Question 4a: Elaborate on the robustness and redundancy of the tracking performance. In particular, how would it deteriorate with a missing layer?

Impact of a Missing Layer on Detector Resolution

We first consider the impact of a missing layer on detector resolution. We have used the Weight Matrix Fitter, which is based on the SLD track fitting algorithm, to estimate track parameter uncertainties. The covariance matrix for the track parameters is calculated by the fitter in the process of fitting tracks. This matrix does not depend on the actual hit positions, only on the amount of material along the track, the track momentum, and the spatial resolution of detector elements. Thus, we don't need a large number of tracks to get accurate calculations of the covariance matrix - for a given set of track parameters only one track needs to be propagated through fitter to determine the covariance matrix for the fitted track parameters. We performed extensive studies to make sure that the covariance matrix gives an accurate estimate of the fitting errors. By comparing the fitted track parameters with the generated Monte Carlo track parameters, we see that the widths of residual distributions are in good agreement with the covariance matrix estimates with deviations at the few percent level.

To estimate the effect of a missing layer on tracker resolution, we have removed hits in the missing layer and then run the Weight Matrix Fitter to see the effect of the missing layer on track parameter resolutions. We selected 8 different polar angles for study, corresponding to cos(θ) values of 0., 0.5, 0.75, 0.8, 0.89, 0.95, 0.97, and 0.986. The results for missing any one of the 21 tracking layers on the 5 track parameters for these 8 values of polar angle as functions of track momentum in the range from 0.2 GeV/c to 1000 GeV/c can be found at http://www.slac.stanford.edu/~sinev/sid02res.html. There you will find a table with clickable links to each of the 840 variations examined. Each plot contains 2 curves, one showing the resolution with the full set of layers, and a second curve showing the resolution with the designated layer missing. In the table of links some entries are shown in blue and some in red. If the entry is red, then missing hits in that layer does not affect the resolution for that track parameter at that particular dip angle. In total, out of 840 entries in the table, only 283 are blue indicating missing hits in that layer adversely affects the track resolution. There is no row in the table that is completely "red", showing that each layer contributes to improving the tracker resolution for some region of phase space

The track parameters resolutions described above were calculated for the expected spatial resolution of the sensors. The vertex pixel sensors have spatial resolution of 3.5 μm in both coordinates, the barrel tracker strip sensors have spatial resolution of 7 μm in the bend coordinate, and the endcap tracker strip sensors, consisting of 2 planes of strip sensors at a 12° stereo angle, have spatial resolution of 7 μm resolution in the bend coordinate and 35 μm resolution in the radial coordinate.

As one would expect, the vertex detector layers are mostly responsible for impact parameter resolution. In the x-y plane, contrary to naive expectations, not just the first layer, but all five of the barrel layers contribute roughly equally to the d0 impact parameter resolution at high momentum. This is because the "lever arm" in our case is very large due to the excellent spatial resolution in the outer tracker. For high momentum tracks, the d0 resolution is almost a factor of $\sqrt{5}$ better than the spatial resolution of a single layer. Removing any one of the vertex barrel layers leads to ~12% degradation in d0 resolution at high momentum. Things are different for low momentum particles, where the d0 resolution is dominated by multiple scattering. In that case, the first vertex layer plays the dominant role in the d0 resolution. Removing it leads to a ~50% degradation in d0 resolution, while second and subsequent layers have little effect. For the z0 resolution, we don't have a large effective lever arm since the barrel tracker sensors do not measure the z coordinate. Correspondingly, z0 resolution is more than a factor of two worse than d0 resolution (4 μm versus 1.8 μm at high momentum). The absence of first layer measurement degrades the z0 resolution by ~50% for all momenta. Selected impact parameter results are shown in Figure 1.

