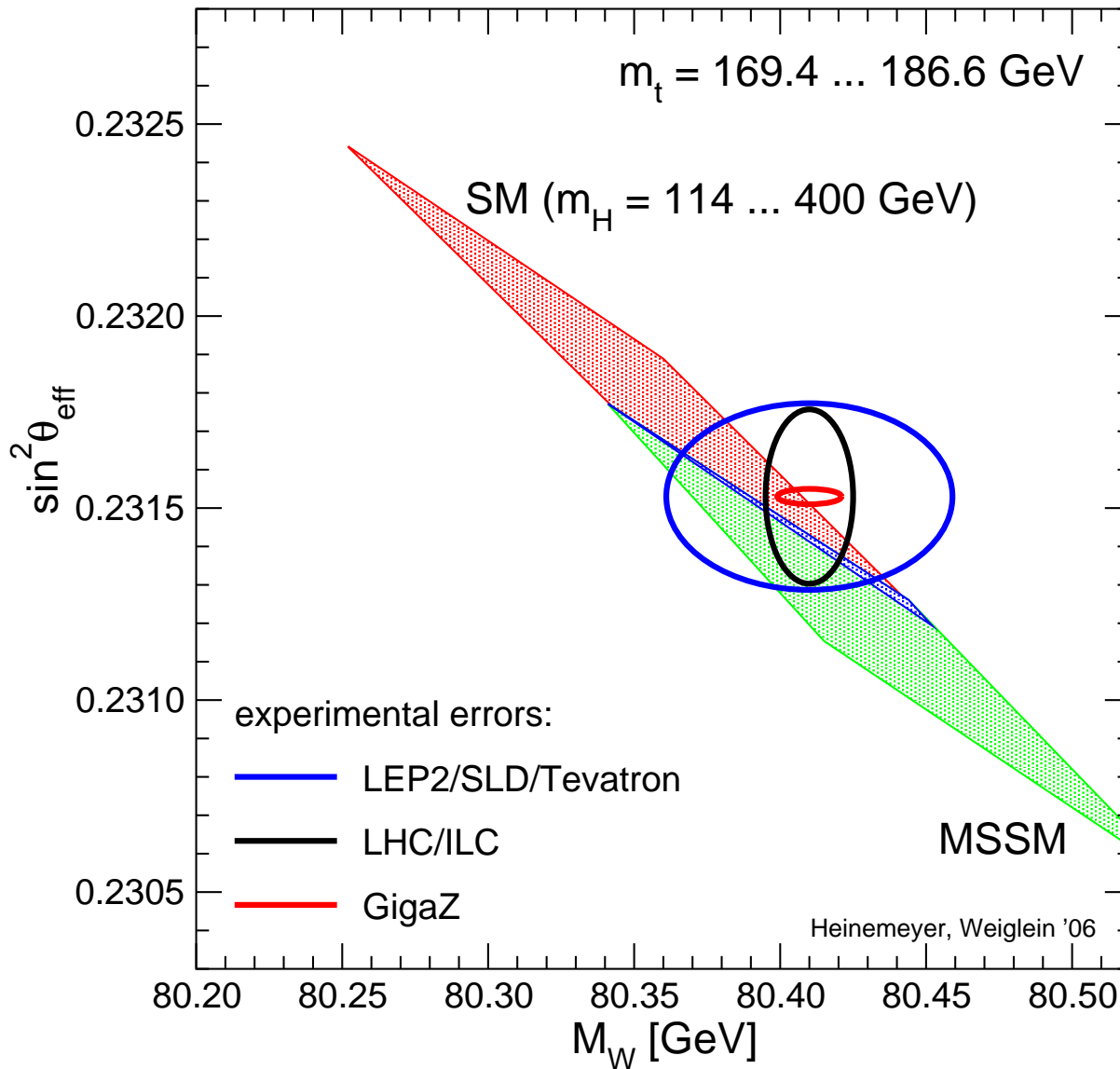


MSSM uncertainty:  
unknown masses  
of SUSY particles

SM uncertainty:  
unknown Higgs mass

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Prediction for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  in the SM and the MSSM :



MSSM uncertainty:  
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SM uncertainty:  
unknown Higgs mass



For  $\chi^2$  fit:

$$\chi_x^2 = \left( \frac{R_x^{\text{exp}} - R_x^{\text{theo}}}{\sigma_x} \right)^2 \quad x = M_W, \sin^2 \theta_{\text{eff}}$$

$R_x^{\text{exp}}$ : experimental value

$R_x^{\text{theo}}$ : theory prediction

$\sigma_x^2$ : (exp. error)<sup>2</sup> + (param. error)<sup>2</sup> + (intr. error)<sup>2</sup>

experimental error

parametric error: from uncertainty in input parameters

intrinsic error: from unknown higher-order corrections

⇒ use **most up to date** calculations and error estimates

[S.H., W. Hollik, G. Weiglein '04]

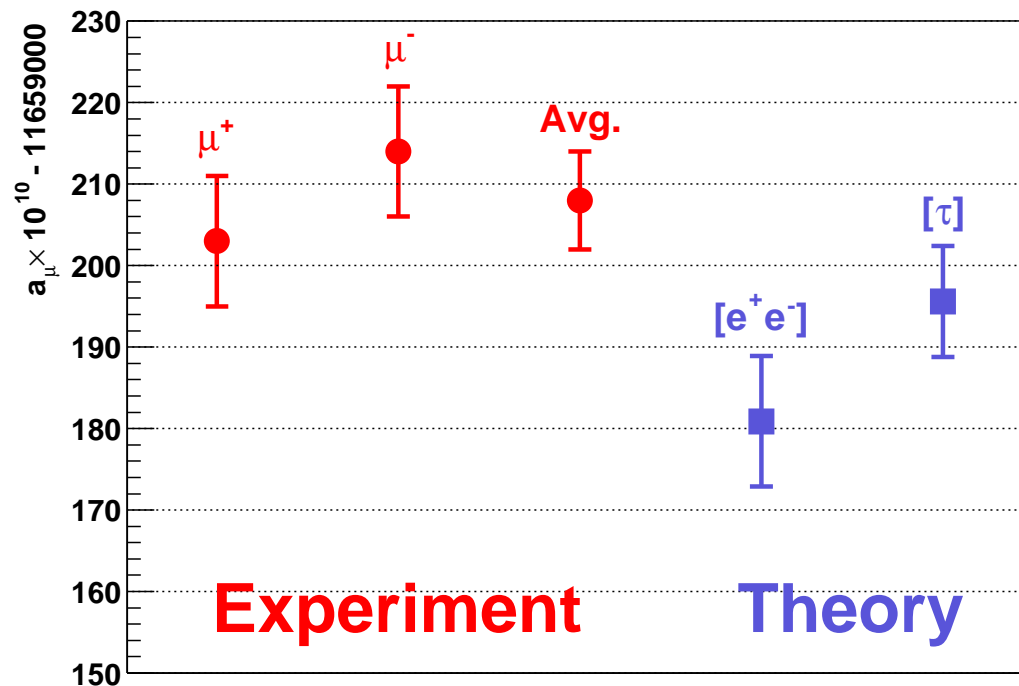
[J. Haestier, S.H., D. Stöckinger, G. Weiglein '05]

[LEPEWWG '05]

## 2 C) Prediction of the anomalous magnetic moment of the muon: $(g-2)_\mu$

Overview about the current **experimental** and **SM (theory)** result:

[*g-2 Collaboration, hep-ex/0401008*]



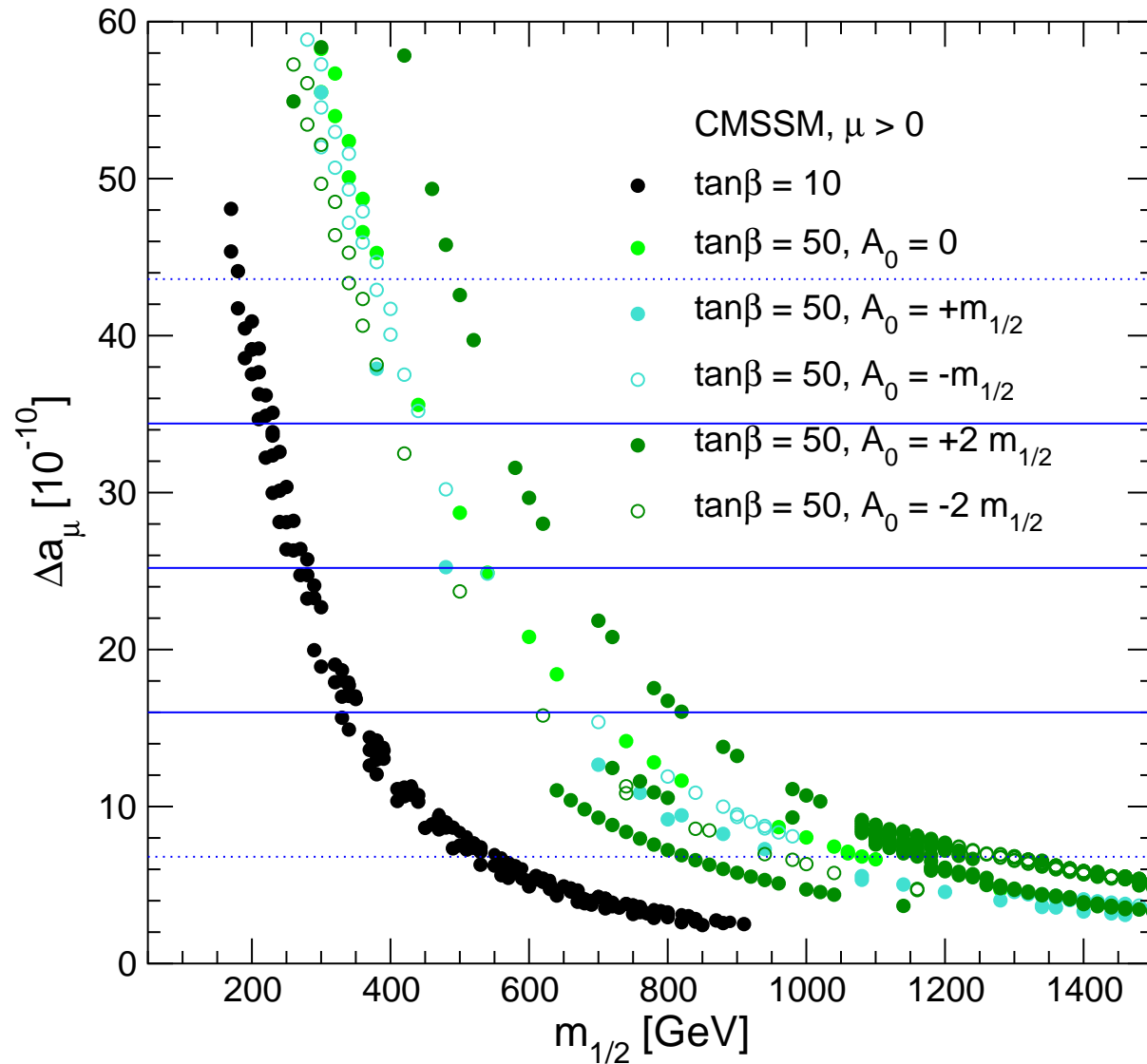
$e^+e^-$  data: no significant changes by new SND, CMD2, KLOE data

$\tau$  data: isospin breaking problem still unresolved

based on  $e^+e^-$  data:

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (25.2 \pm 9.2) \times 10^{-10}$$

## Example: Investigation of mSUGRA with cold dark matter constraint



Scan over  $m_{1/2}, m_0, A_0$   
 $\tan\beta = 10, 50$   
selected points give correct  
amount of cold dark matter

[Ellis, S.H., Olive, Weiglein '04]

Severe bounds on e.g.  $m_{1/2}$

For  $\chi^2$  fit:

$$\chi_x^2 = \left( \frac{R_x^{\text{exp}} - R_x^{\text{theo}}}{\sigma_x} \right)^2 \quad x = (g - 2)_\mu$$

$R_x^{\text{exp}}$ : experimental value =  $(a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}})$

$R_x^{\text{theo}}$ : theory prediction =  $a_\mu^{\text{theo,SUSY}}$

$\sigma_x^2$ : (exp. error)<sup>2</sup> + (param. error)<sup>2</sup> + (intr. error)<sup>2</sup>

experimental error

parametric error: from uncertainty in input parameters

intrinsic error: from unknown higher-order corrections

⇒ use **most up to date** calculations and error estimates

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[S.H., D. Stöckinger, G. Weiglein '03, '04]

[g-2 Collaboration, hep-ex/0401008]

## 2 D) Prediction of the decay $b \rightarrow s\gamma$

$$\chi_x^2 = \left( \frac{R_x^{\text{exp}} - R_x^{\text{theo}}}{\sigma_x} \right)^2 \quad x = \text{BR}(b \rightarrow s\gamma)$$

$R_x^{\text{exp}}$ : experimental value

$R_x^{\text{theo}}$ : theory prediction

$\sigma_x^2$ : (exp. error)<sup>2</sup> + (param. error)<sup>2</sup> + (intr. error)<sup>2</sup>

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[Asatrian, Hovhannisyanyan, Greub, Hurth, Pogosyan '05]

[BaBar, Belle '02, '04]

[HFAG '06]

2 E) Theoretical prediction of the lightest MSSM Higgs boson mass:  $M_h$

Contrary to the SM:  $M_h$  is not a free parameter

MSSM tree-level bound:  $M_h < M_Z$ , excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta m_h^2 \sim G_\mu m_t^4 \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector) f

