Rough estimation of required length of Test Linac (1)

- for cavity alignment test - wakefield effect
- quad vibration test

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revised 20060704

(With correction of a mistake in calculation, which was pointed out by N. Toge)

revised 20060710

(With correction of a mistake, which was pointed out by P. Tenenbaum)

revised 20060720

(for presentation in Vancouver meeting)

Comment on mistakes in the old versions.

The previous version (v4, July 4) had a big mistake in the assumption for detection of cavity misalignment in a test linac. The mistake was pointed out by P. Tenenbaum.

In the previous version, I had assumed the sensitivity of the beam size increase by 10% of nominal and which corresponded to

$$\sqrt{< y_f^2 >} \Big/ \sigma_y = 0.1,$$

where $< y_f^2 >$ is expected squre of position difference between the head and the tail of a bunch and σ_y, the nominal beam size.

It was wrong. The expected beam size should be about $\sqrt{\sigma_y + < y_f^2 >} \Big/ \sigma_y$.

Assuming the sensitivity of 10% beam size increase, then, it correspond to

$$\sqrt{1 + < y_f^2 > \Big/ \sigma_y^2} - 1 = 0.1 \quad \rightarrow \quad \sqrt{< y_f^2 >} \Big/ \sigma_y = \sqrt{0.21} \approx 0.46.$$

Compare with the previous (wrong) result, we need 4.6 times larger position change of the bunch tail, and 21 times longer linac. (We could increase the assumed required rms misalignment (0.26 mm), bur not changed here.)

This new version corrected this mistake.

PT also noted difficulties related to large energy spread increase in the no-acceleration test linac. I think this needs to be considered carefully.

The conclusion is: it will be very difficult to detect cavity misalignment using beams.

Testing Cavity Alignment (offset)?

Detecting wakefield effect
as beam size increase

Cavity offset - Wakefield - Two particle model

Estimation of expected orbit deviation from random misalignment of cavities.

Position change of following particle at linac end due to i-th cavity:

$$y_{i,f} = -eqa_i W_i(z) L_c \sqrt{1/E_i E_f}\sqrt{\beta_f \beta_i}\sin\varphi_i$$

Position change at linac end:

$$y_f = -eq\sum_i a_i W_i(z) L_c \sqrt{1/E_i E_f}\sqrt{\beta_f \beta_i}\sin\varphi_i$$

a_i : offset of i - th cavity,

E_i, E_f : energy at i - th cavity and final,

β_i, β_f : beta at i - th cavity and final beta,

φ_i : phase advance from i - th cavity to final,

q : charge of preceding particle,

z : distance of two particles

L_c : cavity length, N_c : Number of cavities

g : acc. gradient,

Expected Position change square of following particle:

assume all cavities have the same wakefunction, rms misalignment, acceleration

$$< y_f^2 >= \sum_i < a_i^2 > \frac{e^2 q^2 L_c^2}{E_i E_f} W^2(z)\beta_f \beta_i \sin^2\varphi_i \approx \frac{e^2 a^2 L_c \beta_f \bar{\beta} W^2(z)\log(E_f / E_0)}{2gE_f} \quad (g > 0)$$

$$< y_f^2 >= \sum_i < a_i^2 > \frac{e^2 L_c^2}{E_i E_f} W^2(z)\beta_f \beta_i \sin^2\varphi_i \approx \frac{e^2 q^2 a^2 L_c^2 \beta_f \bar{\beta} W^2(z) N_c}{2E^2} \quad (g = 0)$$

Requirement for Main Linac

(Roughly)

Require $\sqrt{< y_f^2 >} \big/ \sigma_y < 0.3, \quad \sigma_y = \sqrt{\beta\varepsilon}$

$$\begin{aligned}
&E_0 = 5\,\text{GeV (including Bunch Compressors)},\\
&E_f = 250\text{GeV},\\
&\gamma\varepsilon = 2\times 10^{-8}\ \text{m},\\
&\beta_i \approx 100\ \text{m},\\
&q \approx 1\times 10^{10} e \quad \text{(half of bunch charge)},\\
&L_c \approx 1\ \text{m},\\
&g \approx 30\ \text{MeV/m, then,}
\end{aligned}$$

$$aW(z) < 7.34\times 10^9\ \text{V/C/m}$$

$$z \approx 2\sigma_z = 0.6\ \text{mm, then,}\ \ \text{W(z)} \approx 3\times 10^{13}\ \text{V/C/m}^2$$

$$a < 250\ \mu\text{m}$$

Test Linac-1

no acceleration for sensitivity to wakefiled

Measure beam size (emittance) at diagnostics-1 and 2 and estimate emittance dilution in the Test Linac.
Assume no acceleration in the Test Linac for good sensitivity to wakefield effect.
No or small acceleration, but, beam loading should be compensated to keep the energy spread small.