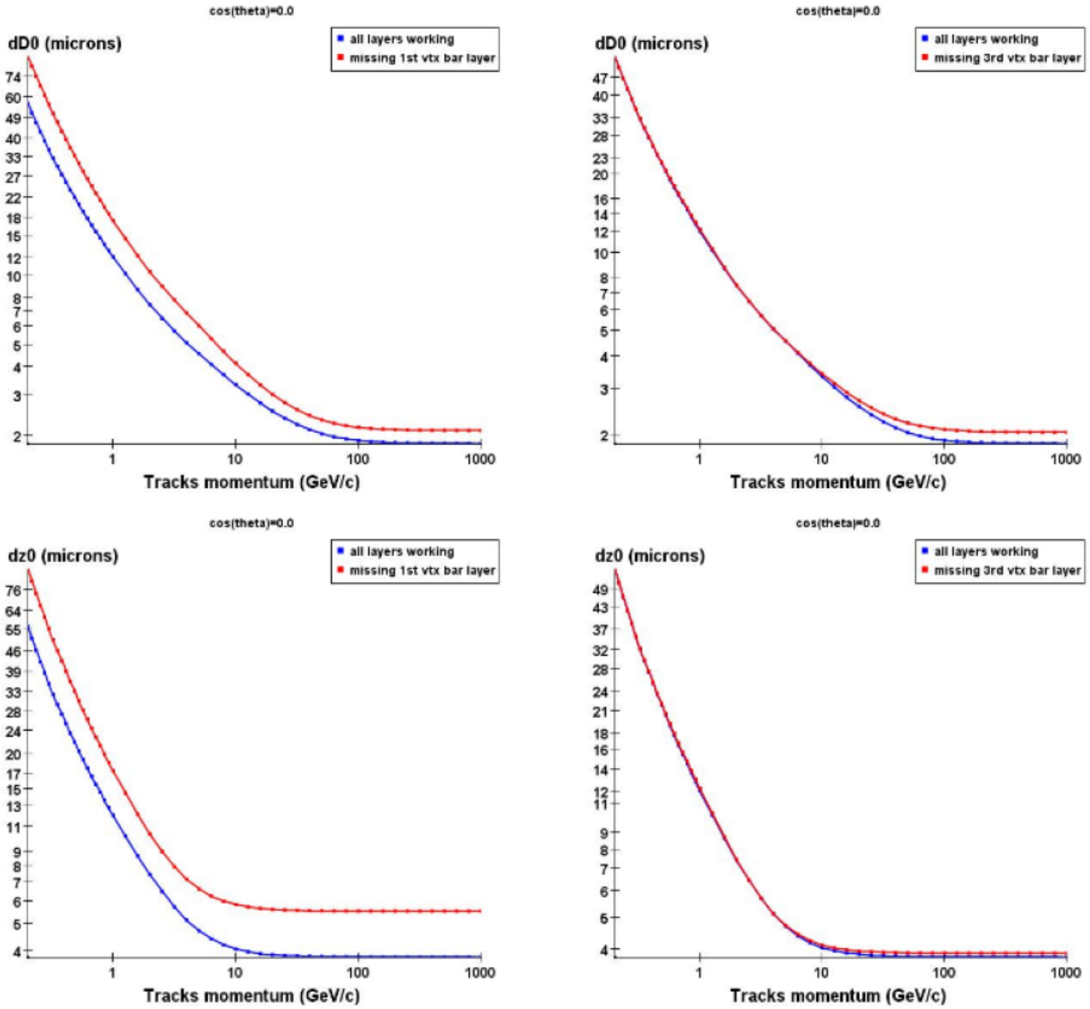

Figure 1: Impact parameter resolution with missing layers in the vertex detector.

The p_T resolution for the SiD tracker depends primarily on the outer tracker measurements. The vertex detector layers have little impact on the momentum resolution except for very low momentum tracks ($p_T < 0.3$ GeV/c) where the absence of the last vertex layer degrades resolution by about ~20%. Excluding hits in the first tracker barrel layer significantly degrades resolution in the low momentum region by as much as a factor of 5. Because such tracks do not reach the next barrel tracker layer, their curvature is measured by vertex detector alone. For $p_T > 1$ GeV/c, the most important tracker layer is the last one, where excluding hits this layer degrades momentum resolution by 15-20%. Selected p_T resolution results are shown in Figure 2.