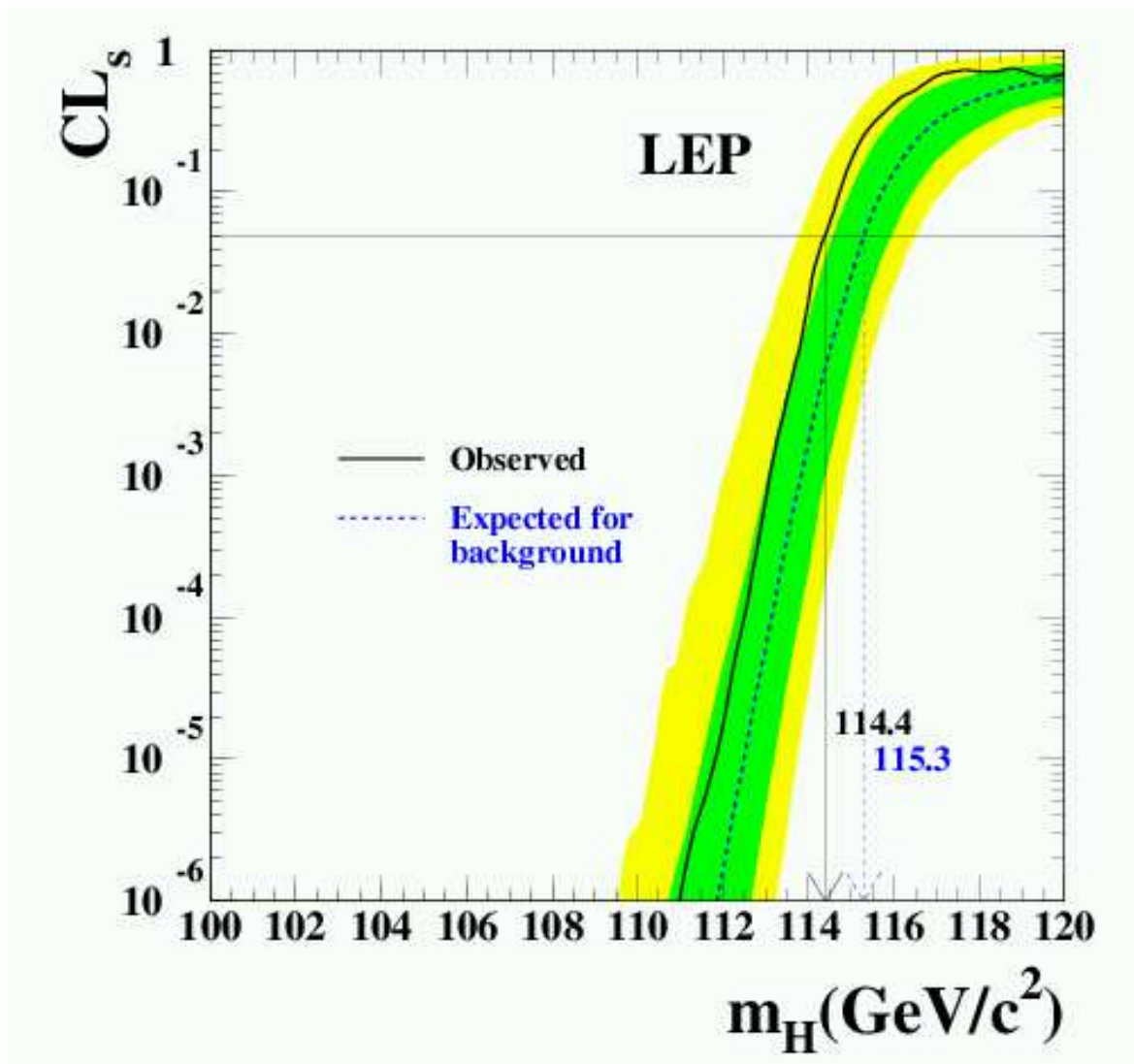
Measurement of  $M_h$ , Higgs couplings  $\Rightarrow$  test of the theory

LHC:  $\Delta M_h \approx 0.2$  GeV

ILC:  $\Delta M_h \approx 0.05$  GeV

$\Rightarrow M_h$  will be (the best?) electroweak precision observable

In CMSSM, NUHM, VCMSSM, GDM: SM bound of  $M_H$  search can be used  
[LEP Higgs Working Group '03]



$CL_s$  can be  
used/transformed  
into  $\chi^2$  values

$\Rightarrow$  additional (unobserved)  
parameter

$$\delta M_h^{\text{intr.}} \approx 3 \text{ GeV}$$

We use *FeynHiggs*  
([www.feynhiggs.de](http://www.feynhiggs.de))

### 3. Fits and ILC reach

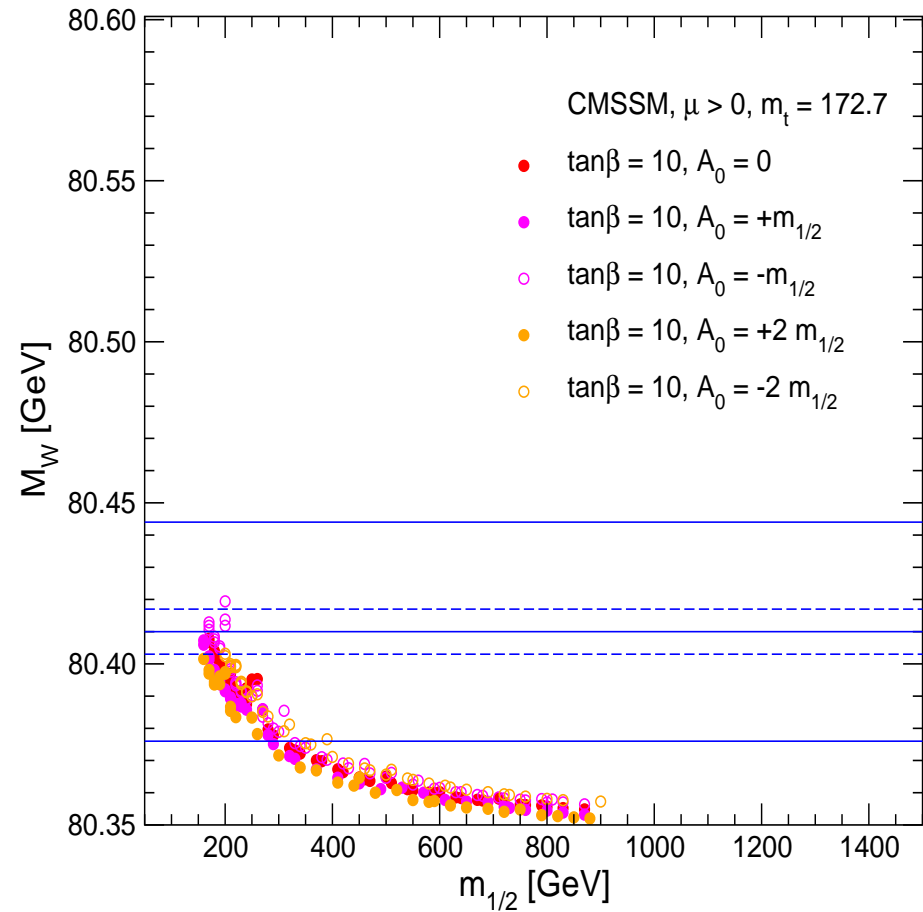
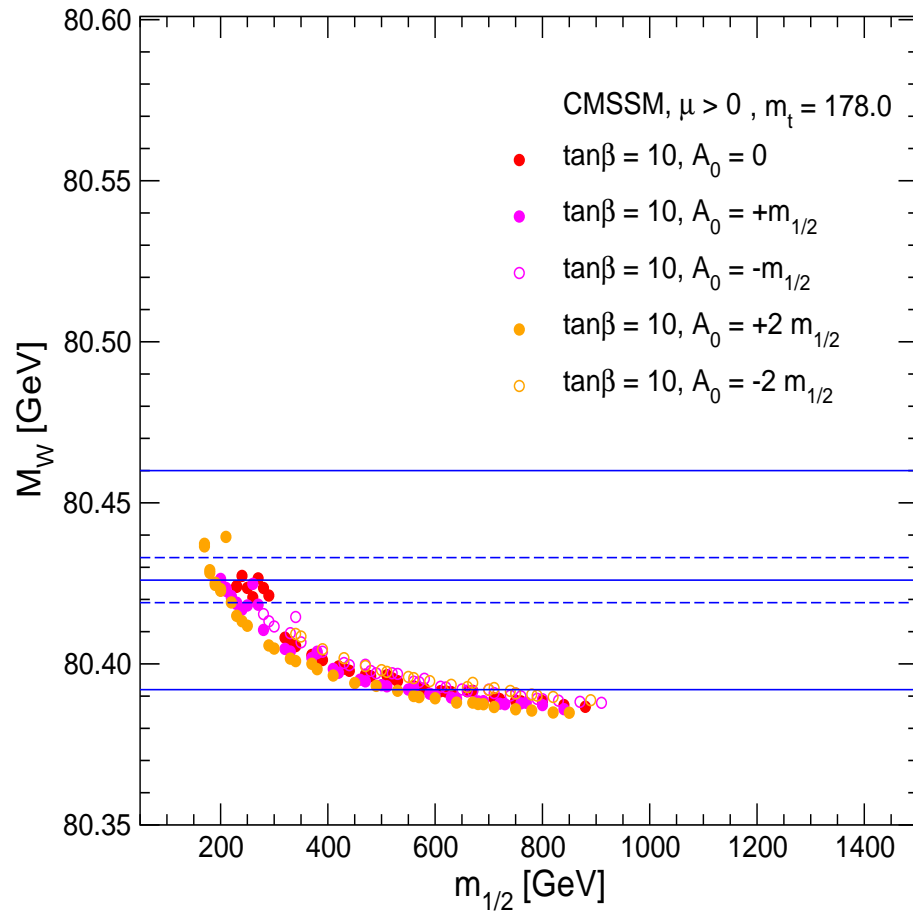
#### Procedure:

1. **Scan** over parameter space:
  - **CMSSM**: for fixed  $\tan\beta = 10, 50$
  - **NUHM**: certain parameter planes,  
corresponding to CMSSM best fit points
  - **VCMSM**: full parameter space ( $A_0/m_0 = 0, 3/4, 3 - \sqrt{3}, 2$ )
  - **GDM (mSUGRA)**: full parameter space ( $A_0/m_0 = 0, 3/4, 3 - \sqrt{3}, 2$ )
2. Perform  $\chi^2$  fit
3. Find **preferred** values for **masses**  
 $\Rightarrow$  ILC reach



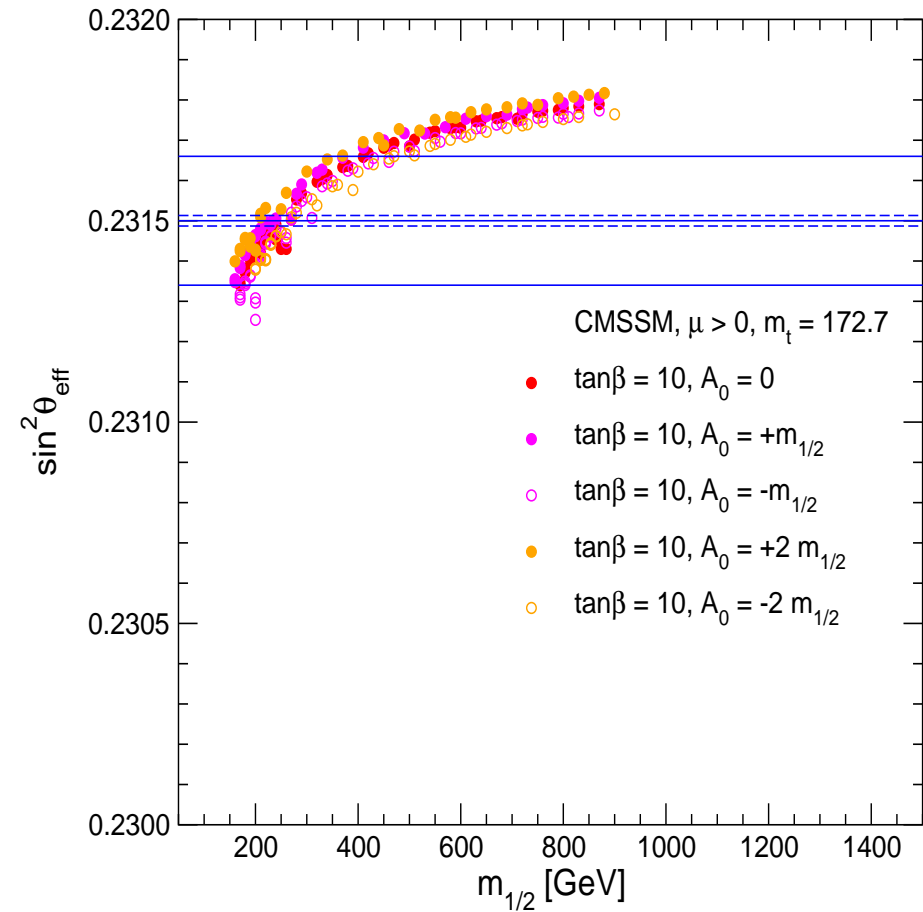
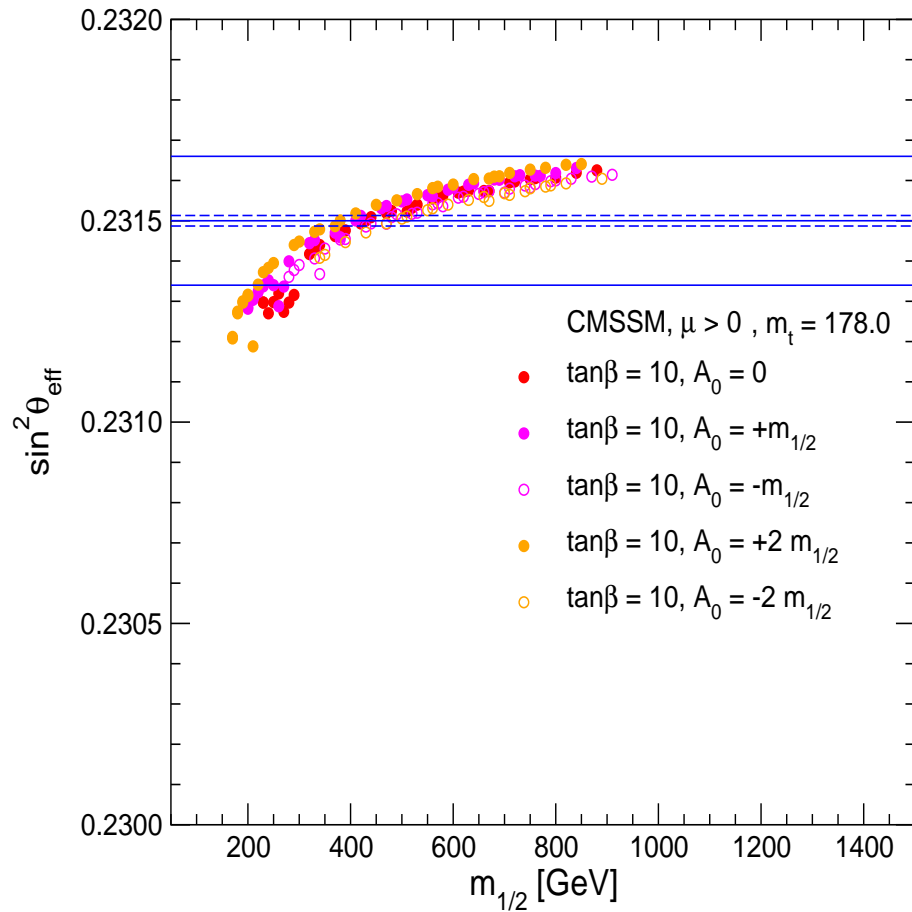
### 3 A) CMSSM:

Compare  $m_t = 178$  GeV with  $m_t = 172.7$  GeV:  $M_W$  for  $\tan\beta = 10$ :



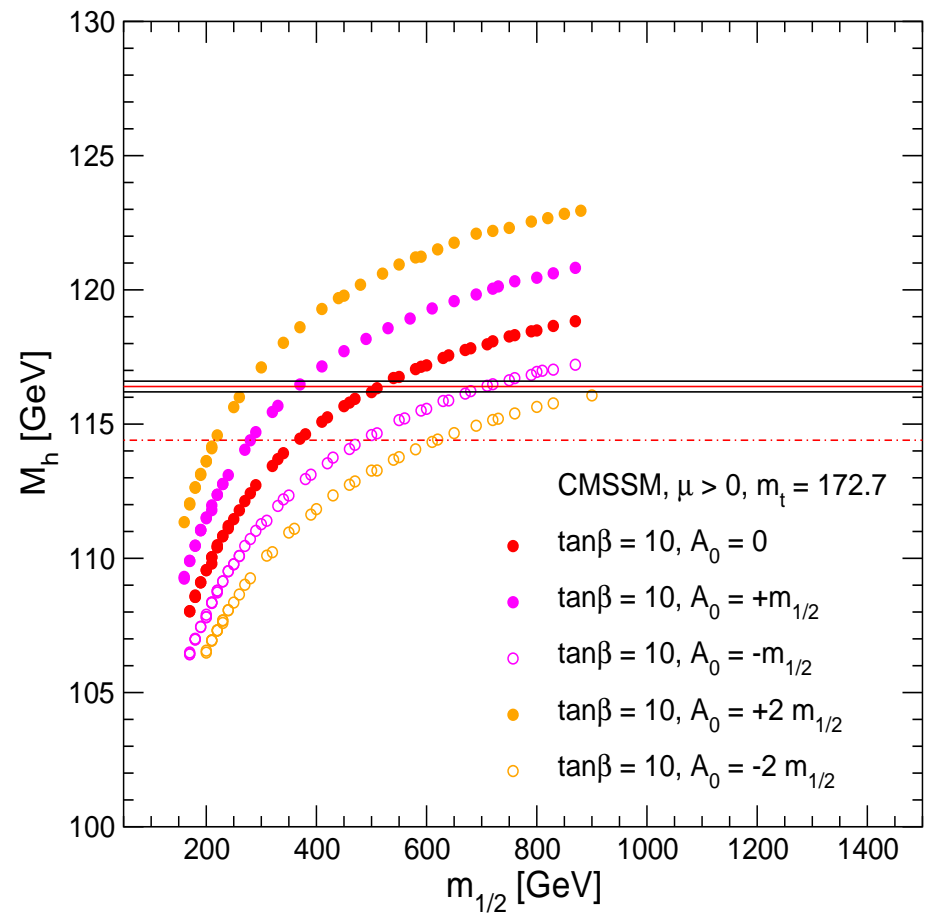
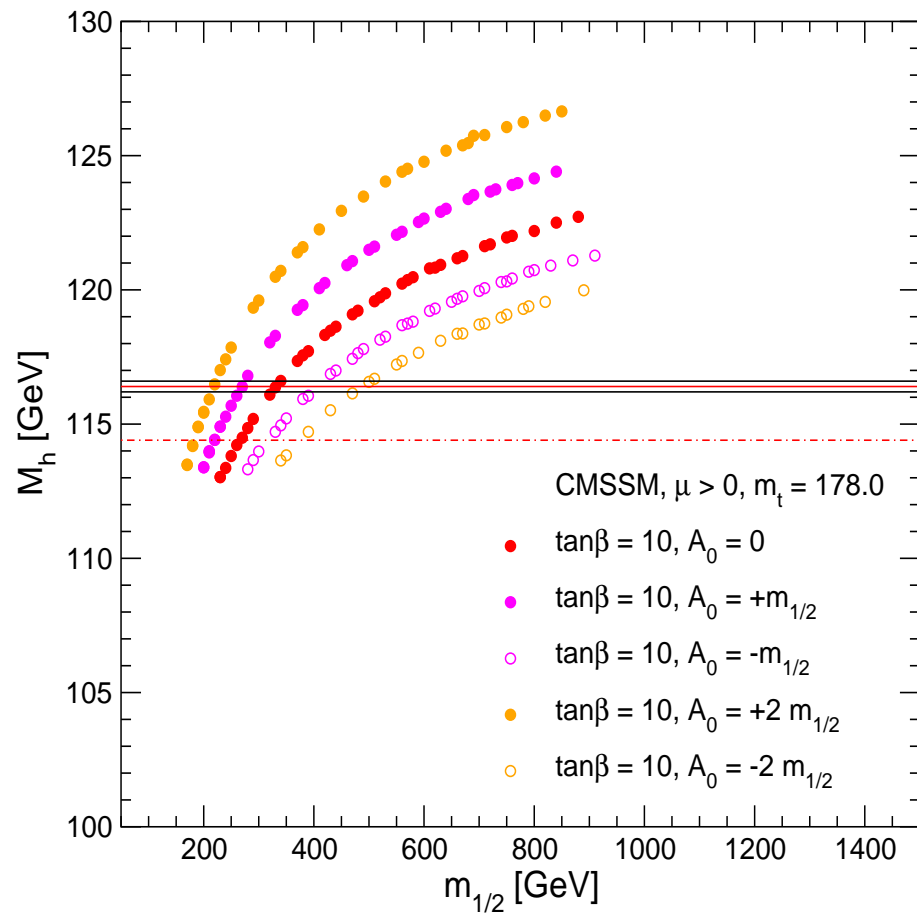
$m_t = 178 \rightarrow 172.7 \Rightarrow m_{1/2}$  lowered

Compare  $m_t = 178$  GeV with  $m_t = 172.7$  GeV:  $\sin^2 \theta_{\text{eff}}$  for  $\tan \beta = 10$ :



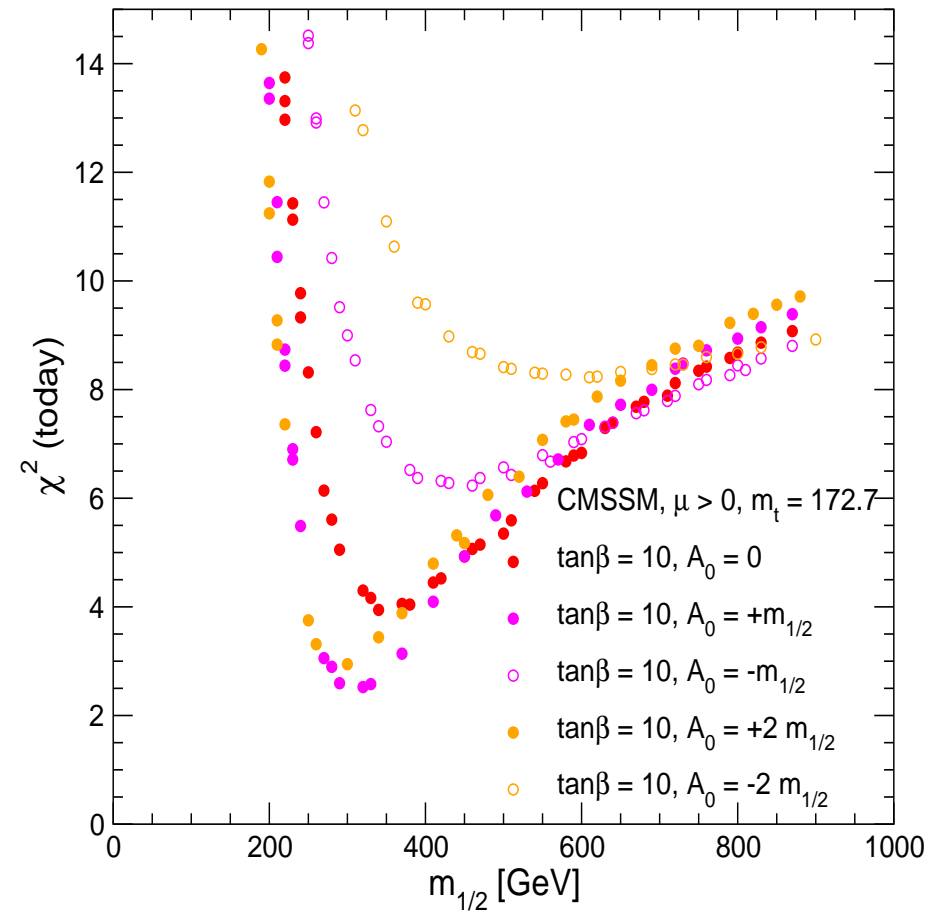
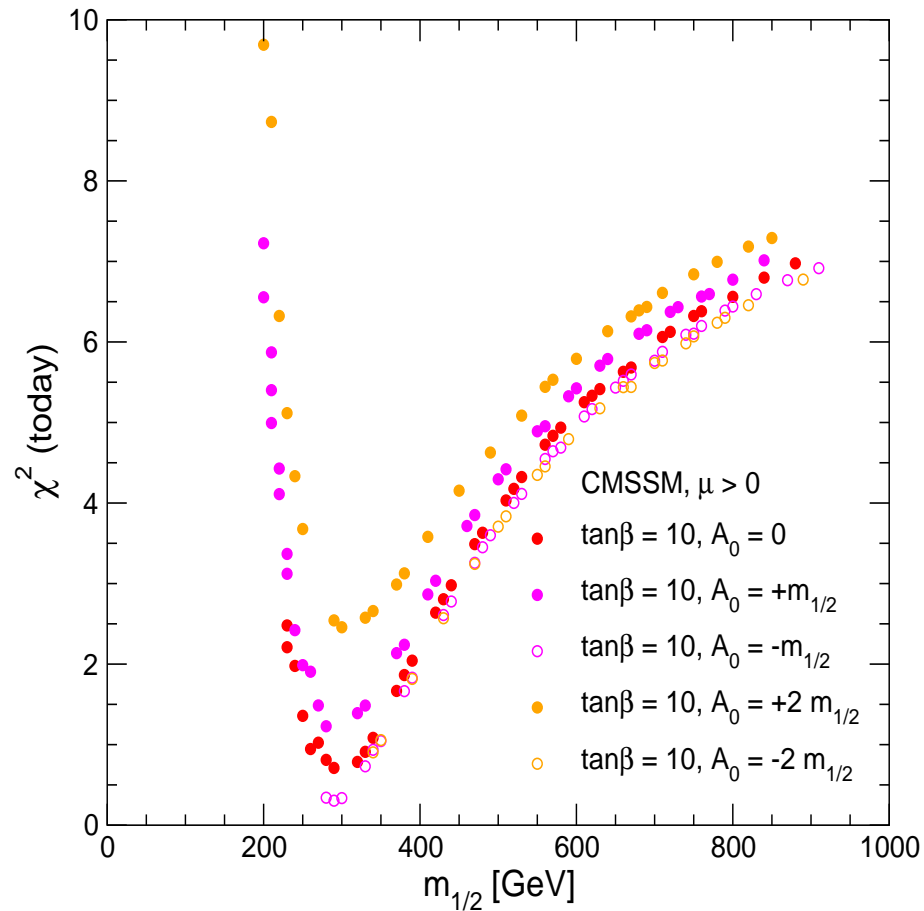
$m_t = 178 \rightarrow 172.7 \Rightarrow m_{1/2}$  lowered

Compare  $m_t = 178$  GeV with  $m_t = 172.7$  GeV:  $M_h$  for  $\tan\beta = 10$ :