Injection beam energy : E

$$< y_f^2 > \approx \frac{e^2 q^2 a^2 L_c^2 \beta_f \overline{\beta} W^2(z) N_c}{2E^2}$$

Requirement for sensitivity to cavity offset

Require $\sqrt{< y_f^2 >}\Big/\sigma_y > s, \quad \sigma_y = \sqrt{\beta_f \varepsilon}$

for the requred rms misalignment in the main linac :

$a < 250\ \mu\text{m} \quad \text{or} \quad aW(z) < 7.34\times 10^9\ \text{V/C/m},$

then,

$$1.18\times 10^{-2} \times \sqrt{\frac{\overline{\beta[m]} N_c}{\gamma\varepsilon[m] E[eV]}} > s$$

If, $\gamma\varepsilon = 1\times 10^{-6}$ m, $\overline{\beta} = 30$ m (average of beta - function),

s = 0.46 (assume 10% beam size increase can be detected)

$E = 100$ MeV, (These parameters should be reviewed.)

Then,

$N_c > 4800$ (4800 cavities, about 200 RF units are needed)

This number is proportional to $\gamma\varepsilon E s^2 \big/ \overline{\beta}$

P. Tenenbaum did tracking simulation, with slightly different, but basically the same condition, and got consistent result with this estimation.

Testing Quad vibration?

Detecting orbit jitter
induced by the quad vibration

Beam position change due to quad offset

Estimation of expected orbit change from random offset of quads.

Final position change due to offset of i-th quad:

$$\Delta y_f = -a_i k_i \sqrt{E_i / E_f} \sqrt{\beta_f \beta_i} \sin \varphi_i$$

a_i : offset of i - th quad, k_i : k - value (f^{-1}) of i - th quad,

E_i : beam energy at i - th quad, E_f : final beam energy k_i : k - value,

β_i : betafunction at i - th quad, β_f : final betafunction

φ_i : phaseadvance from i - th quad to final

Final beam position is sum of all quads' contribution. Assuming random, independent offset, expected beam position offset is:

$$< y_f^2 > \approx a^2 \beta_f \sum_{quad} \beta_i k_i^2 E_i \sin^2 \varphi_i / E_f \approx \frac{a^2 N_q \beta_f \overline{\beta} \overline{k}^2}{4} \frac{E_0 + E_f}{E_f}$$

a : rms of offset, N_q : number of quads, E_0 : initial energy

$\overline{\beta}$: average of betafunction, $\overline{k}^2$: average of k - value square

Requirement in Main Linac

Require $$\frac{\sqrt{<y_f^2>}}{\sigma_y} \approx a\sqrt{\frac{N_q \overline{\beta k^2}(E_0+E_f)}{4mc^2(\gamma\varepsilon)}} < 0.2$$

$E_0 = 5$ GeV (including Bunch Compressors),

$E_f = 250$GeV,

$N_q \approx 320$,

$\gamma\varepsilon = 2\times10^{-8}$ m,

$\overline{\beta k^2} \approx 0.065$ m^{-1}

Distance between quads : $L \approx 35$ m

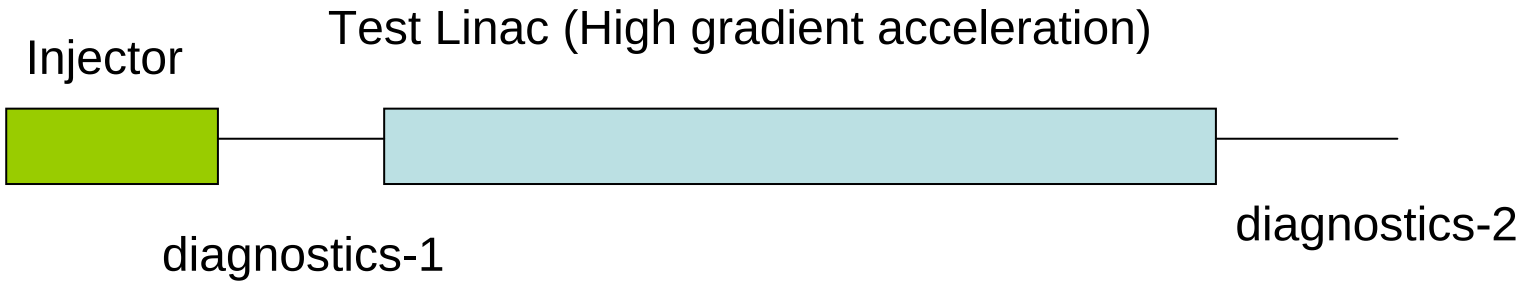
FODO, phase advance per cell : $\mu \approx 60$ deg.