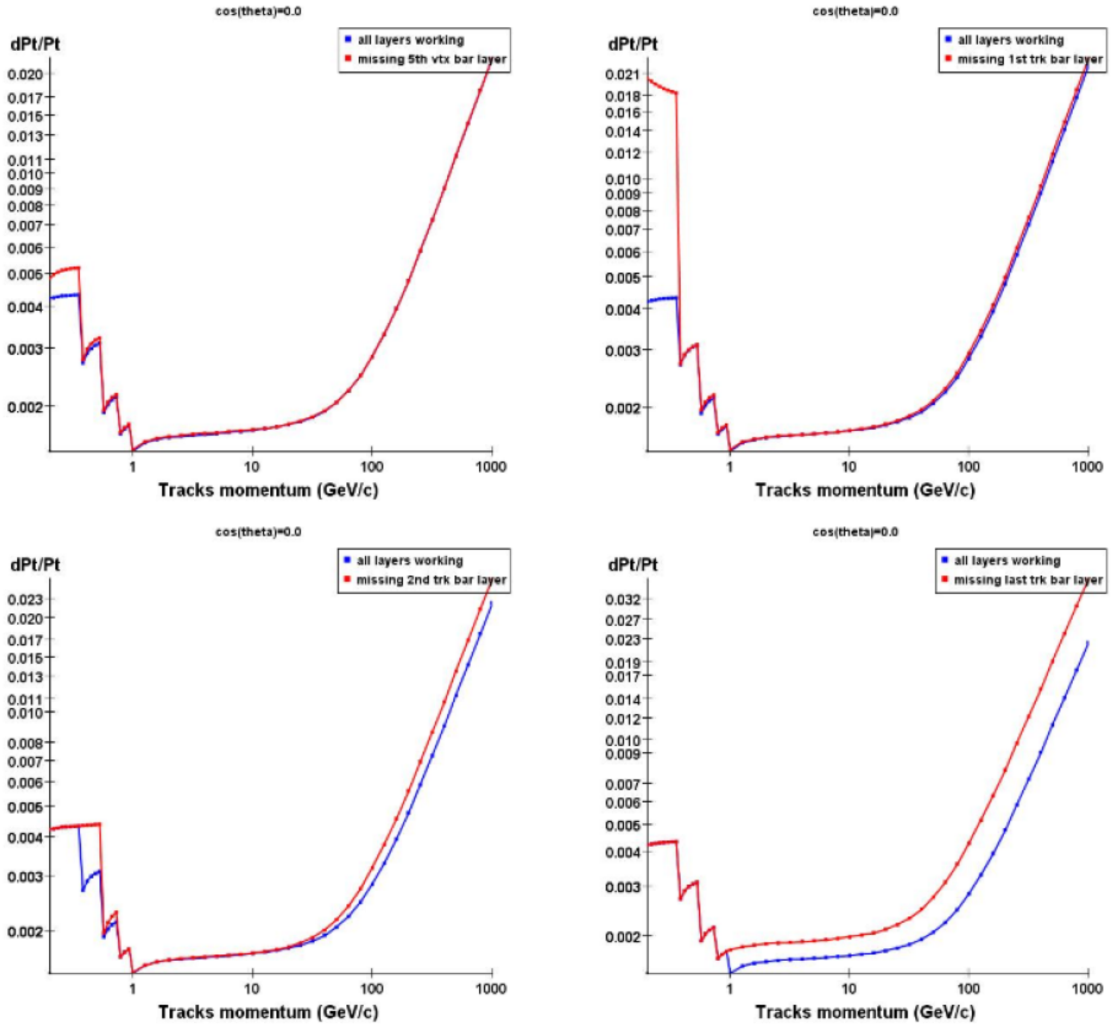

Figure 2: Transverse momentum resolution with missing outer tracker layers.

For far forward tracks ($\cos(\theta) = 0.972$), the most important layers for both impact parameter and momentum resolutions are the first vertex barrel layer and the second forward disk layer. The effect of missing these layers is shown in Figure 3.

In summary, the results presented above demonstrate that there is no single critical layer in SiD, the absence of which would critically impact the detector resolution. The degradation in resolution is either tolerable or limited to narrow regions of phase space (low momentum, very forward tracks) that are not critical to the SiD physics program.

We also observe that no layer is unneeded - every layer leads to improved track resolution in some region of phase space.