$m_t = 178 \rightarrow 172.7 \Rightarrow m_{1/2}$  increased

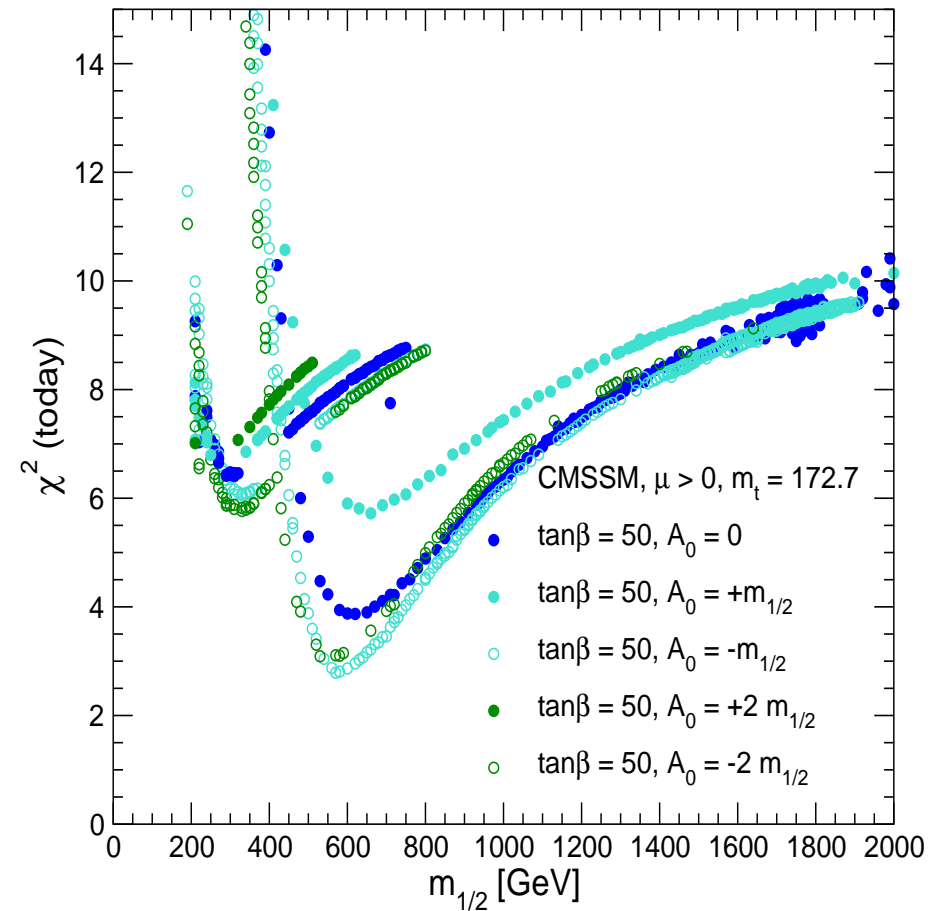
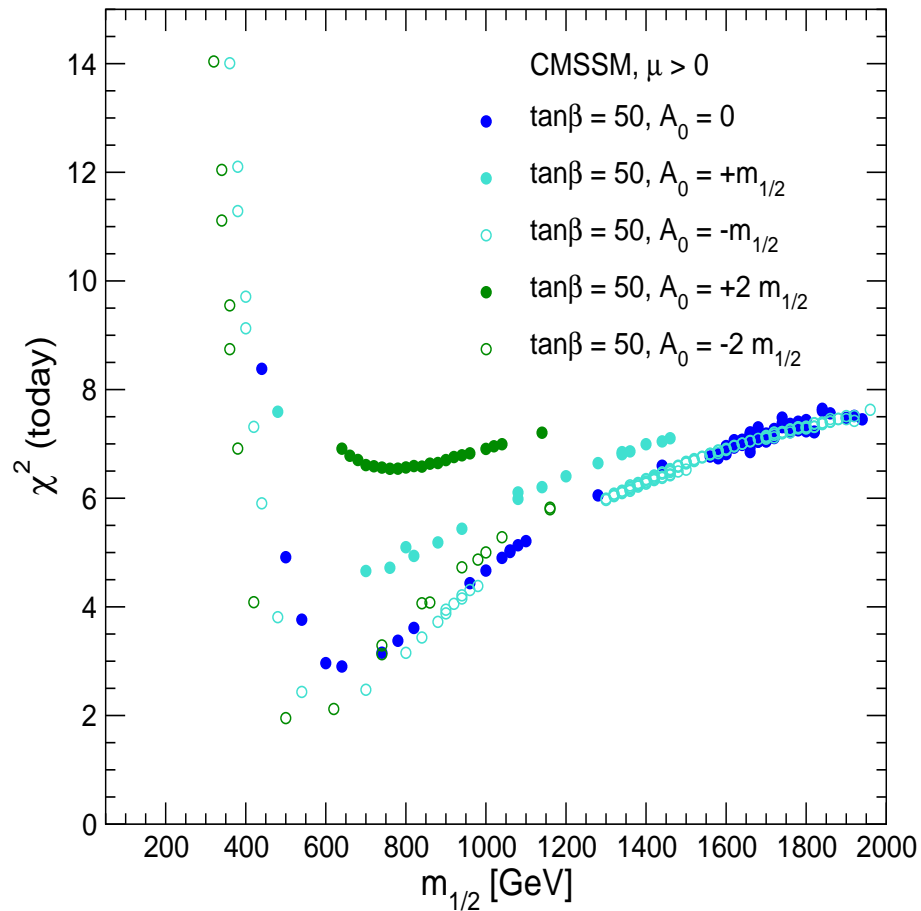
Effect of  $m_t = 178$  GeV with  $m_t = 172.7$  GeV:  $\chi^2$  fit for  $\tan\beta = 10$ :



$m_t = 178 \rightarrow 172.7$  slightly higher  $\chi^2$ ,  $A_0 > 0$  favored

$\Rightarrow M_W$  and  $\sin^2 \theta_{\text{eff}}$  more important,  $(g - 2)_\mu$  less important for low  $m_{1/2}$

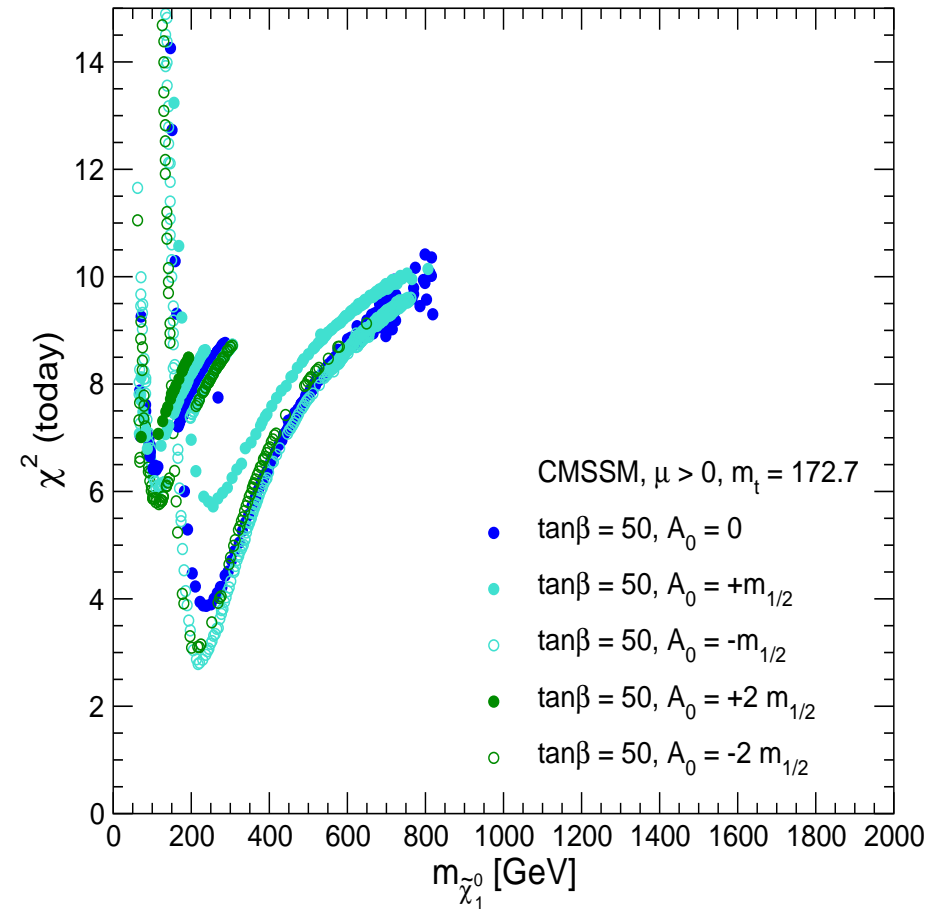
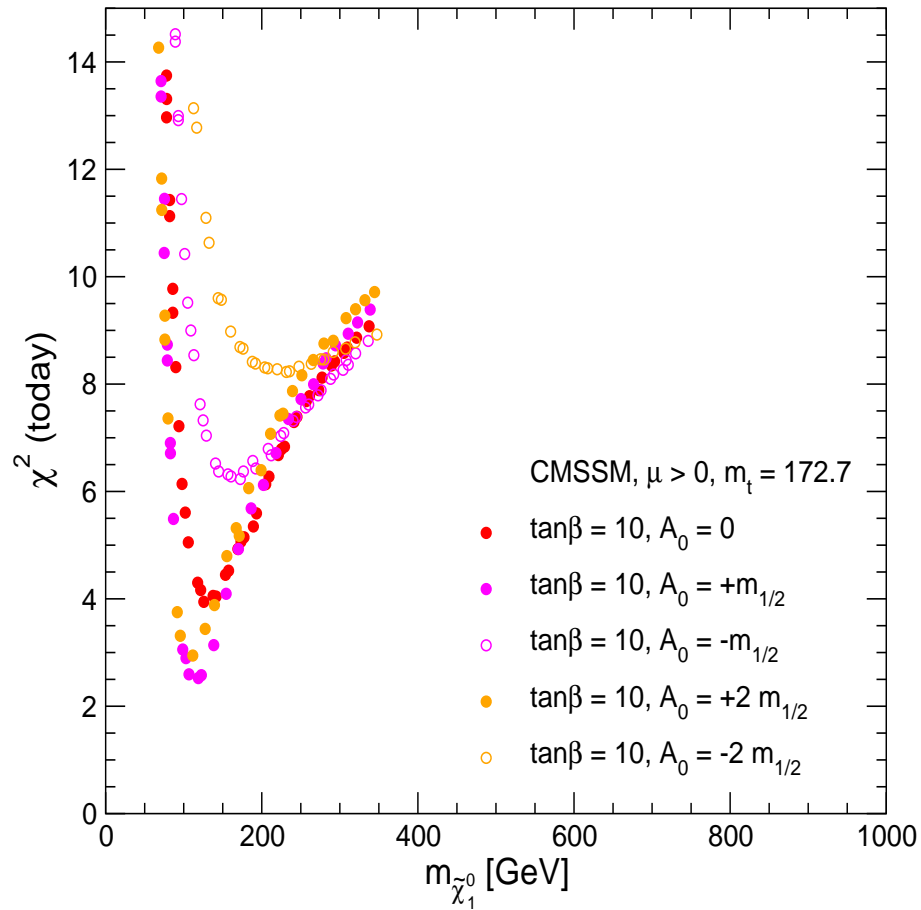
Effect of  $m_t = 178$  GeV with  $m_t = 172.7$  GeV:  $\chi^2$  fit for  $\tan\beta = 50$ :



$m_t = 178 \rightarrow 172.7$  effective change minor

$\Rightarrow$  (re)appearance of focus point region at  $m_{1/2} \approx 200$  GeV

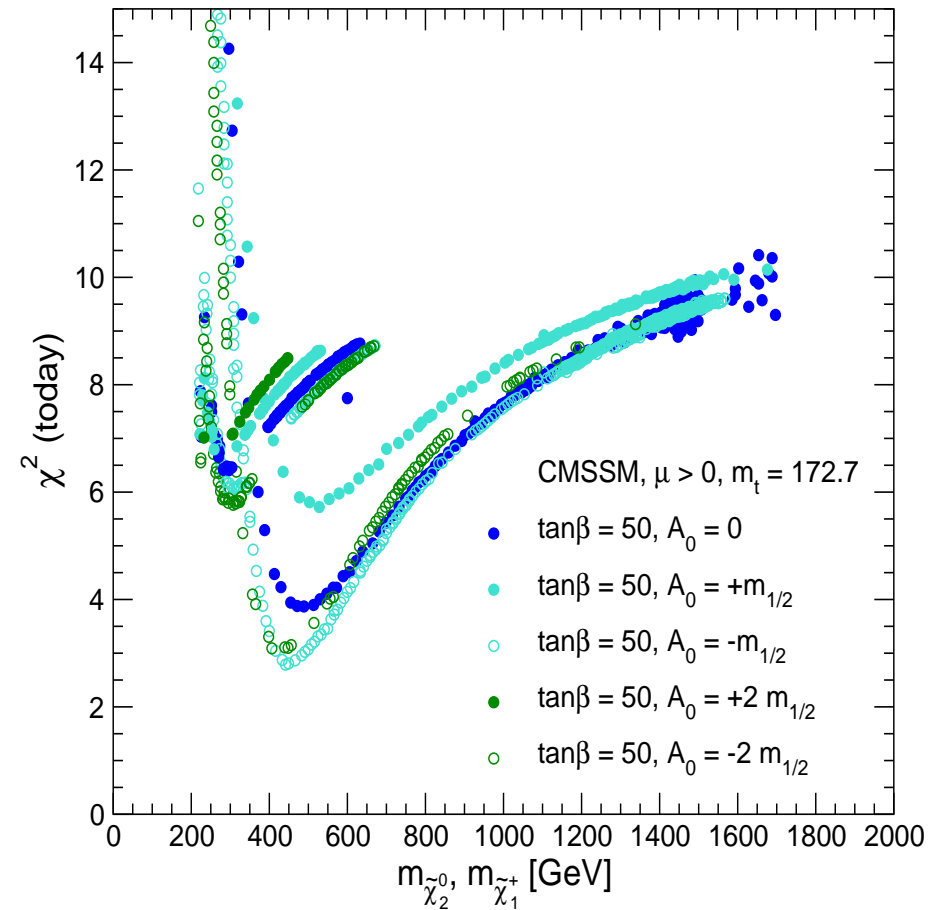
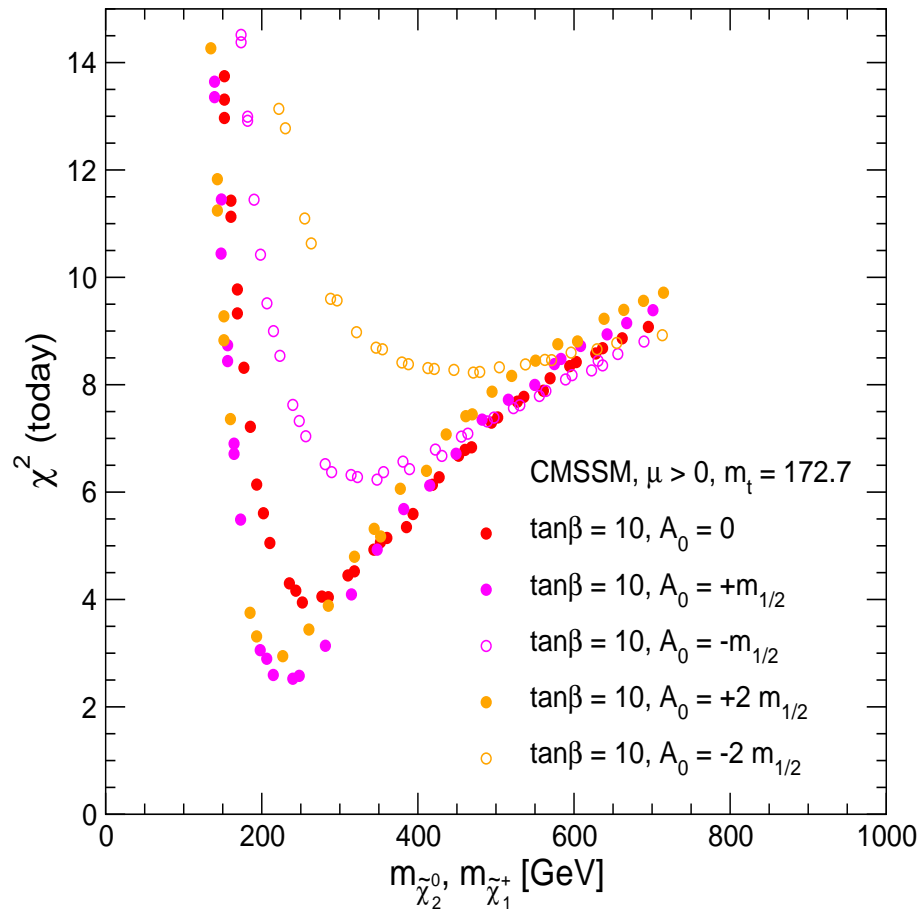
# CMSSM: LSP mass for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$  minimum at 200 GeV