$k = 2\sin(\mu/2)/L, \quad \overline{\beta} = 2L/\sin\mu$

$\rightarrow a < 17$ nm

Test Linac-2

High gradient acceleration for good sensitivity to quad vibration



Measure beam orbit at diagnostics-1 and 2, then evaluate jitters induced in the Test Linac. (subtract injection jitter effects.)

High gradient acceleration is desirable for

- big ratio of position change vs. beam size
- reducing injection jitter effects

Strong quad field.
Many quads: every cryomodule

Requirement for sensitivity to quad vibration in Test Linac - orbit change/beam size ratio

$$\frac{\sqrt{<y_f^2>}}{\sigma_y} \approx a\sqrt{\frac{N_q\overline{\beta k^2}(2E_0+8N_qL_cg)}{4mc^2(\gamma\varepsilon)}} \approx aN_q\sqrt{\frac{2\overline{\beta k^2}L_cN_cg}{4mc^2(\gamma\varepsilon)}}$$

$E_f = E_0 + N_cN_qL_cg, \quad (N_c = 8,$ number of cavities/quad)

$g \approx 30$ MeV/m, $\quad L_c \approx 1$ m,

(Every cryomodule has a quad)

$\gamma\varepsilon \approx 1\times10^{-6}$ m,

$\overline{\beta k^2} \approx 0.56$ m^{-1}

Distance between quads : $L \approx 12$ m

FODO, phase advance per cell : $\mu \approx 120$ deg.

Asuuming a = 17 nm,

$$\frac{\sqrt{<y_f^2>}}{\sigma_y} \approx 2\times10^{-4}N_q$$

$\sqrt{<y_f^2>}\big/\sigma_y \sim 0.1$ level is impossible with reasonable number of cryomodules

Need to measure position jitter much smaller than beam size (<%)

Requirement for sensitivity to quad vibration in Test Linac - example: 30 CM

With the same assumptions as previous page, and assuming

$$E_0 = 100\,\text{MeV}, \quad \beta_f \approx 100\,\text{m}, \quad N_q = 30 \quad (10\,\text{RF unit}),$$

$$\sqrt{< y_f^2 >} \approx a\sqrt{\frac{N_q \beta_f \overline{\beta k^2}(2E_0 + 8N_q L_c g)}{4(E_0 + 8N_q L_c g)}} \approx a\sqrt{\frac{N_q \beta_f \overline{\beta k^2}}{4}} \approx 20a$$

$$\text{Because} \quad \text{a} \approx 17\,\text{nm},$$

$$\sqrt{< y_f^2 >} \approx 360\,\text{nm}, \quad \sqrt{< y_f^2 >} \Big/ \sigma_y \approx 0.004,$$

$$(\text{Beam size}: \ \sigma_\text{y} \approx 83\,\mu\text{m}, \ \leftarrow \ E_f \approx 7.2\,\text{GeV}, \gamma\varepsilon \approx 1\times 10^{-6}\,\text{m}, \ \beta = 100\,\text{m})$$

360 nm should be measured, if *Nq*=30 (30 cryomodules).

Note that $\sqrt{< y_f^2 >} \Big/ \sigma_y \propto N_q \sqrt{g\overline{\beta k^2} \big/ \gamma\varepsilon}$ and $\sqrt{< y_f^2 >} \propto \sqrt{N_q \beta_f \overline{\beta k^2}}$

Summary

- Assumptions in this report should be reviewed carefully. If the assumptions are correct,
- For testing cavity alignment (rms offset 0.25 mm), about 600 cryomodules will be needed. This number looks unreasonably large for a test linac.
- Test linac with about 240 cavities (30 cryomodules, 10 RF units) can be useful for testing quad vibration (rms about 20 nm). If injection orbit is stable, or its effect can be subtracted accurately.