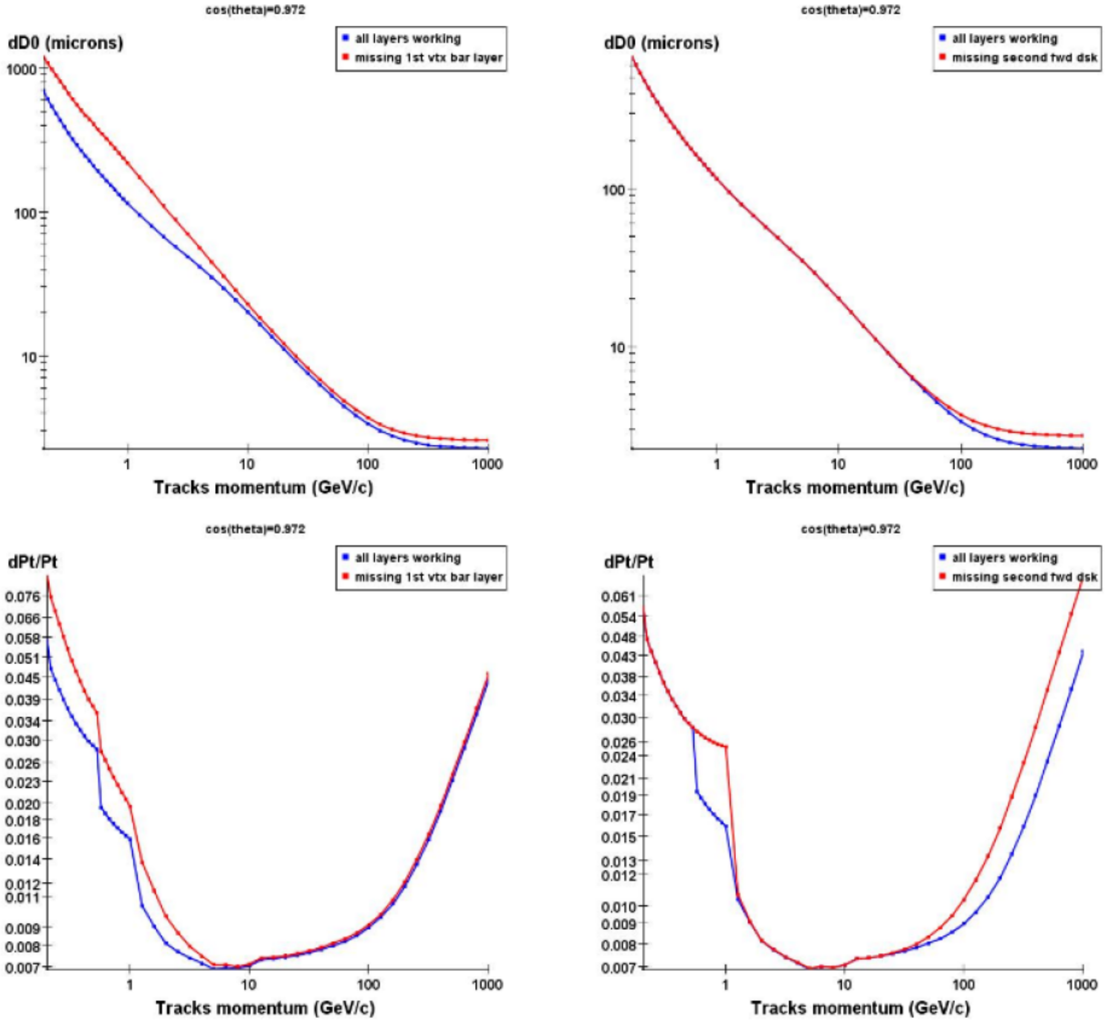

Figure 3: Momentum and impact parameter resolution for forward tracks with missing layers.

Impact of a Missing Layer on Track Reconstruction

We now consider the impact of a missing layer on the track finding efficiency and fake rate. We have studied 5 specific cases for a missing layer in the central (barrel) region: no missing layers (All), missing the innermost barrel vertex detector layer (no VB0), missing the outermost barrel vertex detector layer (no VB4), missing the innermost barrel tracker layer (no TB0), and missing the outermost barrel tracker layer (no TB4). The effect of a missing layer was simulated by forcing the track finding algorithm to ignore hits on the specified layer. For each case, we generated a new set of track finding strategies using the strategy builder (see the LOI for further details on strategies and the strategy builder) using a sample of $e^+e^- \rightarrow t\bar{t}$ events at $\sqrt{s} = 500$ GeV. The tracking performance was measured using an independent sample of top pair events to avoid possible correlations between the strategy building and track reconstruction processes.