$\tan\beta = 50 \Rightarrow$  similar

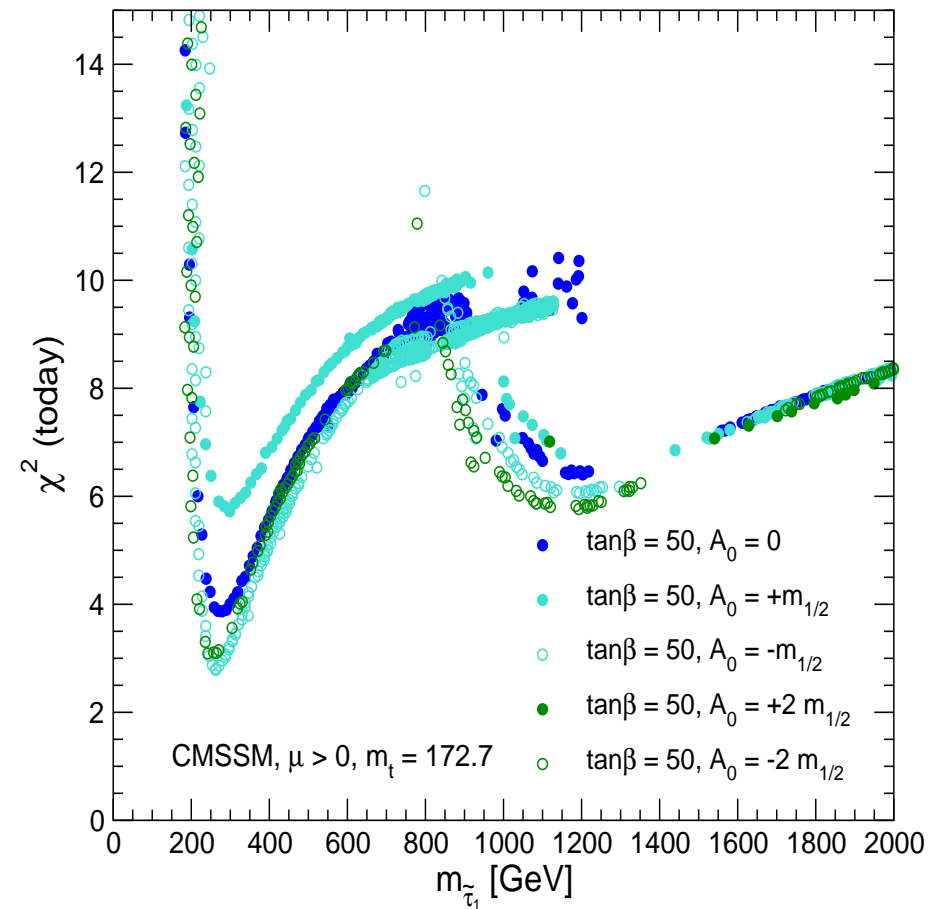
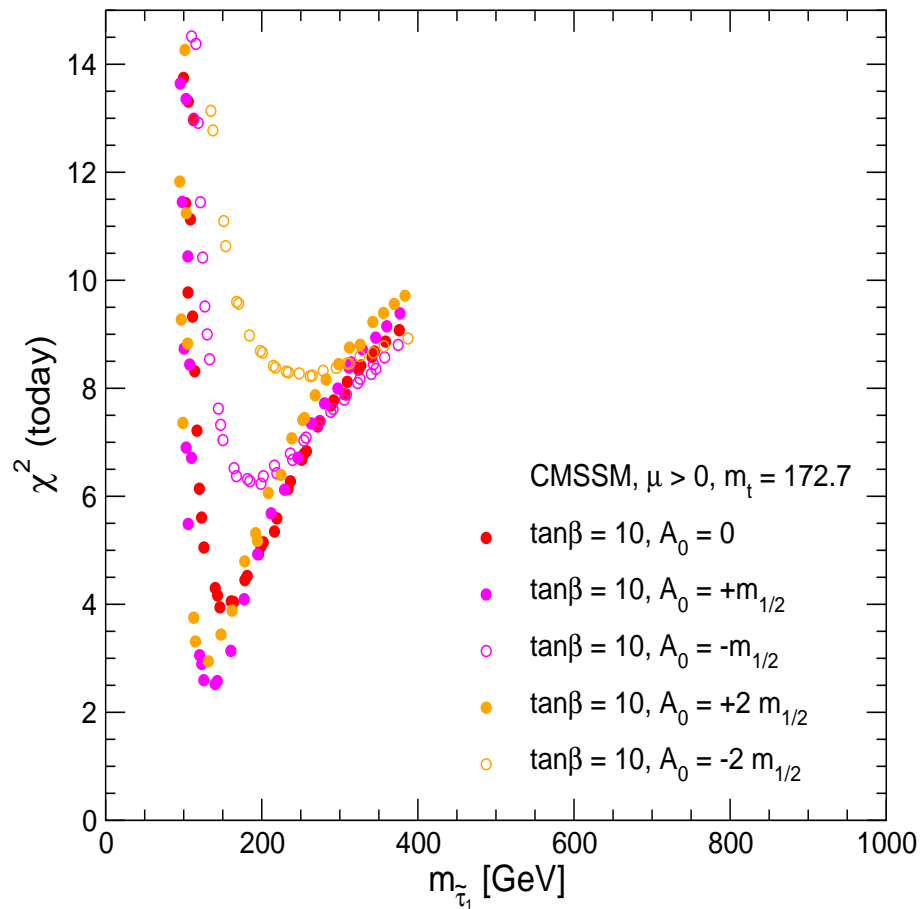
# CMSSM: light neutralino/chargino masses for $\tan\beta = 10, 50$



$\tan\beta = 10 \Rightarrow$  very good prospects for the ILC(1000)

$\tan\beta = 50 \Rightarrow$  good prospects (only) for  $\tilde{\chi}_1^0 \tilde{\chi}_2^0$  production

# CMSSM: lightest stau mass for $\tan\beta = 10, 50$

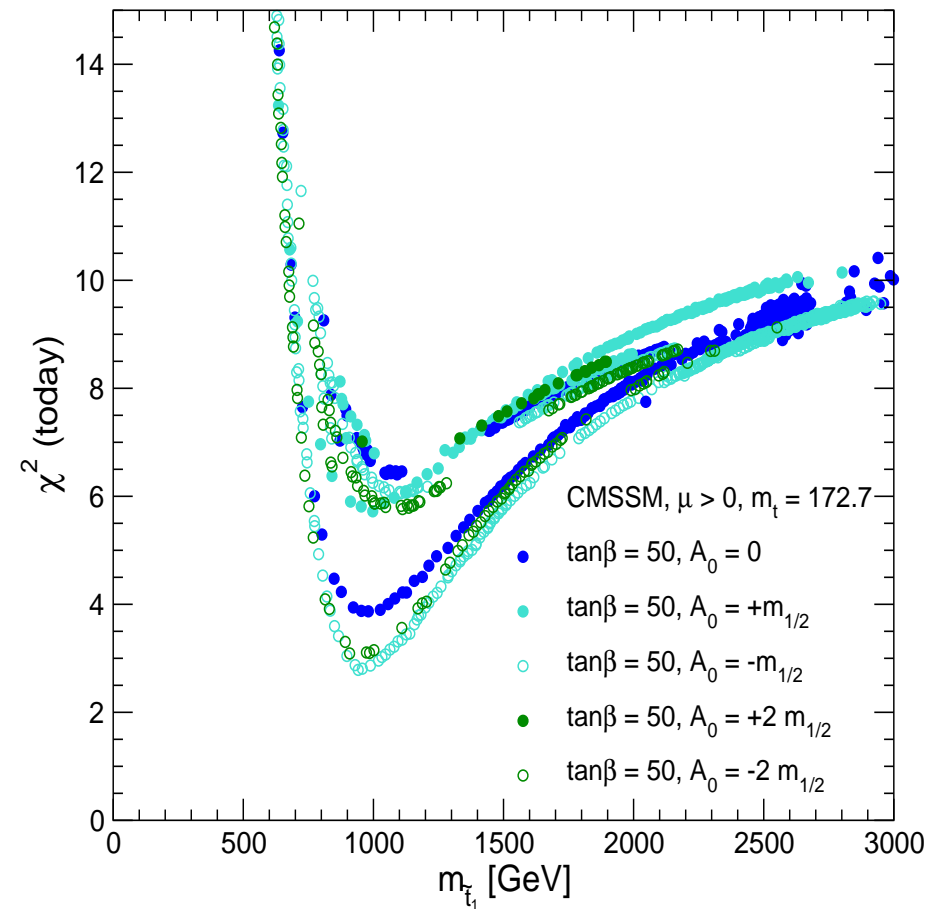
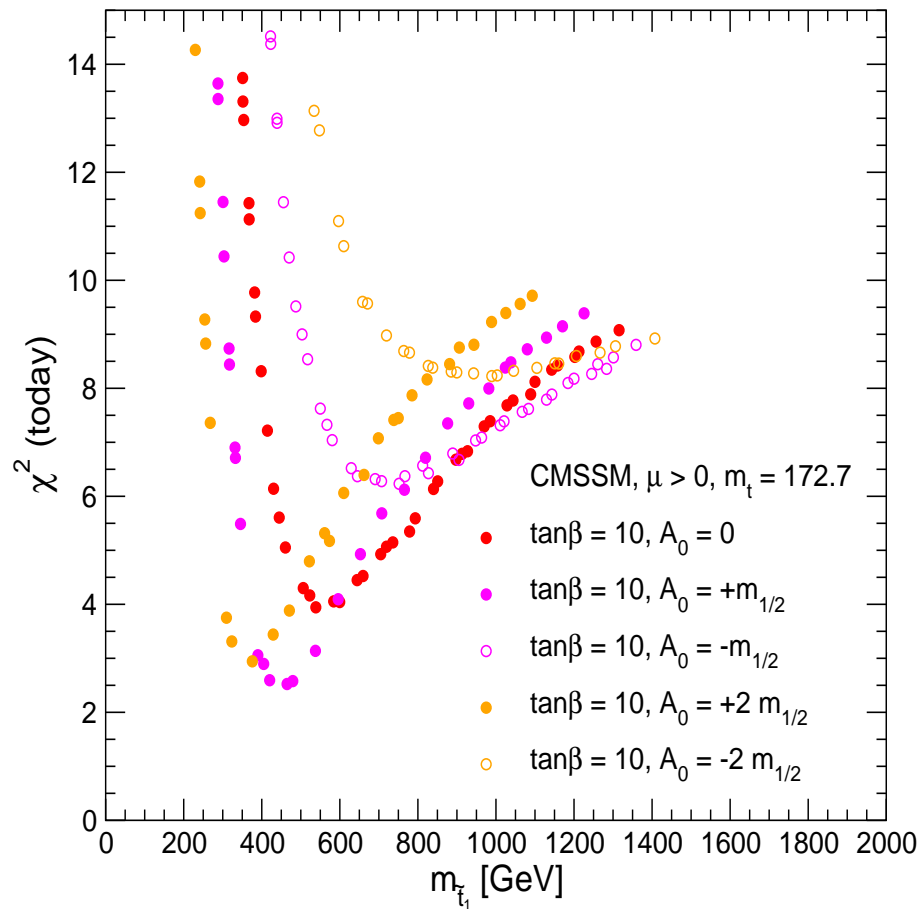