In general, we expect a missing layer to have a negligible effect on tracking performance for high- p_T tracks. Such tracks typically traverse ~10 tracking layers, while the track finding algorithms require only 6 – 7 hits. The situation for low-p_T tracks is more complex. The SiD standard tracking algorithm is not designed to follow “curlers”, and will not necessarily associate hits with a track candidate after the track starts to curl back in towards the origin. Thus, low momentum tracks need to traverse the minimum number of layers (6 for a barrel only strategy, 7 for other strategies) to be reconstructed. If one of the innermost layers is missing, a substantial inefficiency arises in the current algorithm for these low-p_T tracks since the track may no longer traverse sufficient layers to meet the requirements for the standard track finding algorithm.

The distribution for the number of hits associated with a reconstructed track is shown in Figure 4. The distribution has a peak at 10 hits, corresponding to tracks that fully traverse the detector. Most tracks have more hits that the 6 – 7 required by the standard track finding algorithm, but ~15% of the tracks have only 6 – 7 hits and are potentially at risk for not being reconstructed if one of the hit layers is missing.

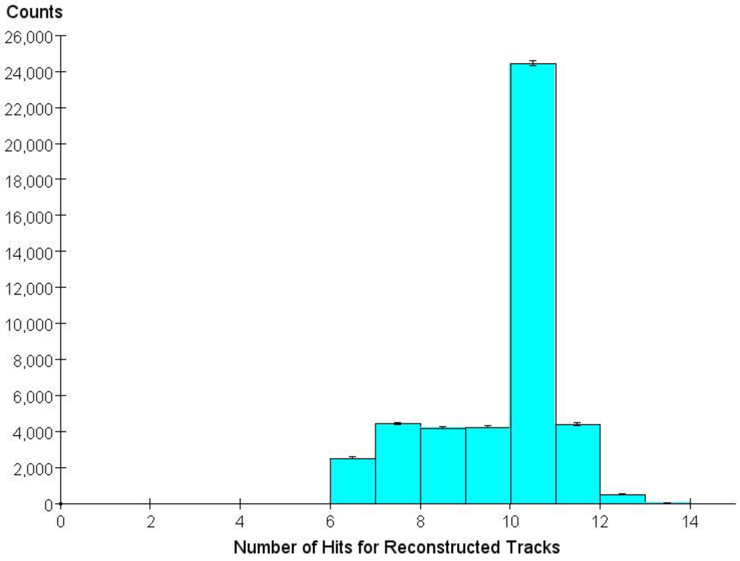

Figure 4: Number of hits associated with reconstructed tracks using the standard track finding algorithm with all tracker layers present.

In Figure 5, we show the track reconstruction efficiency for tracks reconstructed where the innermost vertex detector layer is missing (no VB0). We also show the efficiency where all layers are present for comparison. Both plots are constructed using identical efficiency "denominators" by selecting those tracks that are deemed "findable" with all layers present (see the LOI for a more detailed description of the criteria for a track being findable). Thus, the differences in this plot are entirely due to differences in the efficiency "numerator", namely the number of findable tracks that are reconstructed. A substantial loss in efficiency is observed at low transverse momentum for the case of a missing vertex detector layer, but for transverse momenta above ~0.5 GeV the differences are negligible. Similar results are obtained for the cases where the outermost vertex detector layer is missing (no VB4) and where the innermost tracker layer is missing (no TB0).