$\tan\beta = 10 \Rightarrow$  very good prospects for the ILC

$\tan\beta = 50 \Rightarrow$  still quite good for the ILC



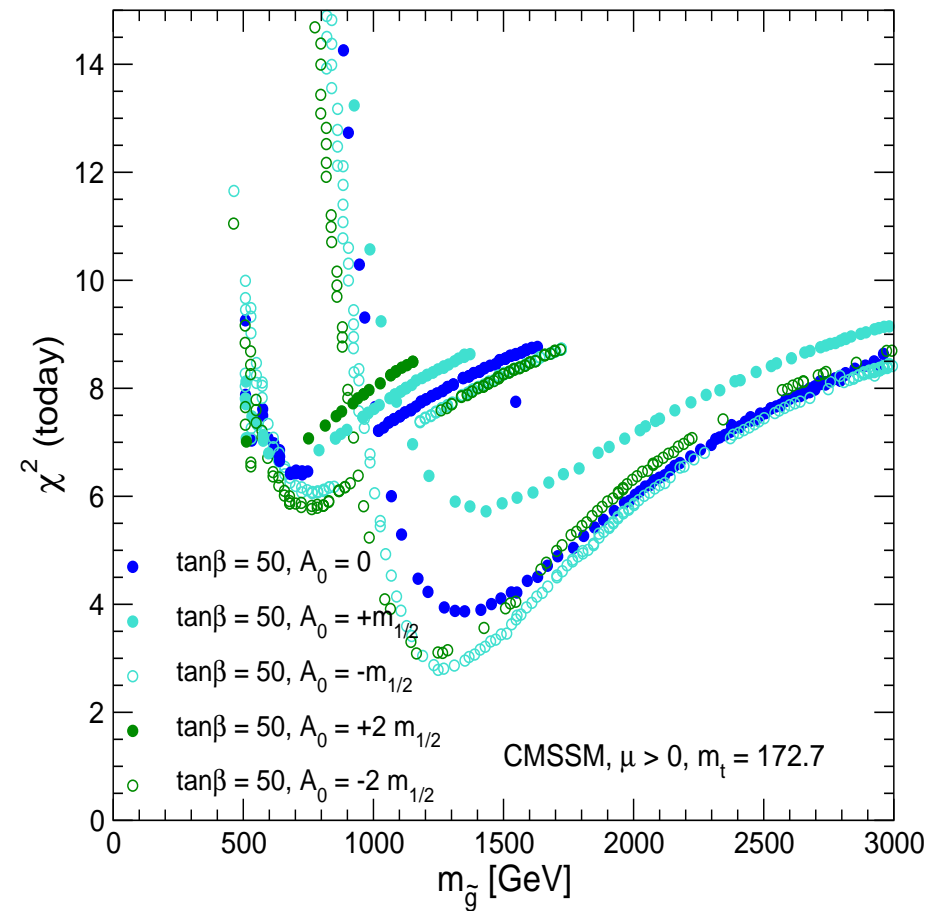
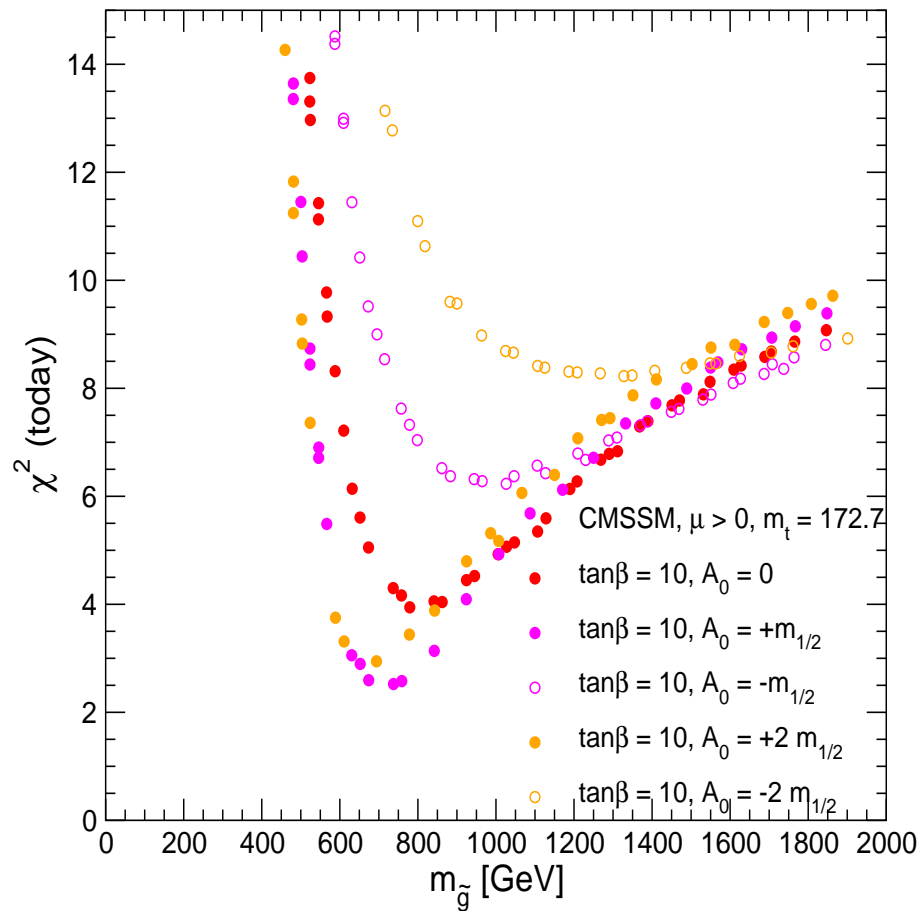
# CMSSM: lightest stop mass for $\tan \beta = 10, 50$



$\tan \beta = 10 \Rightarrow$  moderate prospects for the ILC

$\tan \beta = 50 \Rightarrow$  outside the ILC(1000) reach

# CMSSM: gluino mass for $\tan\beta = 10, 50$

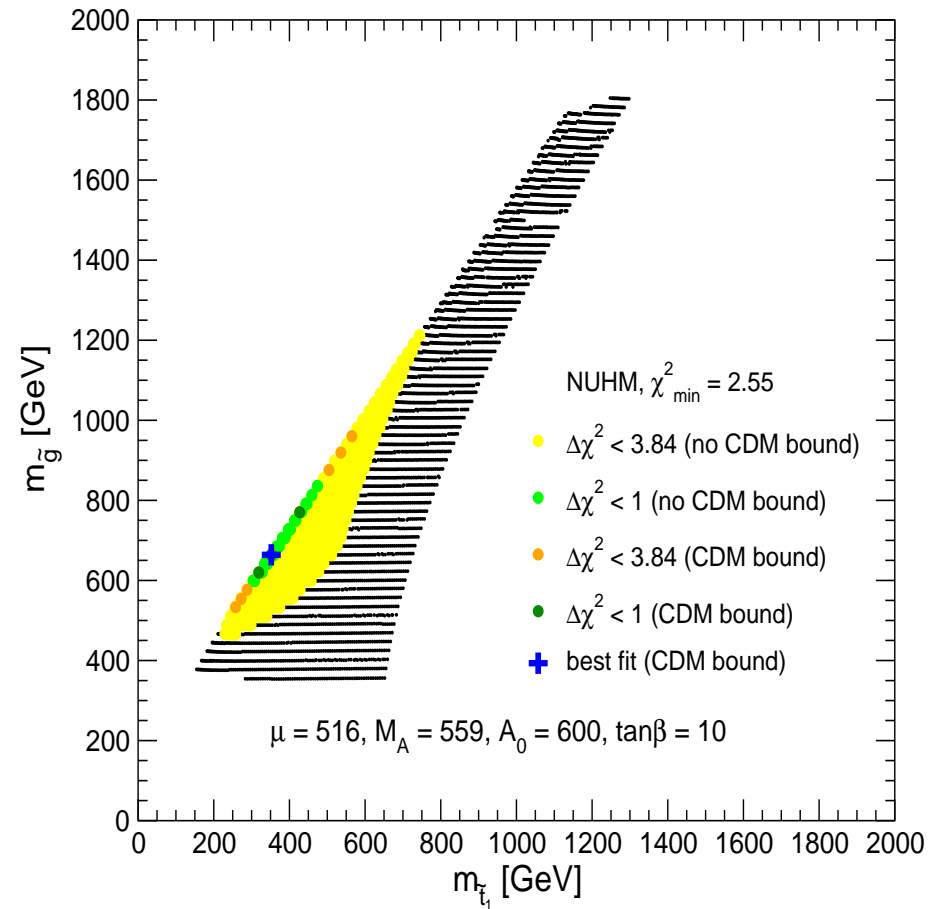
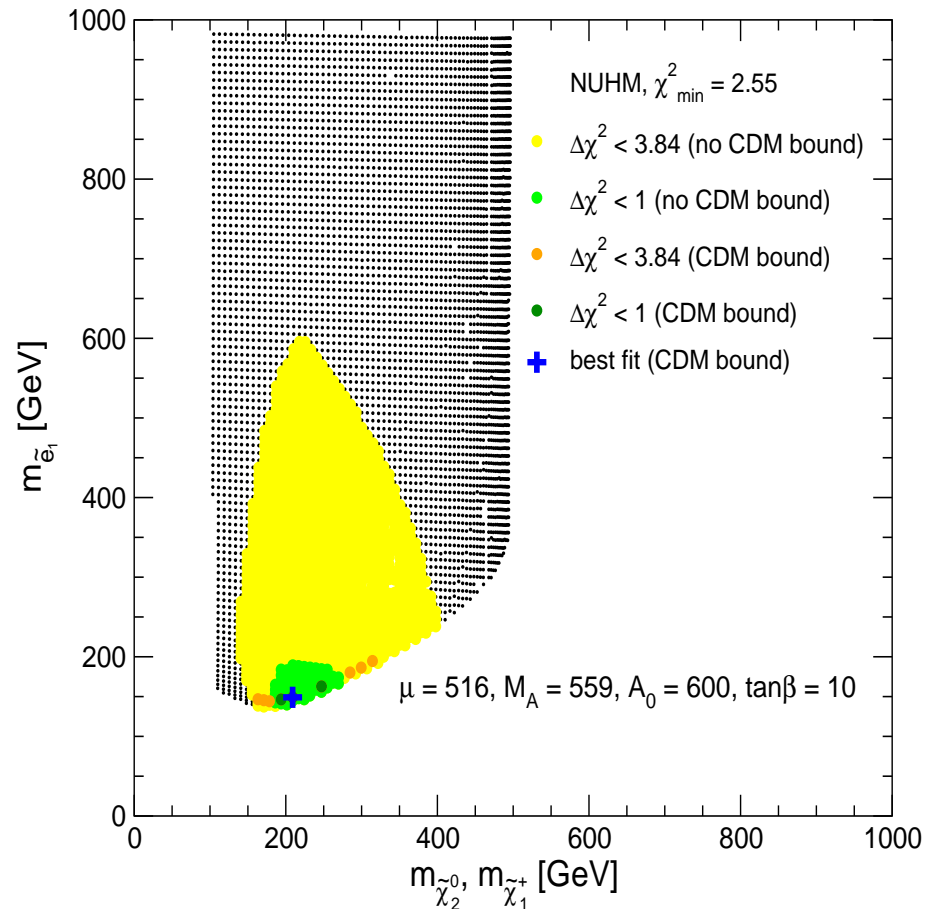


$\tan\beta = 10 \Rightarrow$  outside the ILC(1000) reach

$\tan\beta = 50 \Rightarrow$  chance for focus point region?

### 3 B) NUHM

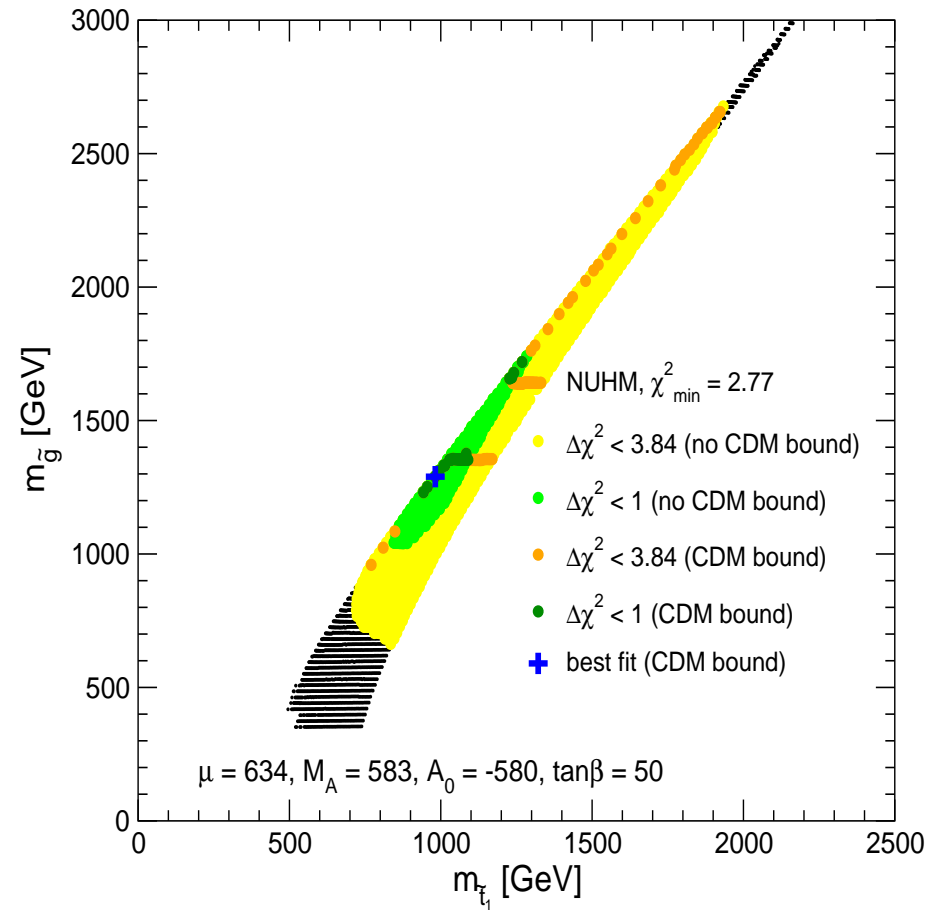
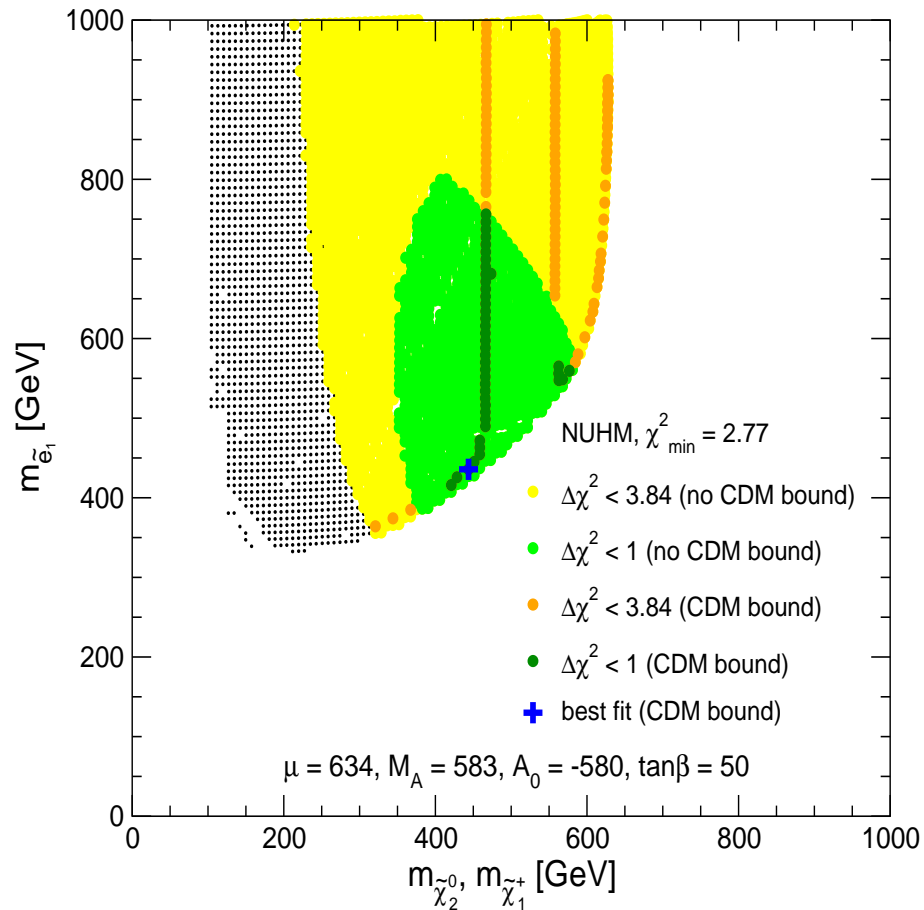
NUHM: vary  $m_{1/2}$  and  $m_0$  around best CMSSM fit point  $\tan\beta = 10$ :



⇒ sleptons, charginos, neutralinos in reach

stops could be, gluinos will be out of reach

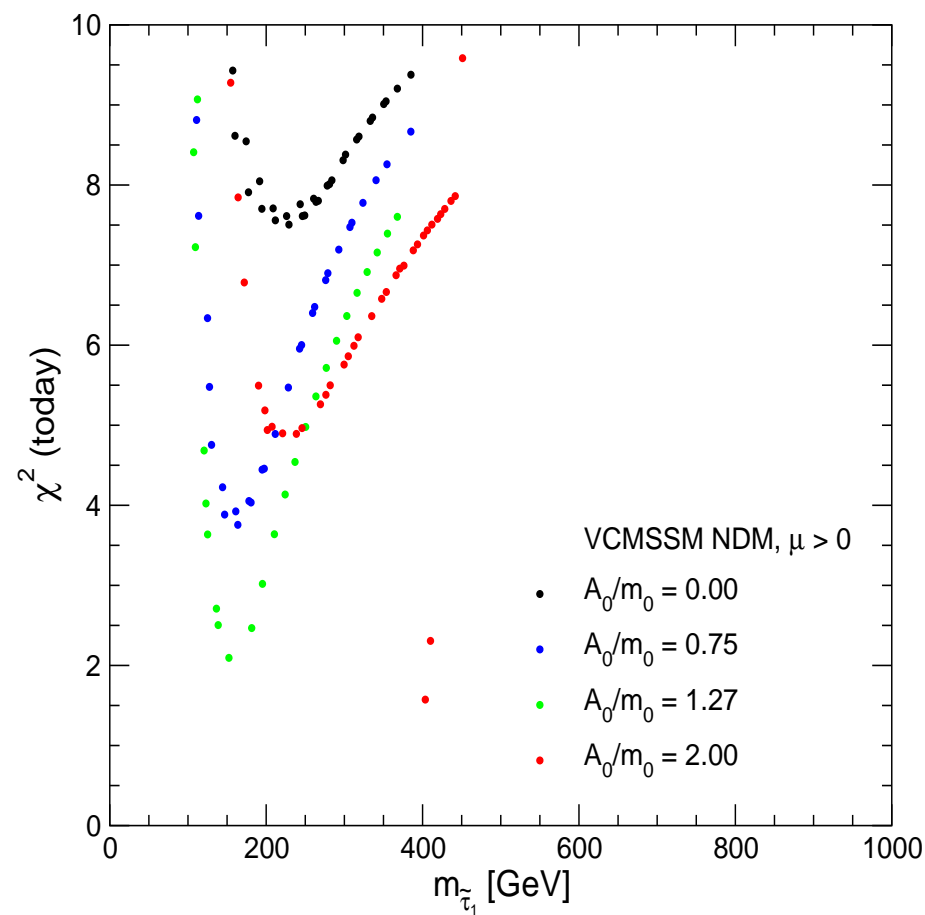
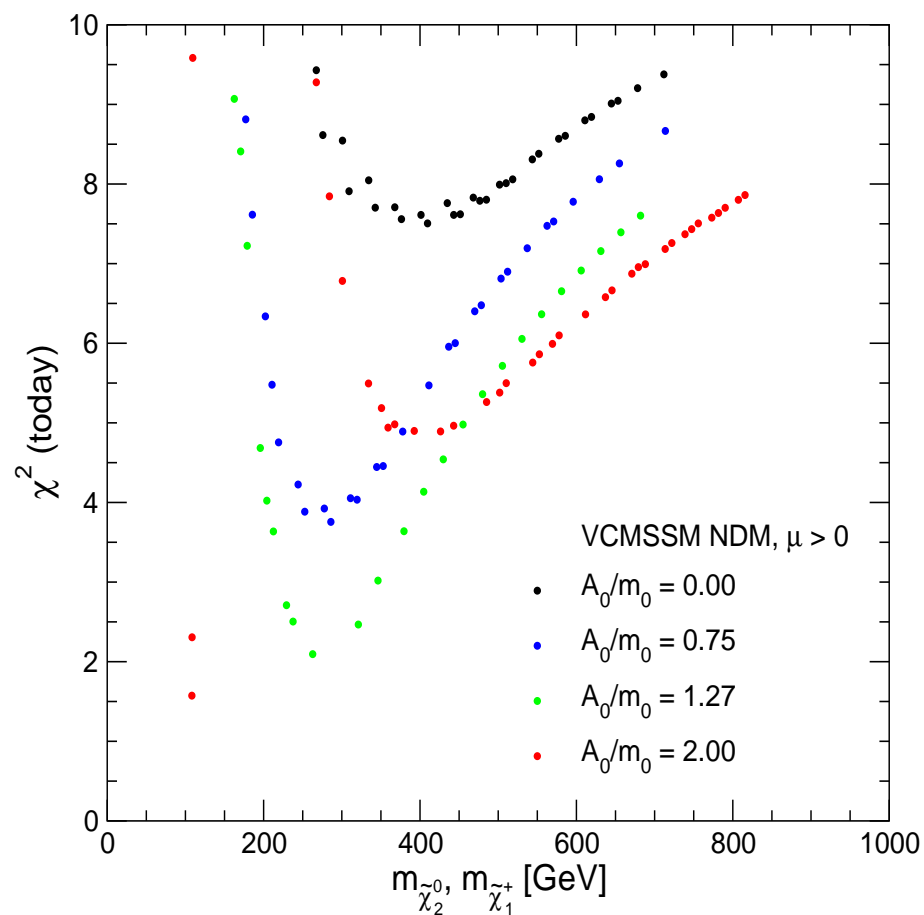
NUHM: vary  $m_{1/2}$  and  $m_0$  around best CMSSM fit point  $\tan\beta = 50$ :



⇒ sleptons, charginos, neutralinos partially in reach for ILC(1000)  
 stops and gluinos will be out of reach

### 3 C) VCMSSM

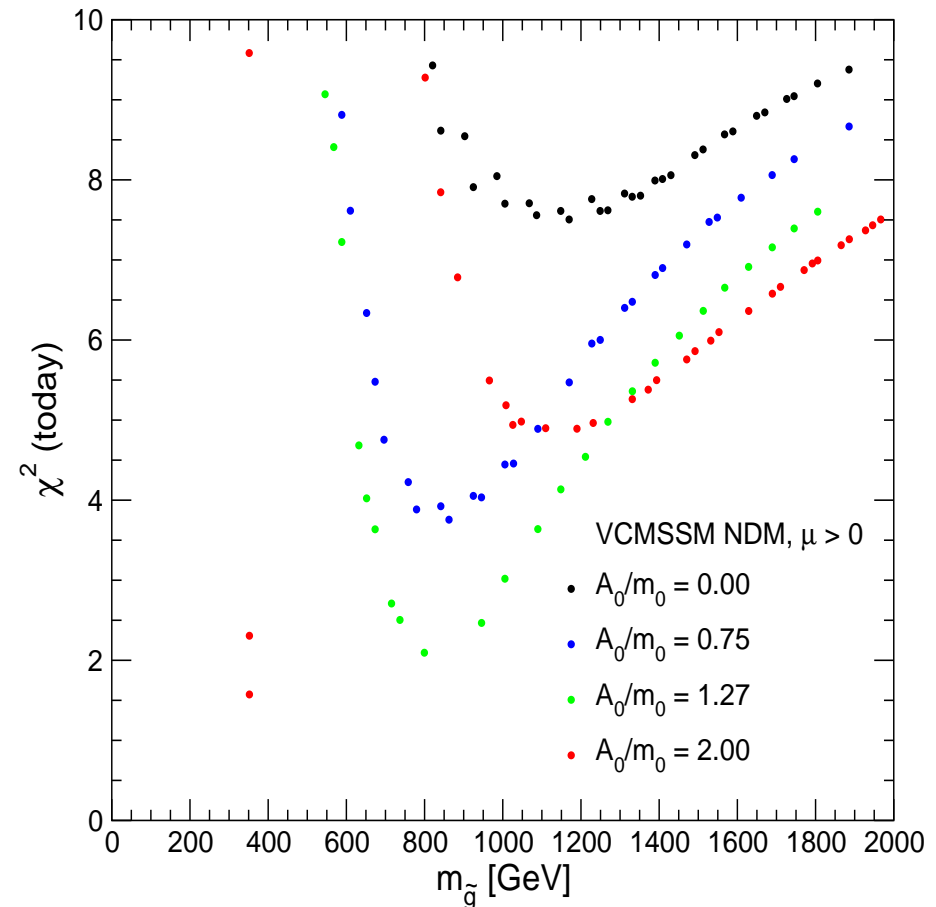
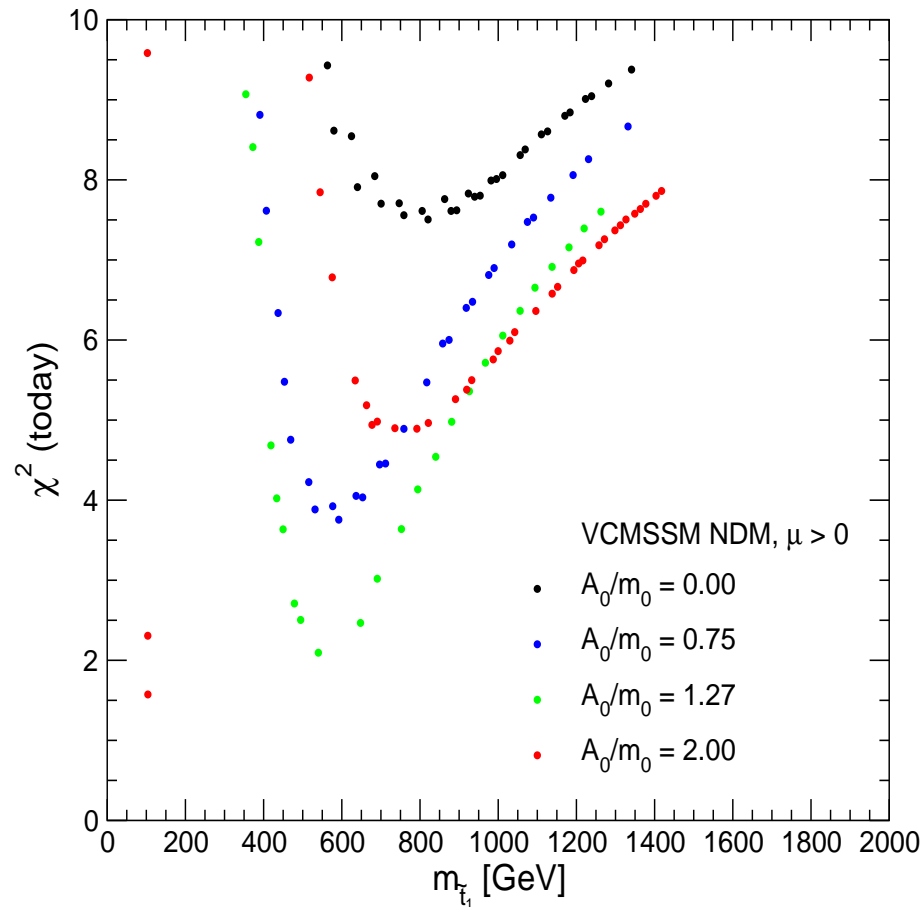
#### VCMSSM: scan over full parameter space