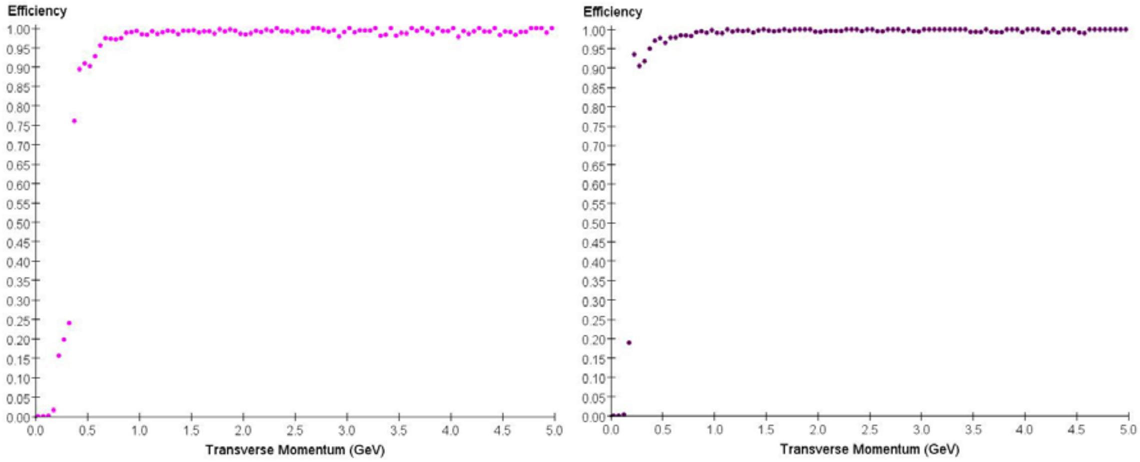

Figure 5: Track reconstruction efficiency with a missing inner barrel layer of the vertex detector (no VB0 - left) and with all layers present (All - right).

In Figure 6, we show the same plots for the case of missing the outermost tracker barrel layer (no TB4). Tracks that make it to the outer tracker layer will generally have 9 hits even without the outer tracker layer, which is more than adequate for track reconstruction in SiD. The loss in tracking efficiency from missing this layer is negligible.

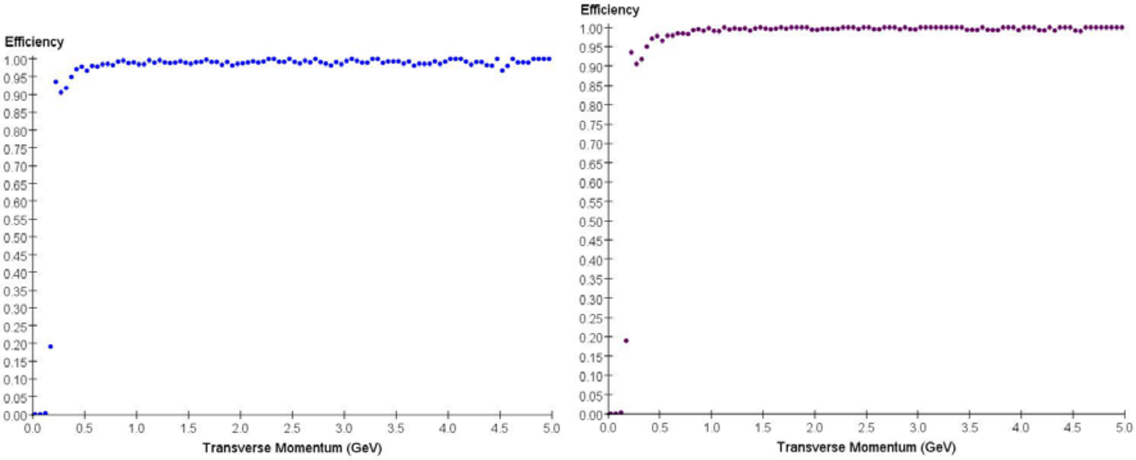

Figure 6: Track reconstruction efficiency with a missing outer barrel layer of the tracker (no TB4 - left) and with all layers present (All - right).

We have also measured the fake rate when a layer is missing. Fake tracks are identified as those having less than half their hits associated with a single Monte Carlo particle. In all cases, the fake rate remains small. The results for track reconstruction efficiency and fake rate for the cases studied are summarized in Table 1.

Table 1: Track reconstruction efficiency and fake rate for the missing layer cases studied. The track reconstruction efficiency is normalized to the efficiency when all layers are present (All).

	All	No VB0	No VB4	No TB0	No TB4
Rel. Eff.	100%	(92.4±0.1)%	(93.8±0.1)%	(92.5±0.1)%	(99.3±0.1)%
Fake rate	(0.15±0.02)%	(0.09±0.02)%	(0.12±0.02)%	(0.03±0.01)%	(0.10±0.02)%

The results of these efficiency measurements are consistent with our expectations. The current tracking algorithm utilizes hits in all vertex detector barrel layers and the innermost tracker layer to find low momentum tracks that curl up before reaching the second tracking layer. Thus, missing one of these layers greatly reduces the track reconstruction efficiency at low momentum. In principle, much of this inefficiency could be recovered with a more sophisticated track reconstruction algorithm that picked up additional hits for curling tracks. On the other hand, high momentum tracks cross most or all layers, and the loss of an outer layer has little effect on the track finding efficiency in the standard tracking algorithm. Note that the calorimeter assisted tracking algorithm relies heavily on the outermost tracking layer, so the loss of this layer would not be without consequences.

Question 4(b): Give the efficiency and the fake track fraction in a jet environment with full background simulation.

The SiD detector concept takes advantage of the fast charge collection in silicon to minimize the impact of beam backgrounds. The silicon strip detectors in the outer tracker “time stamp” the beam crossing they originate from, providing single bunch time tagging. While the SiD vertex detector technology selection has not yet been made, we currently favor technologies that will also provide the ability to time stamp vertex detector hits with the beam crossing. Thus, we anticipate that the SiD tracker will be sensitive only to backgrounds from the same beam crossing as the physics event.

To study the impact of backgrounds, we compare the tracking performance for a physics sample with and without backgrounds. The physics sample used in this study is the same as for the LOI, namely 1000 $e^+e^- \to t\bar{t}$ events at $\sqrt{s} = 500$ GeV. For the sample with backgrounds, the beam-beam backgrounds expected for one beam crossing are added to the physics sample before digitization and reconstruction. The beam-beam background is derived from Guinea Pig simulations of the nominal ILC beam parameters, including the effect of beamstrahlung. Both samples are processed through the same simulation and reconstruction packages used in the LOI. To minimize differences due to statistical fluctuations, the exact same physics sample is used for both the with-background and without-background cases.

Table 1 shows the breakdown of contributions to the track finding efficiency. We find a substantial increase in the number of tracks failing the p_T cut. This is expected due to the large number of very low p_T tracks in the background sample. There is also a ~0.7% increase in the number of tracks found due to the presence of real charged tracks in the background sample that pass all tracking requirements. Except for these differences, the track finding efficiency is essentially unchanged with the addition of backgrounds. In particular, the track reconstruction efficiency for “findable” tracks is 99% independent of whether backgrounds are included or not.

Table 2: Track Finding Efficiency with and without backgrounds.

Selection	$t\bar{t}$ with Background		$t\bar{t}$ without Background	
	Count	Efficiency	Count	Efficiency
All Tracks	188209	-	51871	-
$p_T \geq 0.2$ GeV	48744	(25.90 ± 0.10)%	48472	(93.45 ± 0.11)%
Nhit ≥ 6	44265	(90.81 ± 0.13)%	43997	(90.77 ± 0.13)%
Seed Hits Present	44162	(99.77 ± 0.02)%	43894	(99.77 ± 0.02)%
Confirm Hit Present	44145	(99.96 ± 0.01)%	43877	(99.96 ± 0.01)%
$\lvert d_0 \rvert \leq 1$ cm	44069	(99.83 ± 0.02)%	43801	(99.83 ± 0.02)%
$\lvert z_0 \rvert \leq 1$ cm	43946	(99.72 ± 0.03)%	43261	(99.72 ± 0.03)%
Track Reconstruction	43518	(99.03 ± 0.05)%	43261	(99.05 ± 0.05)%

We have also examined the fake track rate in these samples. Fake tracks are identified as those having less than half their hits associated with a single Monte Carlo particle. The fake rate in the sample with background is found to be 0.006% higher than in the sample without background (0.064% vs 0.058%).

In conclusion, we find that the expected level of background hits has a negligible impact on the SiD track finding efficiency and fake rate. Using the SiD standard track finding algorithm, we observe a tracking efficiency of 99% for findable tracks with a fake track rate of 0.6% for top pair production, independent of whether or not background hits are included in the simulation.