$\Rightarrow A_0/m_0 = 3/4, 3 - \sqrt{3}$  and Higgs pole favored

$\Rightarrow$  sleptons, charginos, neutralinos (partially) in reach for ILC(1000)

# VCMSSM: scan over full parameter space

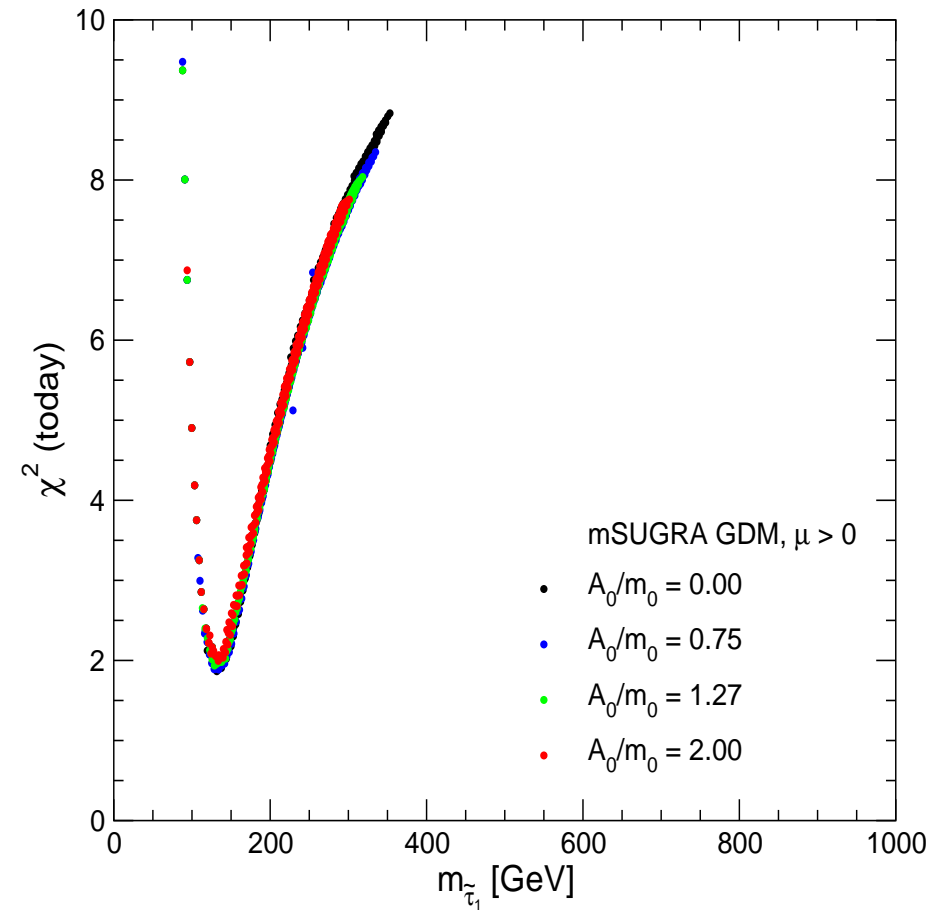
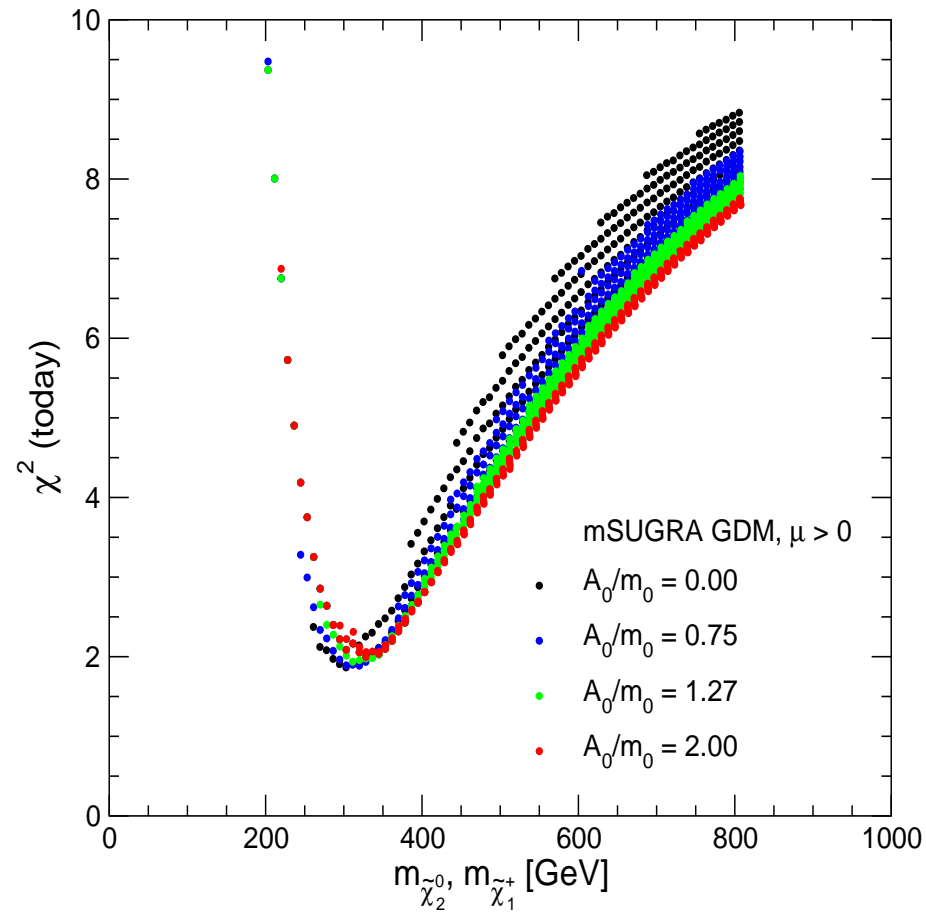


⇒  $A_0/m_0 = 3/4, 3 - \sqrt{3}$  and Higgs pole favored

⇒ stops and gluinos out of reach, except for Higgs pole

### 3 D) GDM (mSUGRA)

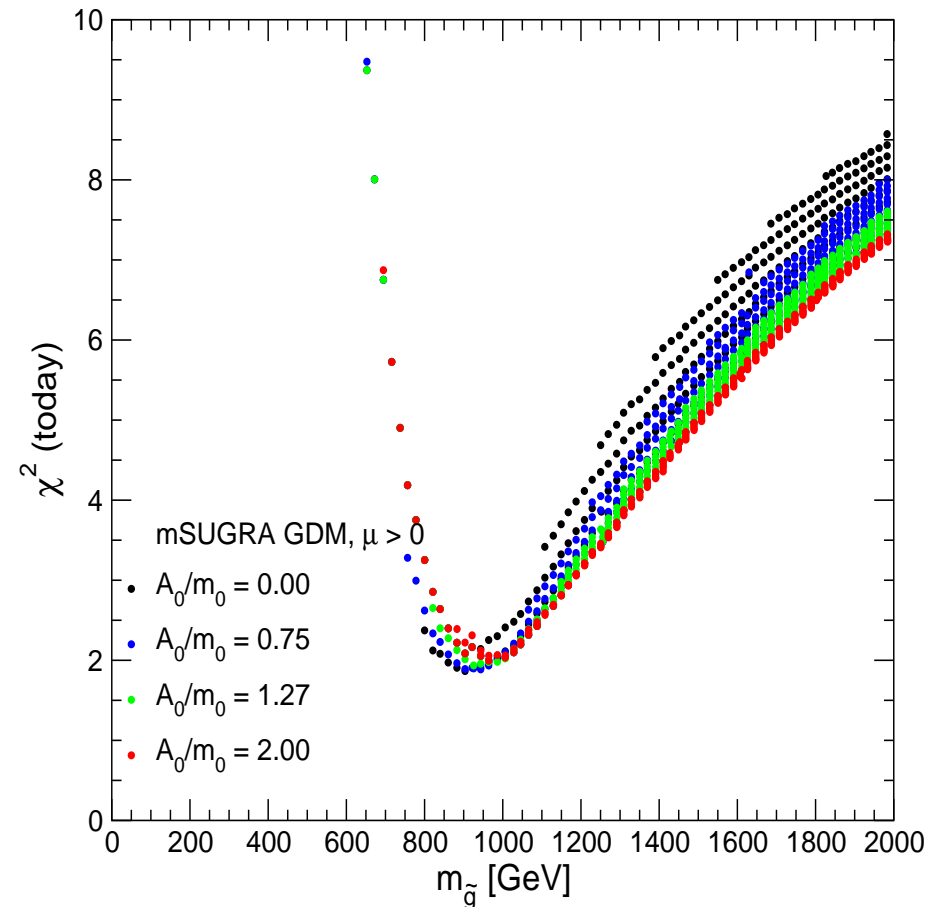
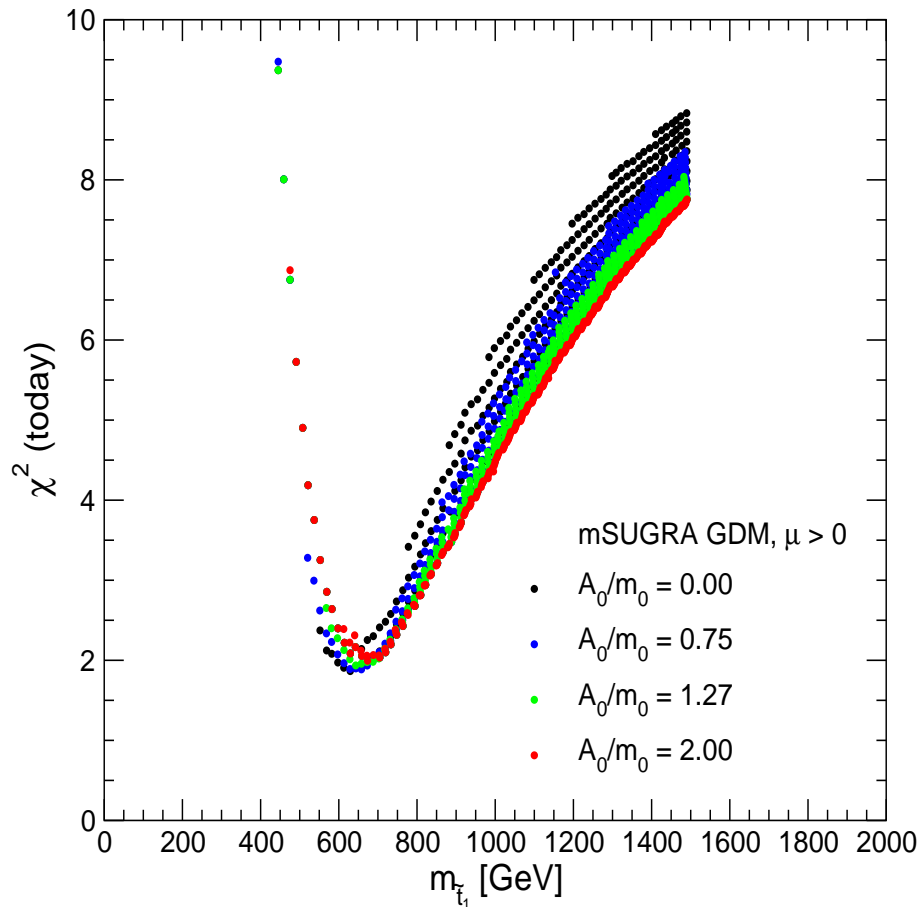
#### GDM (mSUGRA): scan over full parameter space



⇒ all  $A_0/m_0$  values similarly good

⇒ sleptons, charginos, neutralinos in reach for ILC(1000)

## GDM (mSUGRA): scan over full parameter space



⇒ all  $A_0/m_0$  values similarly good

⇒ stops and gluinos out of reach



## 4. Conclusinos

- Precision observables
  - can give valuable information about the “true” Lagrangian
  - can provide bounds on SUSY parameter space
- Most important electroweak precision observables:  
 $M_W, \sin^2 \theta_{\text{eff}}, M_h, (g - 2)_\mu, b \rightarrow s\gamma$
- models under consideration:  
CMSSM, NUHM, VCMSSM, GDM (mSUGRA)
- Current  $\chi^2$  fit: low values,  $\mathcal{O}(2)$  reached
- Evaluation of SUSY spectrum  $\Rightarrow$  ILC reach  
similar results in all scenarios:  
 $\tan \beta = 10$ : sleptons, charginos, neutralinos (partially) in reach  
possibly some chance for light stops  
 $\tan \beta = 50$ : some sleptons, charginos, neutralinos (partially) in reach  
hardly any chance for light stops or gluinos

## 5. Conclusinos

- Precision observables
  - can give valuable information about the “true” Lagrangian
  - can provide bounds on SUSY parameter space
- Most important electroweak precision observables:  
 $M_W, \sin^2 \theta_{\text{eff}}, M_h, (g - 2)_\mu, b \rightarrow s\gamma$
- models under consideration:  
CMSSM, NUHM, VCMSSM, GDM (mSUGRA)
- Current  $\chi^2$  fit: low values,  $\mathcal{O}(2)$  reached
- Evaluation of SUSY spectrum  $\Rightarrow$  ILC reach  
similar results in all scenarios:  
 $\tan \beta = 10$ : sleptons, charginos, neutralinos (partially) in reach  
possibly some chance for light stops  
 $\tan \beta = 50$ : some sleptons, charginos, neutralinos (partially) in reach  
hardly any chance for light stops or gluinos

**In all scenarios the ILC will discover SUSY particles**